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Synchronization Behaviour of the Rotational Dynamics of Enceladus Via Active Control

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Abstract

In this paper, we have investigated the synchronization behaviour of rotational dynamics of enceladus (natural satellite of Saturn) that has fixed elliptical orbit with spin axis perpendicular to the orbit plane with another identical dynamical system evolving from different initial conditions using the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The designed controller, with our own choice of the coefficient matrix of the error dynamics, are found to be effective in the stabilization of the error states at the origin, thereby, achieving synchronization between the states variables of two dynamical systems under consideration. Numerical simulations are presented to illustrate the effectiveness of the proposed control techniques using *mathematica*.

Key words: Chaos; Synchronization; Enceladus

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INTRODUCTION

After the pioneering work on chaos control by Ott et al. (1990) and synchronization of chaotic systems by Pecora & Carroll (1990), chaos control and synchronization has received increasing attention (Kapitaniak 1996, Chen & Dong 1998, Pikovsky et al. 2001, Lakshmanan & Murli 1996, Fradkov & Pogromsky 1996) and has become a very active topic in nonlinear science since last couple of years. Over the last decade various effective methods have been proposed and utilized by Yu & Song (2001), Wang & Ge (2001), Ho & Hung (2002), Yassen (2005), Lei et al. (2005), Vincent (2005), Vincent & Laoye (2007), Tian et al. (2007) to achieve the control and stabilization of chaotic systems like laser, power electronics etc. The idea of synchronization of two identical chaotic systems that start from different initial conditions consists of linking the trajectory of one system to the same values in the other so that they remain in step with each other, through the transmission of a signal.

The control of physical systems is an important subject in engineering and sciences, thus, in some applications, chaos can be useful while in others it might be detrimental for example chaos in power systems and in mechanical systems is objectionable (Wang & Abed 1993). On the other hand, the idea of chaos synchronization was utilized to build communication systems to ensure the security of information transmitted (Rosier et al. 2006, Yang 2004, Lu et al. 2002, Ivarez 2004). Several attempts have been made to control and synchronize chaotic systems (Bai & Lonngren 1997, 2000). Some of these methods need several controllers to realize synchronization. The OGY method, for instance, have been successfully applied to many chaotic systems like the periodically driven pendulum (Baker 1995) and parametric pendulum (Starrett & Togg 1995). Also, Pyragas method (Pyragas 1992, 2001) has been shown to be an efficient method that has been realized experimentally in electronic chaos oscillators (Pyragas 1993), lasers (Bielawski et al. 1994) and chemical systems (Parmanada et al. 1999). In addition, the delayed feedback control, addition of periodic force and adaptive control algorithm has been utilized to control chaos in a symmetric gyro with linear-pluscubic damping (Chen 2002).

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In particular, backstepping design and active control have been recognised as two powerful design methods to control and synchronize chaos. It has been reported (Zhang et al. 2005, Pokotovic 1992, Krstic et al. 1995) that backstepping design can guarantee global stability, tracking and transient performance for a broad class of strict-feedback nonlinear systems. In recent time, it has been employed for controlling, tracking and synchronizing many chaotic systems (Tan et al. 2003, Harb 2004, Harb & Harb 2004, Laoye et al. 2009, Vincent et al. 2007) as well as hyperchaotic systems (Zhang et al. 2005). According to Tan et al. (2003), some of the advantages in the method include applicability to a variety of chaotic systems whether they contain external excitation or not; needs only one controller to realize synchronization between chaotic systems and finally there are no derivatives in the controller. Zhang et al. (2005) states that the controller is singularity free from the nonlinear term of quadratic type, gives flexibility to construct a control law which can be extended to higher dimensional hyperchaotic system and the closed-loop system is globally stable.

In our present study, the aim is to use the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria to study the synchronization behavior of the rotational dynamics of enceladus (natural satellite of Saturn) that is out of round satellite in a fixed elliptical orbit with spin axis perpendicular to the orbit plane with another identical dynamical system evolving from different initial conditions.

1. EQUATION OF MOTION OF ENCELADUS

An approximate model for the 3:1 secondary resonances for synchronous rotation in the spin-orbit problem of enceladus using the standard techniques of Hamiltonian perturbation theory (Wisdom et al. 1984; Wisdom 1987; Sussman & Wisdom 2001) was developed by Wisdom (2004). It is based on approximation for a fixed orbit the planet-to-satellite distance and the true anomaly is periodic. The Hamiltonian governing the rotational dynamics of an out of round satellite in a fixed elliptical orbit with spin axis perpendicular to the orbit plane is

$$H(p,\theta,t)=\frac{p^2}{2C}-\frac{\varepsilon^2n^2C}{4}\cos(2\theta-2nt)+\frac{\varepsilon^2n^2Ce}{8}\{\cos(2\theta-nt)-7\cos(2\theta-3nt)\} \tag{1.1}$$

where $\varepsilon=\sqrt{3(B-A)/C}$, the moments of inertia are $A<B<C$, n is the orbital frequency, θ measures the orientation of the axis of minimum moment from the line to pericenter, p is the angular momentum conjugate to θ and $\varnothing=\theta-nt$.

Using the Hamilton's equations, the equation of motion of the system under study can be written as:

$$\frac{d^2\theta}{dt^2}=\frac{\varepsilon^2n^2e}{4}\sin(2\theta-nt)-\frac{7\varepsilon^2n^2e}{4}\sin(2\theta-3nt)-\frac{\varepsilon^2n^2}{2}\sin(2\theta-2nt) \tag{1.2}$$

2. SYNCHRONIZATION VIA ACTIVE CONTROL

For a system of two coupled chaotic oscillators, the master system ($x=f(x,y)$) and the slave system ($y=g(x,y)$), where $x(t)$ and $y(t)$ are the phase space (state variables), and $f(x,y)$ and $g(x,y)$ are the corresponding nonlinear functions, synchronization in a direct sense implies $|x(t)-y(t)|\to 0$ as $t\to\infty$. When this occurs the coupled systems are said to be completely synchronized. Chaos synchronization is related to the observer problem in control theory (Nimeijer & Ivan 1997). The problem may be treated as the design of control laws for full chaotic observer (the slave system) using the known information of the master system so as to ensure that the controlled receiver synchronizes with the master system. Hence, the slave chaotic system completely traces the dynamics of the master in the course of time.

The system defined by (1.2) together with $\varnothing=\theta-nt$, can be written as a system of three first order differential equations when the three variables are introduced:

$$\theta=x_1,\qquad \dot{x}_1=x_2,\qquad \phi=x_3.$$

The new system is:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=\frac{\varepsilon^2n^2e}{4}\sin(x_1+x_3)-\frac{7\varepsilon^2n^2e}{4}\sin(3x_3-x_1)-\frac{\varepsilon^2n^2}{2}\sin 2x_3\\ \dot{x}_3&=x_2-n\end{aligned} \tag{2.1}$$

Let us define another system as follows:

$$\begin{aligned}\dot{y}_1&=y_2+u_1(t)\\ \dot{y}_2&=\frac{\varepsilon^2n^2e}{4}\sin(y_1+y_3)-\frac{7\varepsilon^2n^2e}{4}\sin(3y_3-y_1)-\frac{\varepsilon^2n^2}{2}\sin 2y_3,+u_2(t)\\ \dot{y}_3&=y_2-n+u_3(t)\end{aligned} \tag{2.2}$$

where (2.1) and (2.2) are called the master and the slave systems respectively, and in the slave system $u_i(t)$ for i=1, 2, 3, are control functions to be determined. Let $e_1=y_1-x_1$, $e_2=y_2-x_2$ and $e_3=y_3-x_3$ be the synchronization errors such that in synchronization state $\lim_{t\to\infty}e_i(t)\to 0$ for i=1, 2, 3 (Liu 2004). From (2.1) and (2.2), we obtain the error

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dynamics:

$$\begin{aligned}
\dot{e}_1 &= e_2 + u_1(t) \\
\dot{e}_2 &= \frac{\varepsilon^2 n^2 e}{4}\{\sin(y_1+y_3)-\sin(x_1+x_3)\}+\frac{7\varepsilon^2 n^2 e}{4}\{\sin(3x_3-x_1)-\sin(3y_3-y_1)\}+\frac{\varepsilon^2 n^2}{2}\{\sin 2x_3-\sin 2y_3\}+u_2(t) \\
\dot{e}_3 &= e_2 + u_3(t)
\end{aligned} \tag{2.3}$$

The error system (2.3) to be controlled is a linear system with control inputs. Therefore the control functions can be redefined in order to eliminate the terms (Youming et al. 2007) in (2.3) which cannot be expressed as linear terms in e_1, e_2 and e_3 as follows:

$$\begin{aligned}
u_1(t) &= v_1(t) \\
u_2(t) &= -\frac{\varepsilon^2 n^2 e}{4}\{\sin(y_1+y_3)-\sin(x_1+x_3)\}-\frac{7\varepsilon^2 n^2 e}{4}\{\sin(3x_3-x_1)-\sin(3y_3-y_1)\} \\
u_3(t) &= v_3(t)
\end{aligned} \tag{2.4}$$

Therefore the linear error system can be written as follows:

$$\begin{aligned}
\dot{e}_1(t) &= e_2(t) + v_1(t) \\
\dot{e}_2(t) &= v_2(t) \\
\dot{e}_3(t) &= e_2(t) + v_3(t)
\end{aligned} \tag{2.5}$$

The error system (2.5) to be controlled is a linear system with control inputs v_1, v_2 and v_3 as the function of the error states e_1, e_2 and e_3. As stated, as long as $\lim_{t\to\infty} e_i(t) \to 0$ for i=1, 2, 3, synchronization between the master (driver) and slave (response) system is realized, that is, the system represented by (2.1) and (2.2) are synchronized under active control. According to active control method, the controllers v_1, v_2 and v_3 can be written as:

$$\begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}^T = A\begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^T \tag{2.6}$$

Where A is a 3×3 constant matrix. As per the Lyapunov stability theory and the Routh-Hurwitz criteria, in order to make the close loop system (2.5) stable, the proper choice of the elements of A is such that the system (2.6) must have all eigen values with the negative real parts.

Let $A = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$, bring (2.6) into (2.5), we may obtain

$$\begin{pmatrix} \dot{e}_1 & \dot{e}_2 & \dot{e}_3 \end{pmatrix}^T = B\begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix}^T$$

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{pmatrix} = \begin{pmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & -1 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} + A\begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix}$$

$$\Rightarrow B = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Now the slave system (2.2) can be defined as

$$\begin{aligned}
\dot{y}_1 &= x_1 + 2y_2 - y_1 - x_2 \\
\dot{y}_2 &= \frac{\varepsilon^2 n^2 e}{4}\sin(x_1+x_3) - \frac{7\varepsilon^2 n^2 e}{4}\sin(3x_3-x_1) \\
&\quad -\frac{\varepsilon^2 n^2}{2}\sin 2x_3 + x_2 - y_2 \\
\dot{y}_3 &= 2y_2 + x_3 - x_2 - y_3 - n
\end{aligned} \tag{2.7}$$

3. NUMERICAL SIMULATION FOR SYNCHRONIZATION

For the parameters involved in system under investigation, e=0.45, ε=0.44 and n=0.3 and the initial conditions for master and slave systems [$x_1(0)$, $x_2(0)$, $x_3(0)$]=[-0.2, 0.1, 0] and [$y_1(0)$, $y_2(0)$, $y_3(0)$]=[0.2, 0.2, 0.2] respectively, the system has been simulated using mathematica. The obtained results show that the system under consideration achieved synchronization. Phase plots and time series analysis of (2.1) and (2.7) (Figures 1 & 2 respectively) and time series analysis of synchronization errors (Figure 3) are the witness of achieving synchronization between master and slave system. Further, it also has been confirmed by the convergence of the synchronization quality defined by

$$e(t) = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)} \tag{3.1}$$

Figure (4) confirms the convergence of the synchronization quality defined by (3.1).

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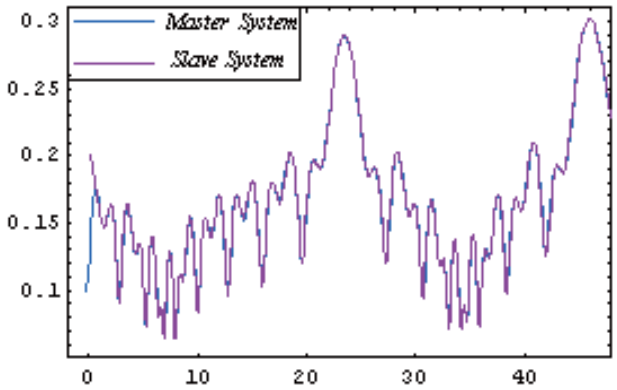

Figure1
Phase Plot of Master & Slave System

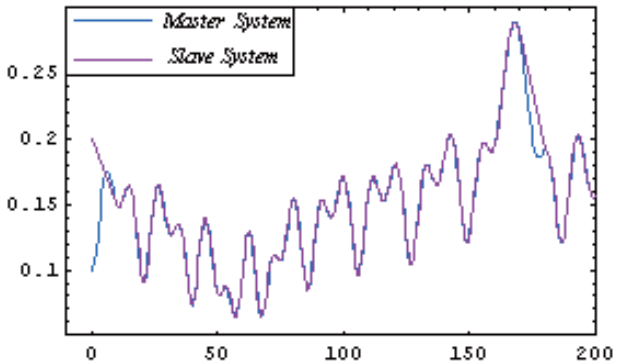

Figure 2
Time Series of Master & Slave System

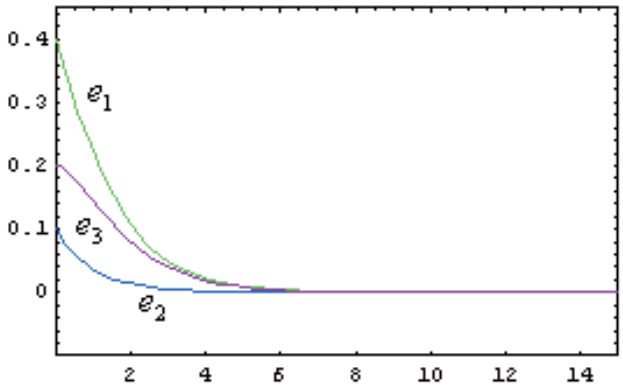

Figure 3
Time Series of Synchronization Errors

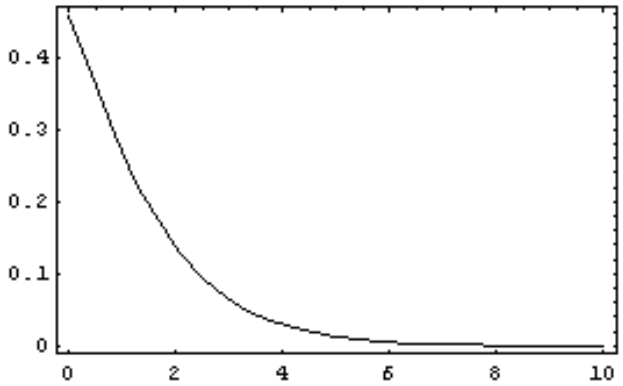

Figure 4
Error Propagation in Synchronization

CONCLUSION

In this paper, we have investigated synchronization behavior of the rotational dynamics of enceladus (natural satellite of Saturn) that is out of round satellite in a fixed elliptical orbit with spin axis perpendicular to the orbit plane with another identical dynamical system evolving from different initial conditions via the active control technique based on the Lyapunov stability theory and the Routh-Hurwitz criteria. The simulated results show that the system under consideration has achieved synchronization. Phase plots and time series analysis of (2.1) and (2.7) (Figures 1 and 2 respectively) and time series analysis of synchronization errors (Figure 3) are the witness of achieving synchronization between the two systems under investigation. Furthermore, it also has been confirmed by the convergence of the synchronization quality (in figure 4).

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