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The Natural Frequency of Elastic Plates with Void by Ritz-Method

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Abstract: To use a new computational method for the problem of the flexural vibration of thin, isotropic rectangular plates with arbitrarily positioned voids. In this paper we use static beam functions under point load as admissible functions and Rayleigh-Ritz method for analysis of discontinuous variation of rigidity of the plates. The voids of plate are expressed continuously by using extended Dirac function, which is defined as Dirac function existing continuously in prescribed region. The governing equation for a plate with voids composed of an isotropic material is formulated without modifying the rigidity of the plates, as done in the equivalent plate analogy. The natural frequencies of simply supported and clamped plates with voids are obtained from the governing equation by this energy method. The model is able to determine the frequency of plates with voids. It is shown that numerical results all converge very fast and are in excellent agreement with other researcher's results. The numerical results obtained from the solution method used here show good agreement with results obtain form the previous equivalent plates analogy.

Key Words: Plates with voids; Static beam function

1. INTRODUCTION

There are many well known publications on free vibration of rectangular isotropic and homogenous and elastic plates. Dickison and Li[4] proposed an alternative set of admissible functions, derived from the mode shapes of vibration of plates having two parallel edges simply supported. Warburton and Edney[11] used Rayleigh-Ritz method to analyze simple plates. Bhat[2] investigated the vibration problem of rectangular plates by using a set of characteristic orthogonal polynomials with Rayleigh-Ritz method. Ng and Araar[7] solved the free vibration and buckling of clamped rectangular plate of variable thickness by the Galerkin method. Belinha and Dinis[1] solved elasto-plastic plates by the element free Galerkin method. Zhou[12] used a new, fast converging series consisting of static beam functions under point load as admissible functions in the Rayleigh-Ritz Method. Zhou[13, 14] and Lee et. al[6] used the same beam shape function to get the natural frequency of rectangular plates on elastic point support or elastically restrained in the Rayleigh-Ritz method. Zhou[15] used static tapered beam functions to get the natural frequency of rectangular plates in the

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Rayleigh-Ritz method. Hao and Kam[5] applied Rayleigh-Ritz method for determining modal characteristics of symmetric laminated composite plates restrained by elastic supports at different locations. Bohinc, Ibrahimbegovic and Brank[3] applied model adaptivity for finite element analysis of thin or thick plates.

For reasons of lightness and structural efficiency and in order to guarantee enough space for equipment, plates with voids are often used. These are called multi-cell slabs with transverse diaphragms, or voided slabs, or cellular slabs, depending upon the shape and size of the voids used. Takabatake[8, 9] had demonstrated the effectiveness of the extended Dirac function for bending and torsion analyses of tube systems and for lateral buckling of I beam with web stiffeners and batten plate. Although analyses based on the finite element method for plates with voids are effective, much numerical calculation is needed. A general analytical method for plates with arbitrarily positioned voids has not been established. Takabatake[10] used extended Dirac function and Galerkin method to get the frequency of plate. The extended Dirac function is defined as a Dirac function existing continuously in a prescribed region. For the current problem, the extended Dirac function has a value in the region where voids exist, and replaces the discontinuous variation in the rigidity of the plates due to the voids with a continuous function; it is therefore effective in presenting a general analytical method for plates with arbitrarily positioned voids. The theory of plates with voids is formulated without modifying the rigidity of the plates, as done in the equivalent plate analogy.

In this paper, we use static beam function and extended Dirac function to study the free vibration for simple plates or clamp plates with voids and compare the result with that of Takabatake[10]. We will illustrate that our numerical results are in agreement with that of Takabatake[10]. The advantage of our method, however, is that it requires less computation and uses less memory space.

2. THE SET OF STATIC BEAM FUNCTIONS UNDER POINT LOAD

From the structural mechanics theory, the deflection of a beam under an arbitrary point load satisfies the following differential equation

$$EI\frac{d^4y(x)}{dx^4} = P_i\delta(x-x_i) \quad (0 \le x \le l), \tag{1}$$

where $y(x)$ is the function of deflection of a beam, x is the axial distance along the beam's length (Figure 1), EI is the flexural rigidity of beam, l is the beam's length, P_i is a point load applied to the beam at $x = x_i$ and the $\delta(x-x_i)$ is Dirac delta function defined as follows:

$$\delta(x-x_i) = \begin{cases} \infty & x = x_i, \\ 0 & x \neq x_i. \end{cases} \tag{2}$$

Let $x = \varsigma l, x_i = \varsigma_i l$ where ς and ς_i are dimensionless parameters. We can get the general solution of equation (1) as follows:

$$y_i(\varsigma) = C_0 + C_1\varsigma + C_2\varsigma^2 + C_3\varsigma^3 + \frac{P_i l^3}{6EI}\langle \varsigma - \varsigma_i \rangle^3, \tag{3}$$

where $C_j\ (j = 0,1,2,3)$ are undetermined constants, and $\langle \varsigma - \varsigma_i \rangle^3 = (\varsigma - \varsigma_i)^3 U(\varsigma - \varsigma_i)$ is the singular function.

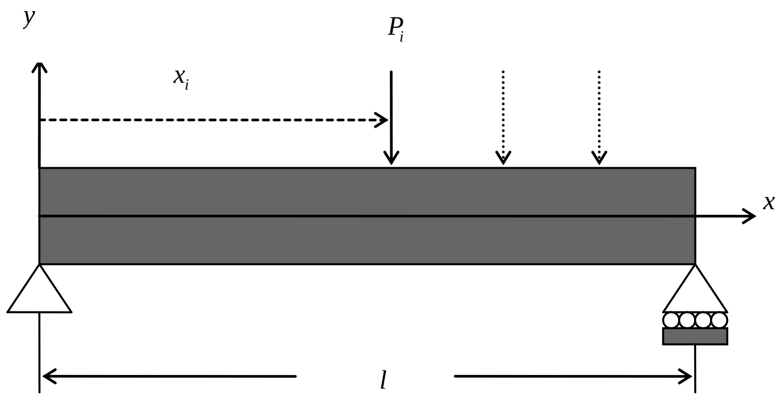


Figure 1: The beam under a point load

There are two types of boundary conditions. For a clamped-clamped beam the deflections and slope are zero at position $\varsigma = 0,1$. The corresponding boundary conditions are as follows:

$$y_i(0) = y_i(1) = 0, \quad \left.\frac{dy_i(\varsigma)}{d\varsigma}\right|_{\varsigma=0} = \left.\frac{dy_i(\varsigma)}{d\varsigma}\right|_{\varsigma=1} = 0, \tag{4}$$

For a simply supported beam the deflections and moments are zero at $\varsigma = 0,1$. The corresponding boundary conditions are as follows:

$$y_i(0) = y_i(1) = 0, \qquad \left.\frac{d^2 y_i(\varsigma)}{d\varsigma^2}\right|_{\varsigma=0} = \left.\frac{d^2 y_i(\varsigma)}{d\varsigma^2}\right|_{\varsigma=1} = 0, \tag{5}$$

Substituting equation (4) and equation (5) into equation (3), respectively, and set point load $P_i = 6EI/l^3$, the constants $C_j\,(j = 0,1,2,3)$ can be determined. The functions of the deflection of a beam with both ends clamped and for the simply supported beam are given by equations (6) and (7) respectively.

$$y_i(\varsigma) = (1-\varsigma_i)^2[3\varsigma_i\varsigma^2 - (2\varsigma_i + 1)\varsigma^3] + \langle \varsigma - \varsigma_i \rangle^3, \tag{6}$$

$$y_i(\varsigma) = (1-\varsigma_i)\varsigma(2\varsigma_i - \varsigma_i^2 - \varsigma^2) + \langle \varsigma - \varsigma_i \rangle^3, \tag{7}$$

If one varies the location of ς_i of point load P_i, the corresponding static beam function is then determined. Generally, we set $\varsigma_i = i/(n+1)$ if the number of the beam function is *n*.

3. GOVERNING EQUATION OF PLATE WITH VOID

Considering an isotropic, homogeneous, elastic and uniform thickness, rectangular plate as shown in Figure 2, the maximum strain energy of the plate is

$$U = \frac{1}{2}\iint D[W_{xx}^2 + W_{yy}^2 + 2\nu W_{xx}W_{yy} + 2(1-\nu)W_{xy}^2]dxdy, \tag{8}$$

where

$$D = \frac{Eh_0^3}{12(1-\nu^2)}, \tag{9}$$

is the flexural rigidity of the plate, h_0 is the thickness of the plate and ν is the Poisson ratio.

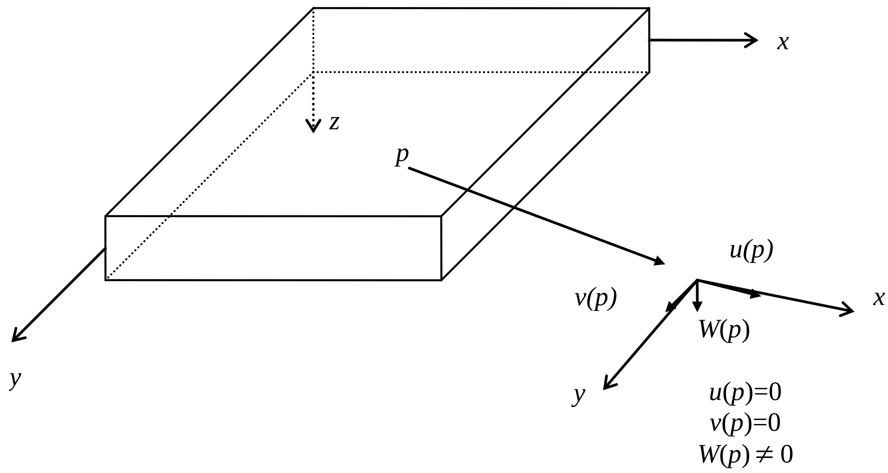


Figure 2: Coordinates of a plate

Consider a rectangular plate with arbitrarily point voids, as shown in Figure 3. Here Cartesian coordinate system *x*, *y*, *z* employed. Assume that each void is a rectangular parallelepiped whose ridgelines are parallel to the x- or y-axis. The position of the m, nth of void is indicated by the coordinate value (X_m, Y_n) of the midpoint of the void. The widths in the x and y directions of the void are $bx_{m,n}$ and $by_{m,n}$, respectively, and its height is $h_{m,n}$ .The size and position of each void are arbitrary except for the assumptions mentioned above.

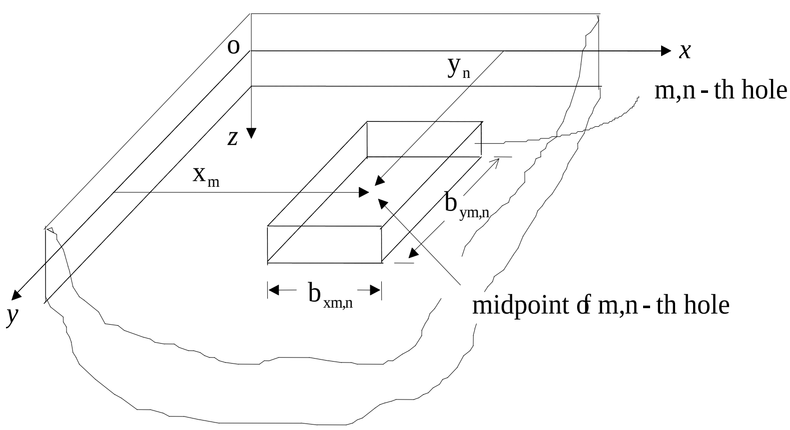


Figure 3: Detail of a void

Considering the bending of isotropic plates subject to small deformations, and assuming the validity of the Kirchhoff-Love plate theory, from equation (8) one can obtain the following at a section where a void exists:

$$\int z^2 dz = \int_{-\frac{h_0}{2}}^{\frac{h_0}{2}} z^2 dz - \int_{-\frac{h_1}{2}}^{\frac{h_1}{2}} z^2 dz, \tag{10}$$

where h_0 is the thickness of solid plates and h_1 is a function of *x* and *y*. At all points in the region where the *m*, *n*th void exists, the relation $h_1 = h_{m,n}$ is valid. Hence, $h_1(x, y)$ can generally expressed by

$$h_1(x, y) = \sum_{m=1}^{m^*} \sum_{n=1}^{n^*} h_{m,n} \bar{\delta}(x - x_m) \bar{\delta}(y - y_n), \tag{11}$$

where $\sum$ is the sum for the total number of voids in the plates, *m** and *n** indicate the final numbers of voids in position counting form $m = 1$ and $n = 1$, respectively.

By utilizing equations (10) and (11), the energy equation becomes

$$U = \frac{D_0}{2} \iint d(x, y)[W_{xx}^2 + W_{yy}^2 + 2\nu W_{xx} W_{yy} + 2(1-\nu) W_{xy}^2] dxdy, \tag{12}$$

$$V = \frac{\omega^2}{2} \rho h_0 \iint h(x, y) W^2 dxdy, \tag{13}$$

where $D_0 = D = \dfrac{Eh_0^3}{12(1-\nu^2)}$ is the flexural rigidity of the plate, ν is the Poisson ratio, ω is the natural frequency of plate, and *d*(*x*,*y*) and *h*(*x*,*y*) are defined as follows:

$$h(x, y) = 1 - \sum_{m=1}^{m^*} \sum_{n=1}^{n^*} \frac{h_{m,n}}{h_0} \bar{\delta}(x - x_m) \bar{\delta}(y - y_n), \tag{14}$$

$$d(x, y) = 1 - \sum_{m=1}^{m^*} \sum_{n=1}^{n^*} \alpha_{m,n} \bar{\delta}(x - x_m) \bar{\delta}(y - y_n), \tag{15}$$

$$\alpha_{m,n} = \left(\frac{h_{m,n}}{h_0} \right)^3, \tag{16}$$

$\bar{\delta}(x - x_m)$ and $\bar{\delta}(y - y_n)$ are extended Dirac function defined as follows:

$$\begin{aligned} \bar{\delta}(x - x_m) &= \begin{cases} 1 & \text{for} \quad x_m - \dfrac{bx_{m,n}}{2} < x < x_m + \dfrac{bx_{m,n}}{2}, \\ 0 & \text{otherwise}, \end{cases} \\ \bar{\delta}(y - y_n) &= \begin{cases} 1 & \text{for} \quad y_n - \dfrac{by_{m,n}}{2} < y < y_n + \dfrac{by_{m,n}}{2}, \\ 0 & \text{otherwise}. \end{cases} \end{aligned} \tag{17}$$

The Rayleigh's quotient for the natural frequency is obtained from equations (12) and (13):

$$\omega^2 = \frac{\dfrac{D_0}{2}\int_0^a\int_0^b d(x,y)\left(\left(\dfrac{\partial^2 W}{\partial x^2}\right)^2 + \left(\dfrac{\partial^2 W}{\partial y^2}\right)^2 + 2\nu\dfrac{\partial^2 W}{\partial x^2}\dfrac{\partial^2 W}{\partial y^2} + 2(1-\nu)\left(\dfrac{\partial^2 W}{\partial x\partial y}\right)^2\right)dxdy}{\dfrac{\rho h_0}{2}\int_0^a\int_0^b h(x,y)W^2 dxdy}, \tag{18}$$

where

$$W = W(x,y) = \sum_{i=1}^{M}\sum_{j=1}^{N} A_{ij}\varphi_i(x)\varphi_j(y), \tag{19}$$

Using the dimensionless space variables ξ and η, where $\xi = x/a$ and $\eta = y/b$ the following equations are obtained

$$\begin{aligned}
\frac{\partial W}{\partial x} &= \frac{\partial \xi}{\partial x}\frac{\partial W}{\partial \xi} = \frac{1}{a}\frac{\partial W}{\partial \xi}, \quad \frac{\partial^2 W}{\partial x^2} = \frac{1}{a^2}\frac{\partial^2 W}{\partial \xi^2},\\
\frac{\partial W}{\partial y} &= \frac{\partial \eta}{\partial x}\frac{\partial W}{\partial \eta} = \frac{1}{b}\frac{\partial W}{\partial \eta}, \quad \frac{\partial^2 W}{\partial y^2} = \frac{1}{b^2}\frac{\partial^2 W}{\partial \eta^2},
\end{aligned} \tag{20}$$

Equation (18) becomes:

$$\omega^2 = \frac{D_0}{a^4\rho h_0}\times \frac{\int_0^1\int_0^1 d(\xi,\eta)\left\{\left(\left(\dfrac{\partial^2 W}{\partial \xi^2}\right)^2 + \dfrac{1}{\beta^4}\left(\dfrac{\partial^2 W}{\partial \eta^2}\right)^2 + \dfrac{2\nu}{\beta^2}\dfrac{\partial^2 W}{\partial \xi^2}\dfrac{\partial^2 W}{\partial \eta^2} + \dfrac{2(1-\nu)}{\beta^2}\left(\dfrac{\partial^2 W}{\partial \xi\partial \eta}\right)^2\right)d\xi d\eta\right\}}{\int_0^1\int_0^1 h(\xi,\eta)W^2 d\xi d\eta}, \tag{21}$$

where $\beta = b/a$.

The deformed shape of the plate can be expressed as

$$W = \sum_{i=1}^{M}\sum_{j=1}^{N} A_{ij}\varphi_i(\xi)\varphi_j(\eta), \tag{22}$$

where $A_{ij}(i=1,2,\cdots,M;\, j=1,2,\cdots,N)$ are the unknown coefficients, and the function $\varphi_i(\xi)$ and $\phi_j(\eta)$ are appropriate admissible functions which must satisfy the corresponding boundary conditions. The functions $\varphi_i(\xi)$ and $\phi_j(\eta)$ are selected, respectively, as follows

$$\varphi_i(\xi) = y_i(\xi), \quad \varphi_j(\eta) = y_j(\eta), \tag{23}$$

where $y_i(\xi)$ and $y_j(\eta)$ are the static beam functions under point load at position (ξ_i, η_j) and satisfy the corresponding boundary conditions of plate.

Defining

$$\frac{\rho h_0\omega^2 a^4}{D_0} = \lambda^2, \tag{24}$$

and substituting equations (22) and (23) into equation (21), one can obtain

$$\lambda^2 = \frac{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{M}\sum_{l=1}^{N} A_{ij}A_{kl}(a_{ijkl}+b_{ijkl}+d_{ijkl}+e_{ijkl})}{\sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{k=1}^{M}\sum_{l=1}^{N} A_{ij}A_{kl}c_{ijkl}}, \tag{25}$$

where

$$a_{ijkl} = \int_0^1\int_0^1 d(\xi,\eta)\frac{d^2\varphi_i(\xi)}{d\xi^2}\frac{d^2\varphi_k(\xi)}{d\xi^2}\varphi_j(\eta)\varphi_l(\eta)\,d\xi d\eta, \tag{26}$$

$$b_{ijkl} = \frac{1}{\beta^4}\int_0^1\int_0^1 d(\xi,\eta)\varphi_i(\xi)\varphi_k(\xi)\frac{d^2\varphi_j(\eta)}{d\eta^2}\frac{d^2\varphi_l(\eta)}{d\eta^2}\,d\xi d\eta, \tag{27}$$

$$d_{ijkl} = \frac{2\nu}{\beta^2}\int_0^1\int_0^1 d(\xi,\eta)\frac{d^2\varphi_i(\xi)}{d\xi^2}\varphi_k(\xi)\varphi_j(\eta)\frac{d^2\varphi_l(\eta)}{d\eta^2}\,d\xi d\eta, \tag{28}$$

$$e_{ijkl} = \frac{2(1-\nu)}{\beta^2}\int_0^1\int_0^1 d(\xi,\eta)\frac{d\varphi_i(\xi)}{d\xi}\frac{d\varphi_k(\xi)}{d\xi}\frac{d\varphi_j(\eta)}{d\eta}\frac{d\varphi_l(\eta)}{d\eta}\,d\xi d\eta, \tag{29}$$

$$c_{ijkl} = \int_0^1\int_0^1 h(\xi,\eta)\varphi_i(\xi)\varphi_k(\xi)\varphi_j(\eta)\varphi_l(\eta)\,d\xi d\eta. \tag{30}$$

Minimizing the Rayleigh's quotient with respect to the coefficient A_{ij}, i.e.

$$\frac{\partial\lambda^2}{\partial A_{ij}} = 0, i = 1,2,\cdots,M;\ j = 1,2,\cdots,N, \tag{31}$$

$$\sum_{k=1}^{M}\sum_{l=1}^{N}\left[\left(a_{ijkl}+b_{ijkl}+d_{ijkl}+e_{ijkl}\right)-\lambda^2 c_{ijkl}\right]A_{kl} = 0, \tag{32}$$

Equation (32) will give a $M \times N$ matrix. A numerical eigensolution was used to extract the eigenvalues and their corresponding eigenvectors. The natural frequencies of the plate and their corresponding mode shapes are then obtained, respectively.

4. NUMERICAL RESULTS

Three types of elastic plates with voids are shown in Table 1. The fundamental frequency of a solid plate, with $h_{m,n}/h_0$ =0, is ω_{01}. The fundamental frequencies of plates with voids are ω_1. The ratios of void sizes along *x* and *y* directions are $bx_{m,n}/a$, $by_{m,n}/b$. For Type 1, $bx_{m,n}/a = 0.05$ 、0.1 、0.15; $by_{m,n}/b$ =0.5, and the aspect ratio of plate is 1 and 2/3. For Type 2, $bx_{m,n}/a$ =0.05、0.1、0.15; $by_{m,n}/b$ =1, and the aspect ratio of plate is 1 and 2/3. For Type 3, the void is square, $bx_{m,n}/a$ and $by_{m,n}/b$ are 0.05、0.1、0.15, respectively, and the aspect ratios of plates are 1 and 2/3.

Convergence studies of the frequency parameter λ are undertaken and the numerical results are shown in Figure 4 and Table 2 to Table 4. From the obtained numerical results, we can say that the proposed method has pretty well convergence when *M*=4 and *N*=4. The natural frequencies from mode 1 to mode 6 reach a steadily converging value with β =2/3 and the Poisson ratio=0.3333 as shown in Figure 4.

According Kirchhoff-Love's assumption, the ratio $h_{m,n}/h_0$ must be restricted less than 0.6 for thin plate model. The fundamental frequencies of plates with voids increase with direct proportion to the ratio,

$h_{m,n}/h_0$ as shown in Table 2 to Table 4 and Figure 5 to Figure 13. Therefore, we can conclude that with M=4 and N=4, the numerical results are satisfactory for all the examples illustrated here.

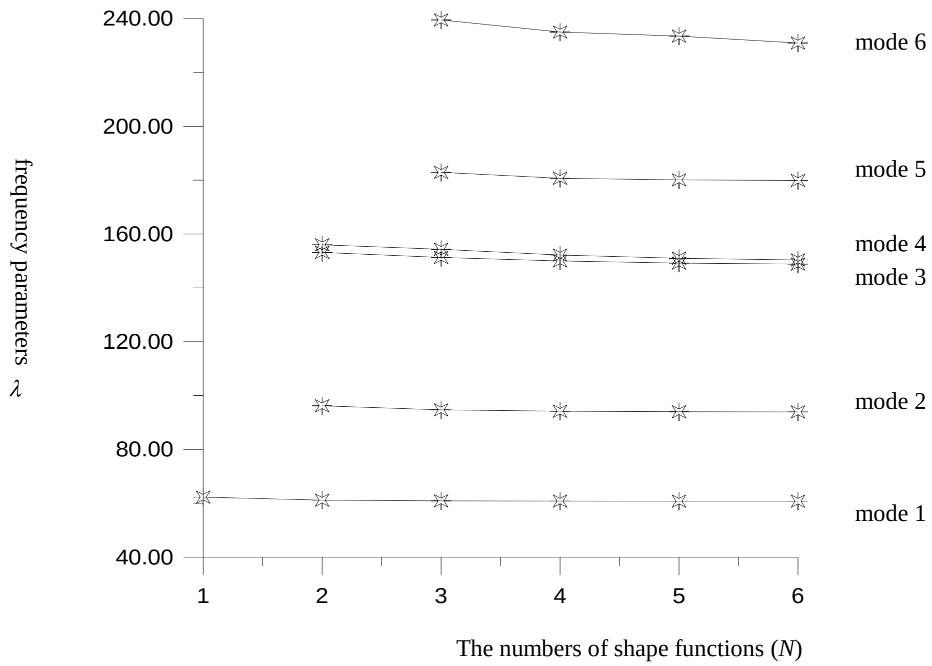


Figure 4: Natural frequency parameters λ for simply supported plate with different numbers of shape functions

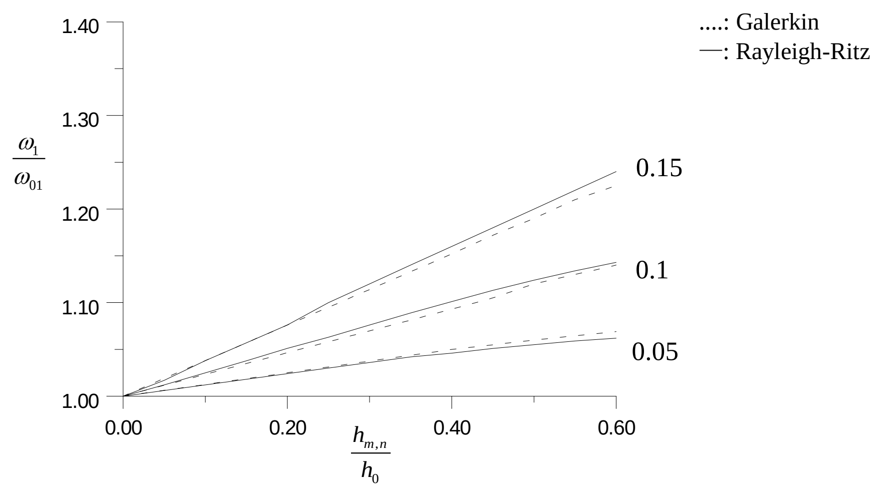


Figure 5: The first frequency for simple supported (type 1) β =1

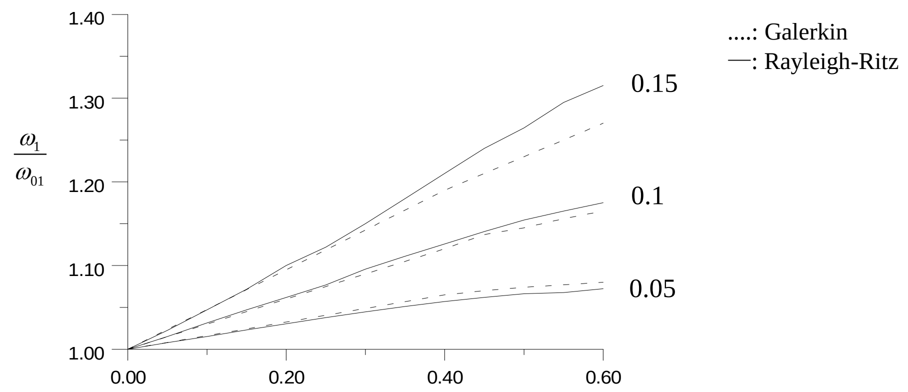


Figure 6: The first frequency for simple supported (type 2) β =1

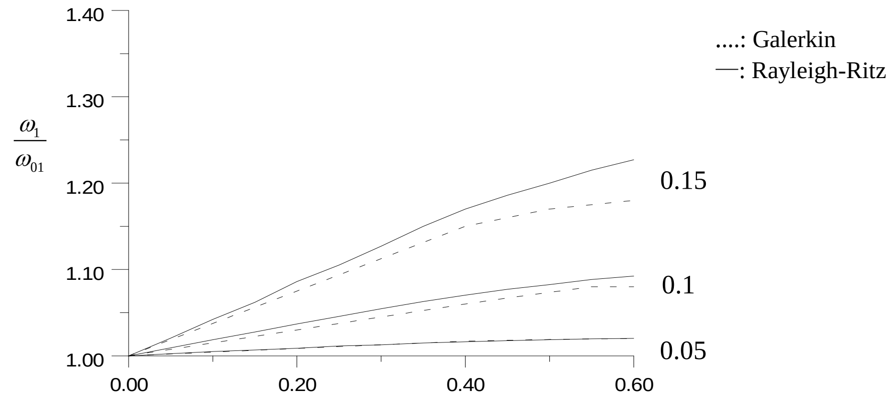


Figure 7: The first frequency for simple supported (type 3) β =1

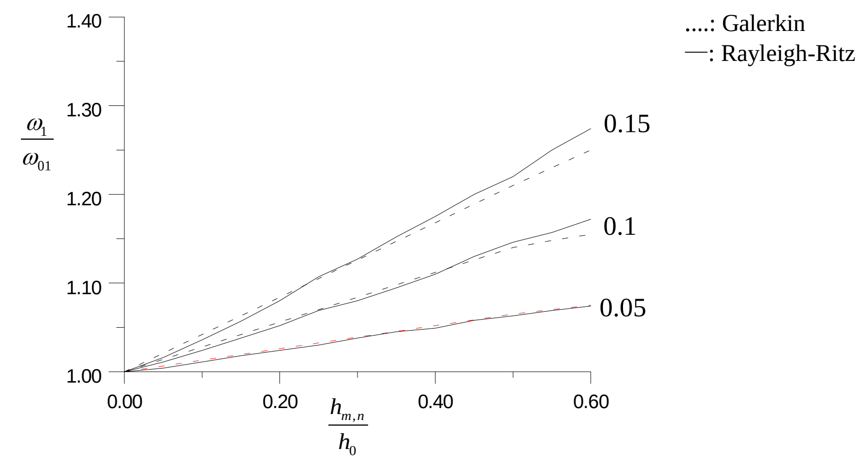


Figure 8: The first frequency for clamp plates (type 1) β =1

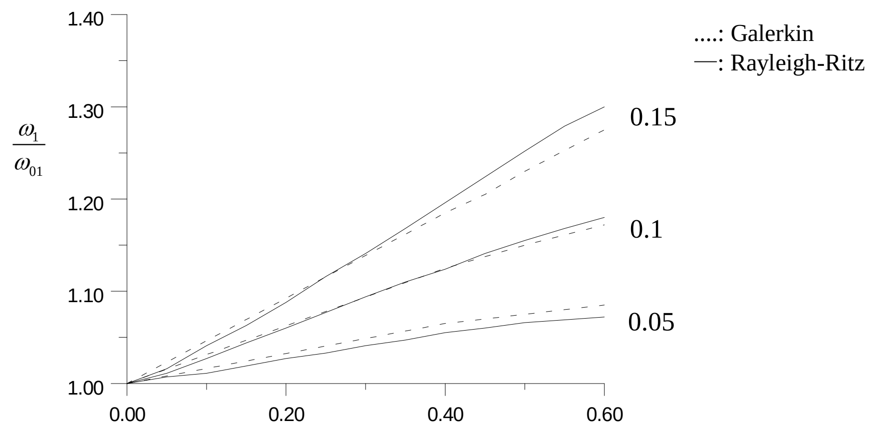


Figure 9: The first frequency for clamp plates (type 2) β =1

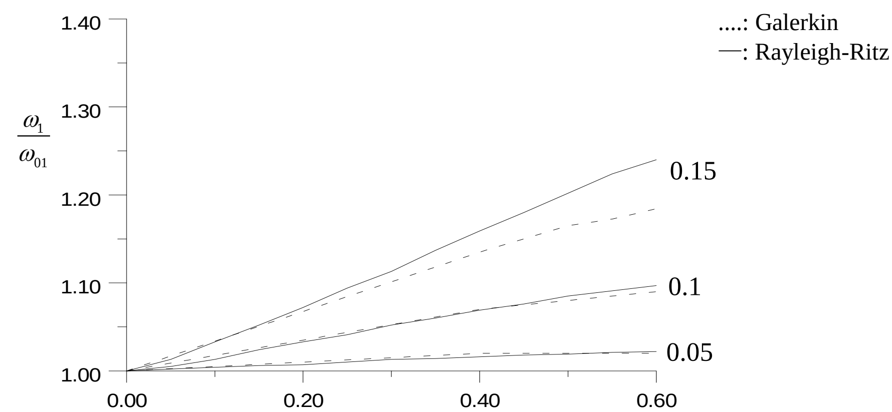


Figure 10: The first frequency for clamp plates (type 3) β =1

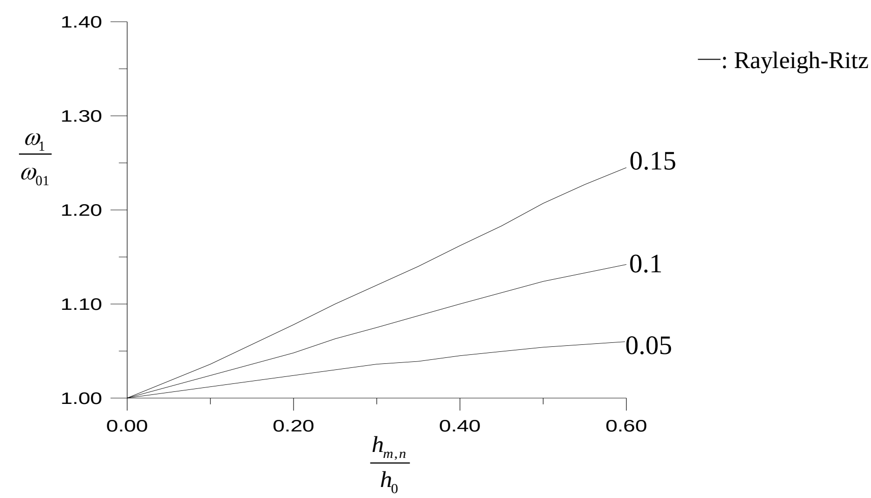


Figure 11: The first frequency for simple supported (type 1) β =2/3

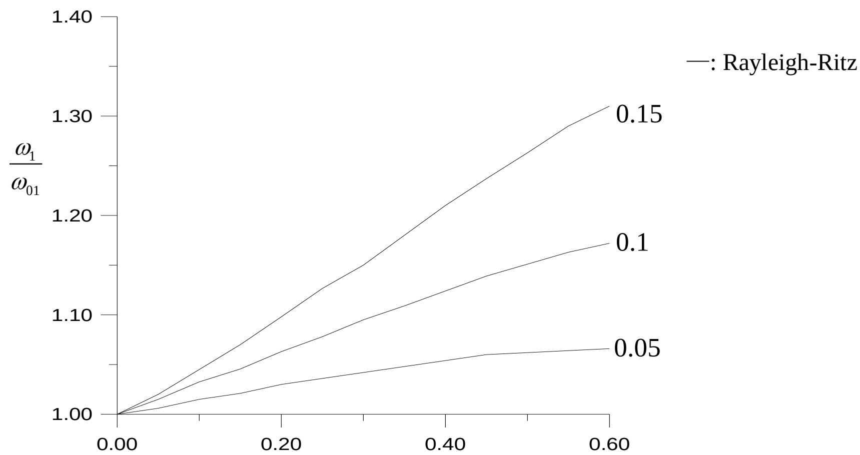


Figure 12: The first frequency for simple supported (type 2) β =2/3

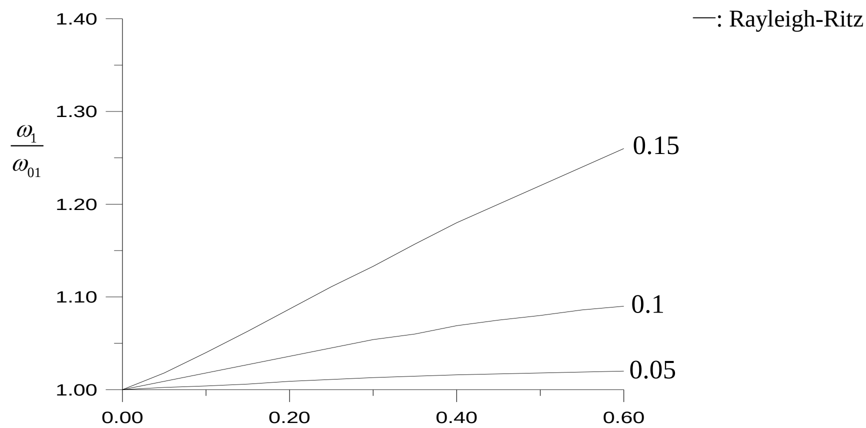


Figure 13: The first frequency for simple supported (type 3) β =2/3

Case 1: A simply supported plate with $\beta = 1, \nu = 0.333$

The fundamental frequencies for all three types of plates with variation of void's height are shown in Figure 5 to 7. The dash lines are the results of previous researcher by using Galerkin method, as in Takabatake (1990). The solid lines are obtained by applying beam functions with Rayleigh-Ritz method. It is shown that the frequencies almost increase linearly with the void ratio of the plates. Slight discrepancy can be observed for void's ratio $h_{m,n} / h_0$ over 0.4 with $bx_{m,n} / a = 0.15$ in Type 2 and Type 3 plates.

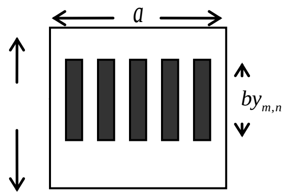
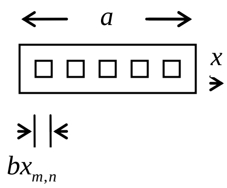
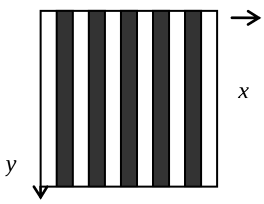
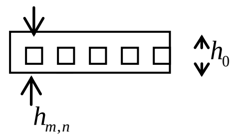
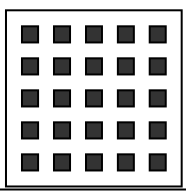
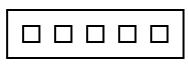
Case 2: A clamped plate with $\beta = 1, \nu = 0.333$

The fundamental frequencies for all three types of clamped plates with variation of void's height are shown in Figures 8 to 10. The fundamental frequencies of plates with voids also increase linearly with the void's height ratio which is very similar to Case 1.

Case 3: A simply supported plate with $\beta = 2/3, \nu = 0.333$

The fundamental frequencies for all three types of clamped plates with a different aspect ratio of $\beta = 2/3$ are shown in Figures 11 to 13. As expected, the fundamental frequencies also increase linearly with the void's ratio $h_{m,n}/h_0$ which is very similar to the previous two cases.

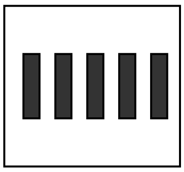
Table 1: Three types of isotropic rectangular plates with voids

TYPE	PLANE	SECTION	$\frac{h_{m,n}}{h_0}$	$\frac{bx_{m,n}}{a}$	$\frac{by_{m,n}}{b}$	$\beta = \frac{b}{a}$
1			0.5	0.1	0.5	1.0
2			0.5	0.1	1.0	1.0
3			0.5	0.1	0.1	1.0

5. CONCLUSIONS

A general analytical method for isotropic rectangular plates with arbitrarily positioned voids has been proposed by means of an extended Dirac function. By using this type of shape functions which were obtained from beams with point loads, excellent approximate values of the natural frequency of plates can be easily obtained. With just a few shape functions, the convergence of natural frequencies can be easily achieved. The results of this research are in good agreement with previous researchers' work and the method illustrated here requires less calculation effort in solving for the frequency parameters λ^2. For further research on a related area, the voids in plates can be considered as internal cracks which influence the stiffness and the natural frequencies of the plate structure. By applying this method, the size and location of crack may be determined corresponding to the variations of natural frequencies.

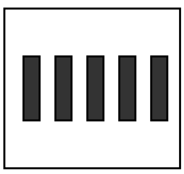
Table 2: ω_1/ω_{01} for a for a plate with voids, *M*, *N*=1,2,3,4 (type 1)



$bx_{m,n}/a = 0.05$
$by_{m,n}/b = 0.5$

h/h_0	ω_1/ω_{01} *M*, *N* =1	*M*, *N* =2	*M*, *N* =3	*M*, *N* =4
0	1	1	1	1
0.05	1.006	1.006	1.004	1.006
0.1	1.012	1.012	1.011	1.012
0.15	1.018	1.018	1.016	1.018
0.2	1.025	1.024	1.023	1.024
0.25	1.031	1.030	1.029	1.030
0.3	1.037	1.036	1.033	1.036
0.35	1.042	1.041	1.039	1.042
0.4	1.047	1.047	1.043	1.046
0.45	1.052	1.051	1.049	1.051
0.5	1.056	1.055	1.054	1.055
0.55	1.059	1.058	1.058	1.059
0.6	1.061	1.061	1.062	1.062

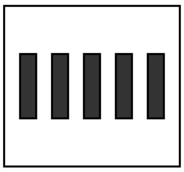
Table 3: ω_1/ω_{01} for a simply supported plate with voids, *M*, *N*=1,2,3,4 (type 1)



$bx_{m,n}/a = 0.1$
$by_{m,n}/b = 0.5$

h/h_0	ω_1/ω_{01} *M*, *N*=1	*M*, *N* =2	*M*, *N* =3	*M*, *N* =4
0	1	1	1	1
0.05	1.016	1.015	1.016	1.013
0.1	1.026	1.025	1.024	1.025
0.15	1.039	1.038	1.034	1.037
0.2	1.052	1.051	1.048	1.051
0.25	1.065	1.064	1.061	1.063
0.3	1.078	1.077	1.072	1.076
0.35	1.091	1.089	1.084	1.089
0.4	1.103	1.101	1.097	1.101
0.45	1.114	1.113	1.109	1.113
0.5	1.125	1.121	1.117	1.124
0.55	1.135	1.129	1.131	1.134
0.6	1.143	1.142	1.144	1.143

Table 4: ω_1/ω_{01} for a simply supported plate with voids, *M, N*=1,2,3,4 (type 1)



$bx_{m,n}/a = 0.15$
$by_{m,n}/b = 0.5$

h/h_0	ω_1/ω_{01}			
	M, N=1	*M, N* =2	*M, N* =3	*M, N* =4
0	1	1	1	1
0.05	1.015	1.015	1.015	1.017
0.1	1.038	1.038	1.034	1.038
0.15	1.059	1.058	1.056	1.057
0.2	1.081	1.079	1.075	1.076
0.25	1.101	1.111	1.097	1.111
0.3	1.125	1.119	1.112	1.121
0.35	1.147	1.144	1.138	1.141
0.4	1.169	1.164	1.158	1.161
0.45	1.192	1.189	1.176	1.178
0.5	1.215	1.209	1.201	1.202
0.55	1.237	1.234	1.225	1.221
0.6	1.256	1.254	1.246	1.241

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