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Simulation of Exchange Rates of Nigerian Naira Against US Dollar, British Pound and the Euro Currency

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Abstract: In this paper a simulation is carried out to study the exchange rate of the Nigerian Naira against the exchange rates of the US dollar, British pound and the Euro currency. Monte Carlo simulation is made for three factors exchange rate model using Euler–Maruyama forward difference approximation. The model was calibrated using polynomial regression analysis. The Naira is found to be appreciated more in value when compared to the dollar, the pound and euro. The result obtained shows that the interest does not have much influence on the value of the exchange rates.

Key words: Exchange rate model; Random walk model; Euler–Maruyama scheme; Regression analysis; Monte Carlo simulation

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1. INTRODUCTION

In the recent years many investors are interested in investing in the foreign exchange market but due to constant changes foreign exchange market, many investors are

guiding against lost in investment. Risk is inevitable in any investment because one may not know if the exchange rate would increase or decrease in future. An increase in exchange rate would certainly have effect in an investment. Therefore, how to avoid or reduce risk involved in foreign exchange market and its attendant effect on investment is very important to investors.

Furthermore, there is the need to have models for forecasting precisely the future exchange rate of currencies. Exchange rate forecasts are drown-up through the computation of a currency value vis-à-vis other currencies over a period of time [7,8]. Exchange rate forecasts are very important to Central Banks of Countries, Financial Institutions, International Investors or Firms.

Furthermore, there is indeed the need to evaluate the foreign denominated cash flows structure involved in the international transactions, evaluation of benefits, risk attendant to business environment and the return on investments, forecast needed for deciding whether or not expenses on a foreign currencies needed to be hedged. Foreign subsidiaries expenses and revenues, and annual budgets required foreign exchange and hence foreign exchange forecast becomes essential [14].

There are several models for predicting exchange rates, examples are the purchasing power parity (PPP) model and interest rate parity (IRP) model and the celebrated random walk class of model. There is an ongoing debate about exchange rate predictability in time series data. A large body of empirical literature, reviewed by Frankel and Rose [11], Meese and Rose [17], focused on the question on whether existing theoretical and econometric models of exchange rate determination represent good descriptions of the empirical data. Some authors have stressed that the poor forecasting performance of fundamental-based models is not related to the weak informative power of fundamental based models [2].

The superiority of random-walk forecasts is instead related to the weakness of the econometric techniques used in producing out-of-sample forecasts [28]. In a recent study, Sarno and Valente [23] analyzed how to optimally select the correct number of fundamental based models to be used in computing the best forecasting model in each period. One of the most widely studied and still unanswered question in the literature involves why monetary models of exchange rate determination cannot forecast much of the variation in exchange rates [15].

Nigeria as a developing nation, the price of foreign exchange plays significant role in ability of the economy to attain optimal productivity capacity [19]. The country undergone several exchange regimes in the last four decades starting from the fixed exchange rate system (1990–1988) to a flexible exchange system (1986–1995). Naira was pegged against British pound or dollar due to devaluation of pound or dollar.

In reforming the exchange rate management system in Nigeria, various policies have been and are still being put in place by the Central Bank of Nigeria (CBN) to stabilize the exchange rate of naira. In 1995, the Autonomous Foreign Exchange Market (AFEM) was introduced and later followed by the Interbank Foreign Exchange Market (IFEM) on October 1999. In July 22, 2002 the Wholesale Dutch Auction system (WDAS) was adopted and later replaced by the Inter-Bank Foreign Exchange Market (IFEM).

It would be recalled that the IFEM was designed as a two-way quote system, and intended to diversify the supply of foreign exchange in the economy by encouraging the funding of the inter-bank operations from privately-earned foreign exchange [24]. The IFEM is expected to broaden and deepen the foreign exchange market on daily basis and discourage speculative activities.

In July 22, 2002, the Wholesale Dutch Auction System (WDAS) was adopted to replace the Inter-Bank Foreign Exchange Market (IFEM). It created room for the licensing of more Bureau de Change thereby giving both the CBN and authorized dealers to participate in the foreign market for devaluing the Naira. At the same time, the WDAS has assisted in narrowing the arbitrage premium from double digit to a single digit, enhanced the relative stability of the naira, vis-à-vis the US dollar-the intervention currency and has also assisted in stemming the spate of capital flight and curbing rent-seeking amongst market operators [24].

Currency exchange rates changes from moment to moment in a manner that may seem chaotic though regularly being monitored by the CBN. Between July, 2002 and February 20, 2004, the naira depreciated by 11.5 percent against the dollar [24]. From the CBN statistical report, the average AFEM intervention rate which closed at 82.33 to a dollar in 1995 appreciated to 81.48 per dollar in 1996. The rate depreciated continuously to 81.98, 84.84 and 91.83 in 1997, 1998 and 1999 respectively. From 1960 until 2011 the USD-NGN exchange averaged 47.25 reaching an historical high of 157.85 in September of 2011 and a record low of 0.53 in October of 1980 with a depreciation rate of 4.78 percent against the US Dollar during the last 12 months of the year 2011 [29].

Different currencies are in use in different countries and their relation to each other is known as the foreign exchange rate. Since the riskless interest rate of each country can influence the monetary policy, the rate of return of an investment, inflation and so on, it can also influence the movement of foreign exchange markets. Therefore, it becomes an important variable in modeling exchange rate of currencies.

The exchange rate model [25] is one of such models that describe the relationship between the exchange rate and the difference in the riskless rate of currencies of the two countries. In finding an estimate of the expected financial value of the exchange rate, the Monte Carlo simulation technique [20,22] will be use to generate the sample paths and make basic comparison between naira and dollar,pound and euro currencies.

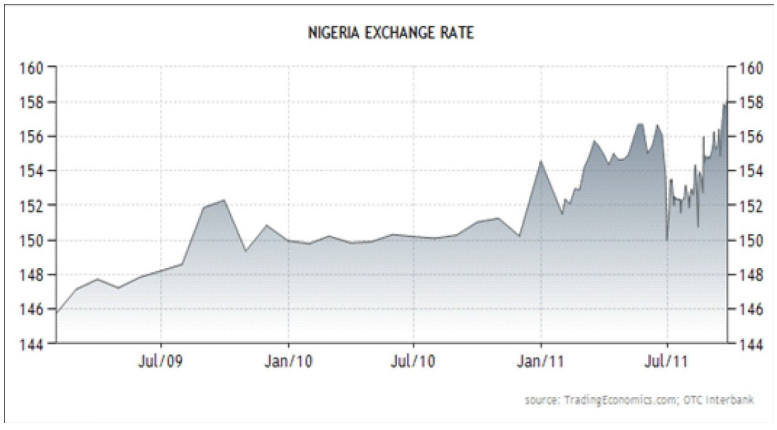

Figure 1
Naira-Dollar Exchange Rates

2. STATEMENT OF THE PROBLEM

The following notations would be used throughout this paper:

We will call the probability system (Ω, Σ, F, p) a filtered probability space, Ω being the set of points with events, which is a δ-algebra of subsets of Ω such that $\Omega \in F$ and p denotes the probability measure.

The filtration $F_t \in F$ is the information available up to time t such that if $s < t$ than $F_s \subset F_t$.

We will also denote the expectation of a random variable x by $E[x]$ and the norm of the vector y by $|y|$ and make use of the standard Euclidean norm $|y| = \left(\sum_{i=1}^{n} y_i^2\right)^{\frac{1}{2}}$. $N(0,1)$ will be the standard normal variate and $w(t)$ the standard Brownian process.

We say that a discretization X has strong order of convergence $\beta > 0$ if

$$E|\bar{X}(nh) - X(T)| \leq ch^{\beta}$$

for some constant c and sufficiently small h. where $E(.)$ is the expectation of $(.)$. X has weak order of convergence β if $|E[f(\bar{X}(nh))] - E[f(X(T))]| \leq ch^{\beta}$ for some constant c all sufficiently small h, for all f in a set $C_p^{2\beta+2} = \{g \in C^{2\beta+2} : |g(x)| \leq k(1 + |x|^q)\}$ for some constants k and q and $x \in R^d$.

Furthermore, Let the domestic and foreign risk-less interest rates be $R(t)$ and $R^f(t)$ respectively, the exchange rate volatility between the domestic and foreign currency be $\sigma(t)$.

3. METHODS

Let $R_i(t), i = 1, 2, 3$ be the exchange rates of the the foreign currency-the US dollar, the UK pound or the European euro against the base currency (naira). The exchange rate is measured as the price of a unit of the foreign currency compared the Nigerian naira.

Then the model will be considered as a continuous time model with risk-neutral diffusion process $Q_i(t), i = 1, 2, 3, t \geq 0$ which are the exchange rates that satisfies the following stochastic differential equations:

$$\mathrm{d}Q_1(t) = Q_1(t)[(R(t) - R_1^f(t))\mathrm{d}t + \sigma_1(t)\mathrm{d}\tilde{W}(t)],\ Q_1(0) = Q_{10} \tag{1}$$

$$\mathrm{d}Q_2(t) = Q_2(t)[(R(t) - R_2^f(t))\mathrm{d}t + \sigma_2(t)\mathrm{d}\tilde{W}(t)],\ Q_2(0) = Q_{20} \tag{2}$$

$$\mathrm{d}Q_3(t) = Q_3(t)[(R(t) - R_3^f(t))\mathrm{d}t + \sigma_3(t)\mathrm{d}\tilde{W}(t)],\ Q_3(0) = Q_{30} \tag{3}$$

where

$\tilde{W}(t)$ is a standard Brownian motion,

$R(t)$ denote the riskless rate in the Nigerian market,

$R_k^f(t), k = 1, 2, 3$ denote the riskless rate in the UK, USA and European respectively,

$\sigma_k\ (t)$ is the each pair exchange rate's volatility.

And the parameters $R(t)$ and $R_i^f(t), i = 1, 2, 3$ would be calibrated using a sample data which is base parametric polynomial regression analysis, which provides a polynomial time series estimators for the parameters. Implementation will be done by the use of the built-in MATLAB function "polyfit" and "polyval" and $\sigma(t)$ is the

volatility of the exchange at the time t would be obtained from historical market data from foreign exchange market.

To simulate the model we need to discretize the model,carry out parameters' calibration and estimation and design a simulation experiment to obtain the exchange rates for various currencies from historical data. When solving differential equations numerically, it is essential to reduce the continuous problem to a discrete one [26,27]. It helps in making the numerical evaluation and computer implementation (simulation) of the continuous-time model easier. It also facilitates the understanding of the link between continuous time models and discrete time models.

Different methods of discretization of continuous-time models are in existence but this research employs the Euler–Maruyama forward difference approximation method [5] in discretization the exchange rates model. This is because It is the most straight forward scheme, easy to implement and almost universally applicable. It is based on the approximations of the time derivatives of the differential equations (time-valued function) by a finite difference recursion at some discrete time-points [4]. According to [26], it is a formula that relates each discrete solution value to the solution at the preceding t-point. Setting $\bar{Q}_i(t)$ as the time-discretized approximation to $Q_i(t)$, we have the recursion

$$\bar{Q}_1(t_{i+1}) - \bar{Q}_1(t_i) = \bar{Q}_1(t_i)[(R(t) - R_1^f(t)))(t_{i+1} - t_i) + \sigma_1(t)(\tilde{W}(t_{i+1}) - \tilde{W}(t_i))] \tag{4}$$

$$\bar{Q}_2(t_{i+1}) - \bar{Q}_2(t_i) = \bar{Q}_2(t_i)[(R(t) - R_2^f(t)))(t_{i+1} - t_i) + \sigma_2(t)(\tilde{W}(t_{i+1}) - \tilde{W}(t_i))] \tag{5}$$

$$\bar{Q}_3(t_{i+1}) - \bar{Q}_3(t_i) = \bar{Q}_3(t_i)[(R(t) - R_3^f(t)))(t_{i+1} - t_i) + \sigma_3(t)(\tilde{W}(t_{i+1}) - \tilde{W}(t_i))] \tag{6}$$

where

$\bar{Q}_i(t_i) = \bar{Q}_i(i), i = 0, 1, 2, \ldots$ are the value of variables $\bar{Q}_i$ at time t_i,

$t_{i+1} - t_i = h$ is the time-step size at $i = 0, 1, 2, \ldots$.

From Ito's distribution rule, $\tilde{W}(t_{i+1}) - \tilde{W}(t_i) = \sqrt{[t_{i+1} - t_i]} Z_i, Z_i \sim N(0, 1)$, i.e., Z_i is a standard normal variable with mean 0 and variance 1 for all i.

Therefore equations (4–6) become

$$\bar{Q}_1(t_{i+1}) - \bar{Q}_1(t_i) = \bar{Q}_1(t_i)[(R(t) - R_1^f(t)))(t_{i+1} - t_i) + \sigma_1(t)(\sqrt{h} Z_i)] \tag{7}$$

$$\bar{Q}_2(t_{i+1}) - \bar{Q}_2(t_i) = \bar{Q}_2(t_i)[(R(t) - R_2^f(t)))(t_{i+1} - t_i) + \sigma_2(t)(\sqrt{h} Z_i)] \tag{8}$$

$$\bar{Q}_3(t_{i+1}) - \bar{Q}_3(t_i) = \bar{Q}_3(t_i)[(R(t) - R_3^f(t)))(t_{i+1} - t_i) + \sigma_3(t)(\sqrt{h} Z_i)] \tag{9}$$

where $Q_{i0} = \bar{Q}_i(0), i = 1, 2, 3$ and $Z_{i+1}, i = 0, 1, 2, \ldots$. are independent normal random variables. Therefore considering the times $0 = t_0 < t_1 < \cdots < t_N = T$ such that $h = T/N$, then $\bar{Q}_i(t_0), \bar{Q}_i(t_1), \bar{Q}_i(t_2), \cdots, \bar{Q}_i(t_N)$ are sequence of approximations, defined on the set of discrete equally-spaced t-points, with each $\bar{Q}_i(i)$, an approximation of the actual random variable at $\bar{Q}_i(t_i); \bar{Q}_i(i) \simeq \bar{Q}_i(t_i), t_i = ih$ for $0 \leq i \leq N$.

Therefore, from equations (7–9) are the discrete time versions of the exchange rate model of the NGN-GBP, NGN-USD and NGN-EUR exchange rate in equations (1–3) respectively.

3.1. Description of Data

In this study, the data used includes

1. The selling price of the currencies of each of the European euro (€), British pound sterling (£) and the US dollar ($) to the Nigeria naira (₦) which was downloaded from the web site of the Central Bank of Nigeria. The data used include the daily average (Mon-Fri, excluding holidays) foreign exchange rate for the three currency pairs- Nigerian naira-European euro (NGN-EUR), Nigerian naira-British pound sterling (NGN-GBP), and Nigerian naira–U.S. dollar (NGN-USD). The data cover the period-January 2, 2008 through December 31, 2011, with the GBP, USD and EUR having a total of 740, 742 and 735 observations respectively.

2. The monthly average interest rates of United State, United Kingdom, European countries and Nigeria were downloaded from the Federal Reserve Bank of America, the Bank of England, European Central Bank and the Central Bank of Nigeria websites respectively. Plotting the graphs of the series NGN-GBP, NGN-USD and NGN-EUR in order to show their natural relationship and the time path of each exchange rate over the sample period. The Figures 2–4 show the time series plots of the exchange rates for the three countries' currencies using MS Excel 2007.

The Figures 3 and 4 display the time path of the Nigerian exchange rate with the three foreign currencies. Their graph clearly display a wavy trend in the exchange rate, showing that the value of the naira fluctuates (though the degree of fluctuations is different) throughout the sample period for the three currencies.

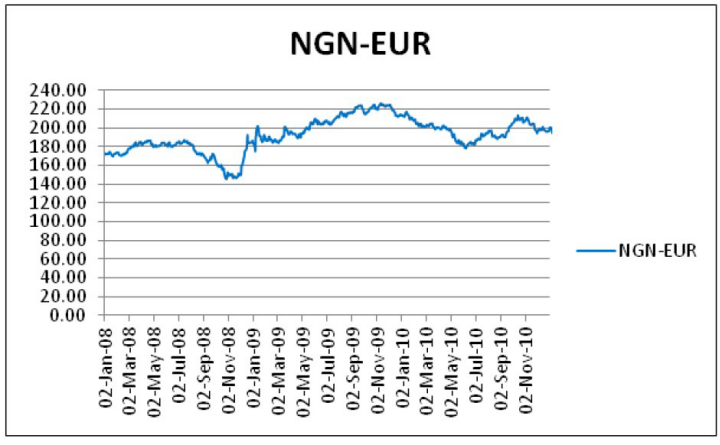

Figure 2
The Naira-Euro Exchange Rates over the Sample Period

4. RESULTS

The forecasting period of 36 trading period with a time step of one day, starting with the last day in the observed data, December 31, 2010 with value 231.57, 149.17, 199.74

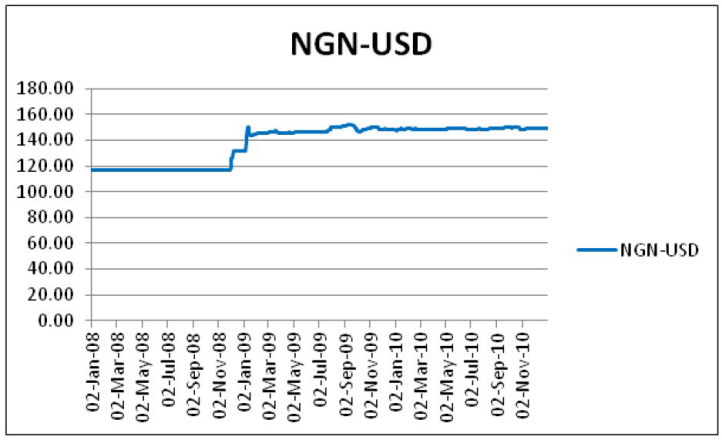

Figure 3
The Naira-Dollar Exchange Rates over the Sample Period

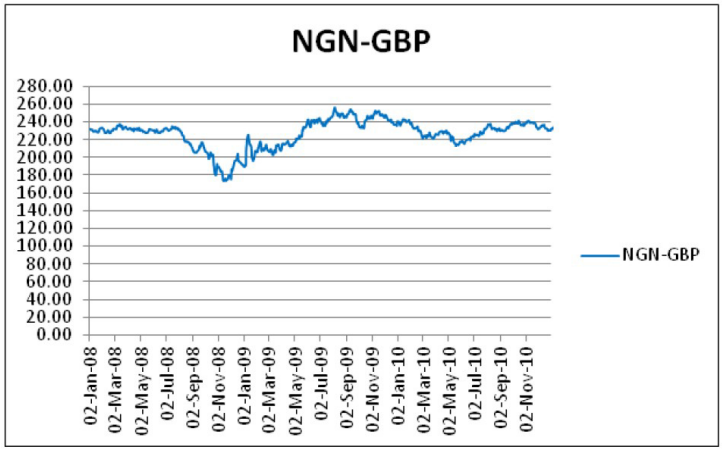

Figure 4
The Naira-Pound Exchange Rates over the Sample Period

for the NGN-GBP, NGN-USD and NGN-EUR respectively. The simulation of the exchange rates of the descretized model as described in equations (7)–(9), was implemented in MATLAB, based on the simulation and the prediction for the 36 days period was made. In this procedure, new observations are added to the sample day by day and the results below are recorded. The model parameters were estimated (calibrated) using forecasting period of 36 trading period with a time step of one day, starting with the last day in the observed data, December 31, 2010 with value 231.57, 149.17, 199.74 for the NGN-GBP, NGN-USD and NGN-EUR respectively. The calibration of the parameters was based on polynomial regression analysis(PRA), which provides a polynomial time series estimators for the parameters $R(t)$, $R_1^f(t), R_2^f(t)$ and $R_3^f(t)$. Implementation of the PRA was done in MATLAB environment with the use of the built-in MATLAB function "polyfit" and "polyval"

and the data used was from 720 sample data from historical market data from the websites of Central Banks of nations whose currencies are being simulated.

4.1. Calibration of the Model

Table 1
Estimates of the Interest Rates' Polynomial Regression

Interest rates	Polynomial of degree 3
UK Pound (p_1)	$0.0001t^3 + 0.0022t^2 - 0.3805t + 6.2766$
USA Dollar (p_2)	$-0.0002t^3 + 0.0178t^2 - 0.4466t + 3.6878$
European Euro (p_3)	$-0.0002t^3 - 0.0073t^2 - 0.1150t + 4.5011$
Nigerian Naira (p)	$0.0008t^3 - 0.0382t^2 + 0.3324t + 9.3620$

Table 2
Estimates of the Difference in the Interest Rates' Polynomial Regression

Difference in rates	Polynomial of degree 3
$n = p - p_1$	$0.0006t^3 - 0.0404t^2 + 0.7129t + 3.0854$
$n = p - p_2$	$0.0010t^3 - 0.0560t^2 + 0.7790t + 5.6742$
$n = p - p_2$	$0.0005t^3 - 0.0309t^2 + 0 : 4473t + 4.8609$

Table 3
Volatility of Exchange Rates

Parameter	Volatility
NGN-GBP	2.1982
NGN-USD	0.5823
NGN-EUR	1.7921

From Table 3, the explanation of the high instability of the NGN-GBP and NGN-EUR is seen from their volatility estimates. The volatility shows that the NGN-GBP is the most volatile among the three foreign exchange rates, followed by the NGN-EUR and the NGN-USD, the least volatile.

The result from Table 4 which is the simulated value for the NGN-GBP, NGN-USD and NGN-EUR exchange rates over the 36 days period show minimum value of 197.10, 149.17 and 184.38 respectively and maximum value 329.47, 196.23 and 277.84 respectively. Also the graph 2 of Figures 5–6 displayed the estimated Nigerian interest rate and the movement is found to be undulating in nature with a peak rate of 10 and least rate of about 5.5.

4.1.1. NGN-GBP Exchange Rate

Graph 4 of Figure 5 displays an unstable exchange rate of the NGN-GBP over the simulation period, with a large difference between the highest and lowest value (197.10–329.47).

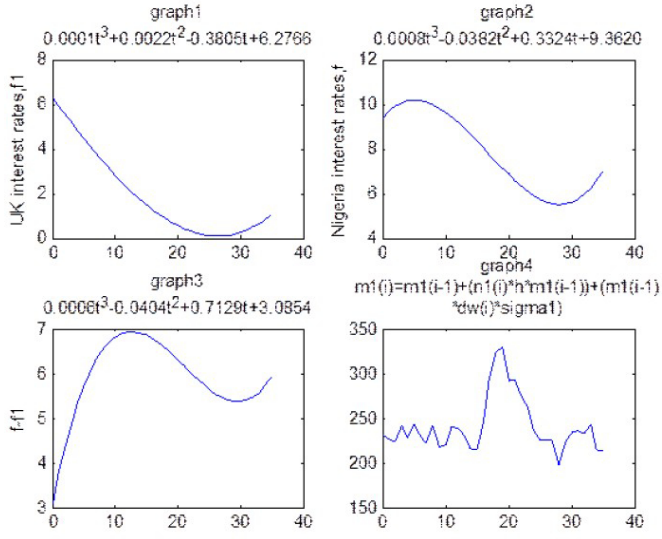

Figure 5
Graph Plots of the Simulated Results of the NGN-GBP Exchange Rate

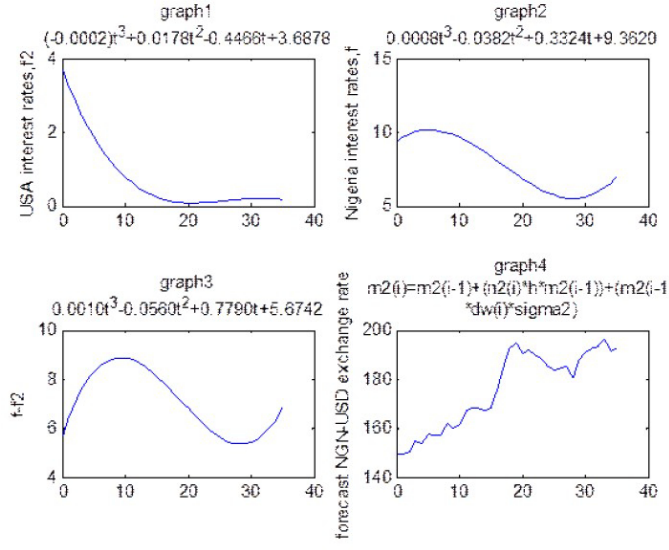

Figure 6
Graph Plots of the Simulated Results of the NGN-USD Exchange Rate

Graph 1 of Figure 6 display the estimated US interest rate has downward slope, from 6.28% to 0.08% in the first 27 periods, followed by an upwards movement. Graph 3 which illustrates the difference in the interest rate of the Naira and the Pound also shows an upward movement over the first 14 periods, then a downward slope to the 31st period. This explains that either there was a more increase of the Nigeria interest rate than the US or a more decrease in the UK interest rate while that of the naira is constant.

Table 4
Simulated Result for the NGN-GBP, NGN-USD and NGN-EUR Exchange Rates

Periods (Days)	NGN -GBP	NGN -USD	NGN -EUR	Periods (Days)	NGN -GBP	NGN -USD	NGN -EUR
1	231.57	149.17	199.74	19	323.92	192.59	273.76
2	226.38	149.37	196.68	20	329.47	194.85	277.84
3	223.99	150.13	195.54	21	292.24	190.37	252.38
4	242.24	154.63	209.10	22	292.72	191.70	252.90
5	227.54	153.51	199.25	23	275.22	189.87	240.72
6	243.40	157.75	211.09	24	260.85	188.39	230.60
7	230.01	156.93	202.05	25	237.85	185.09	214.12
8	222.22	157.01	196.88	26	224.96	183.46	204.76
9	241.75	162.16	211.41	27	224.84	184.43	204.80
10	218.15	159.52	194.91	28	225.35	185.52	205.30
11	221.03	161.60	197.34	29	197.10	180.34	184.38
12	240.67	166.91	212.00	30	223.27	187.63	204.52
13	238.11	167.98	210.46	31	234.05	191.03	212.75
14	229.06	167.80	204.20	32	235.83	192.47	214.22
15	215.97	166.73	194.91	33	232.82	192.93	212.14
16	214.41	167.82	193.99	34	242.50	196.23	219.52
17	246.42	175.82	217.88	35	214.96	191.62	199.32
18	293.60	186.11	252.23	36	213.02	192.52	198.04

Comparing graph 3 and graph 4 in the Figure 5, the NGN-GBP exchange rate appreciates more in value when there is a continuous increase in the differences of the naira and pound interest rates and vice versa.

4.1.2. NGN-USD Exchange Rate

From Figure 6, graph 4, the simulated NGN-USD exchange rate has upward slope with a little or no changes except from period 17 to 18. Its value varies between 149.17 and 196.23. The US interest rate graph (Figure 6, graph 1) shows a downward slope over the first 22 periods from 3.69 to 0.07 and then moved up to 0.22.

The interest rates differences graph (Figure 7, graph 3) displays an undulation behavior with a high and low peak periods of 10 and 29 respectively.

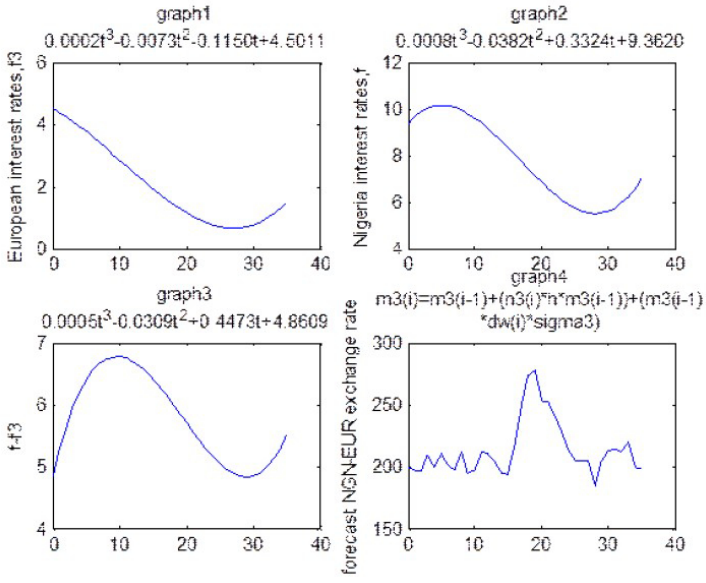

Figure 7
Graph Plots of the Simulated Results of the NGN-EUR Exchange Rate

4.1.3. NGN-EUR Exchange Rate

There is similarity in the NGN-EUR and NGN-GBP both followed the same pattern of movement from the simulated results. The Euro interest rates decrease over the first 28 periods from 4.50% to 0.64%. The Pound interest rate also shares same trend as the Euro. Moreover,the graph of their interest rates differences begins with an upward movement to the 11th period and then downward to the 30 period.

4.1.4. Comparison of the Three Exchange Rates

From the simulation, except with the dollar, as the difference in the interest rates increases, the value of the exchange rate appreciates and as it decreases, the value depreciates. This implied that if the interest rate of the naira is higher than the pound or euro, the naira appreciates and if is lower than, it depreciates in value.

Also the range of simulated exchange rates value of the NGN-USD is small compared to the NGN-EUR and the NGN-GBP. The NGN-USD is between 149.17 and 196.23 while the NGN-GBP is between 197.10 and 329.47; the NGN-EUR is between 184.38 and 277.84. Thus the naira to the dollar is more appreciable in value than the pound and euro which can also been seen from the range of value of the differences between their interest rates and that of the naira.

The dollar is on the higher side (5.33–8.87) which shows that the interest rate of the naira is high relative to the dollar while that of the pound and euro is between 3.09–6.93 and 4.84–6.78 respectively.

The simulation has strong order of convergence equal to $\beta = \frac{1}{2}$ (Desmond, 2001) such that there exists a constant C such that $E|Q_n - Q(T)| \leq Ch^{\beta}$ for fixed $T = nh, h \in [0, T]$ here $n = 50000,\ h = 0.00072$ and $T = 36$.

5. CONCLUSION

The exchange rate model considered in the paper does not include the option pricing of the currencies and does not approach the simulation from the econometric perspective, but rather from stochastic differential equations perspective. The simulation was studied on short time horizon which most simulation of exchanges are found to be most accurate. We recommend that more research work should be exploited using Milstein methods and option pricing of exchange rates on long horizons.

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