Elementary Discussion of the Distribution of Prime Numbers

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Abstract: Propose The interval number distributed theorem. And Proof that the interval number distributed theorem. Then get the fundamental theorem of prime number distribution. This proved a new theorem of prime number. Given the detailed empirical calculation.

Key words: Prime number; Density; Theorem

1. INTRODUCTION

In 1849, in the letter to Encke the German mathematician Gauss wrote that, he found a large $x$ by examining the number of prime numbers in the period of 1000 adjacent integers from 1792 to 1793, primes average distribution density should be as follows [1–3]

$$
\frac{1}{\ln x}.
$$

(1)

which points [1–3,5,6]:

$$
Li(x) = \int_2^x \frac{1}{\ln u} du,
$$

(2)
The following equation can be obtained from (2):

\[ Li(x) = \frac{x}{\ln x} + \frac{x}{\ln^2 x} + \frac{2!}{\ln^3 x} + \cdots + \frac{(k-1)!}{\ln^k x}, \quad k \leq \ln x, \quad (3) \]

Here (3) has different forms. Mathematicians have proven \[ \pi(x) \sim Li(x), \quad (x \to \infty). \quad (4) \]

Equation (4) is called the prime number theorem. \( \pi(x) \) means the number of prime numbers less than \( x \).

In this paper, the interval prime number theorem is proposed and proved in order to prove the prime number theorem. Thus a new prime number theorem is proved.

2. THE REGIONAL DISTRIBUTION OF PRIME NUMBERS THEOREM

Set a large number \( x \), the prime number \( p \), the number of prime numbers is \( \pi(x, x/\lambda) \), by ignoring the remainder, table as an integer, get \[ \pi(x, x/\lambda) = s(x, x/\lambda), \quad s(x, x/\lambda) = \frac{x(\lambda - 1)}{\lambda \ln \lambda} \sum_{x/\lambda \leq p \leq x} \frac{1}{p}, \quad (5) \]

Equation (5) is called regional distribution of prime numbers theorem. For example:

Let \( x = 100000000, \ \lambda = 1.000048 \), from (5) we can get

\[ \pi(x, x/\lambda) = 259 + 0.00028, \]

Ignore 0.00028, the integer is 259, so \( \pi(x, x/\lambda) = 259 \).

3. PROOF REGIONAL DISTRIBUTION OF PRIME NUMBERS THEOREM

Proof. Set a large number \( x \), there is only one prime number \( p \) from \( x \) to the interval \( x/\lambda \), label the integer by ignoring the remainder. apparently:

\[ \frac{x - x/\lambda}{p \ln \lambda} = 1, \]

For example:

Let \( x = 108, \ \lambda = 1.018, \ x/\lambda = 106.09037328, \ p = 107 \). Calculate:

\[ 1 + 0.0003957, \]

Ignore 0.0003957, the integer is 1.

Generally speaking, get a prime number \( p \) from \( x \) to the interval \( x/\lambda \), ignore the remainder, table as an integer, get:

\[ \frac{x - x/\lambda}{\pi(x, x/\lambda) \ln \lambda} \sum_{x/\lambda \leq p \leq x} \frac{1}{p} = 1, \quad (6) \]
For example:
Let \( x = 100000000, \lambda = 1.000048 \), by (6) Calculate:

\[
1 + 0.0000010810530266,
\]

Ignore 0.0000010810530266, the integer is 1.

By (6), the following can be obtained:

\[
\pi(x, x/\lambda) = \frac{x(\lambda - 1)}{\lambda \ln \lambda} \sum_{x/\lambda \leq p \leq x} \frac{1}{p},
\]

Then (5) is proved.

\[\square\]

4. THE DISTRIBUTION OF PRIME NUMBERS FUNDAMENTAL THEOREM

Set prime number \( \pi(x, x^{1/2}) \) in the range from \( x \) to \( x^{1/2} \)

\[
\pi(x, x^{1/2}) = \sum_{n=1}^{a/2} \pi(x\lambda^{1-n}, x\lambda^{-n}),
\]

So that:

\[
\pi(x) = \pi(x, x^{1/2}) + \pi(x^{1/2}) = \sum_{n=1}^{a/2} \pi(x\lambda^{1-n}, x\lambda^{-n}) + \pi(x^{1/2}),
\]

(7)

By setting the real numbers from (5), we can get:

\[
\pi(x) = s(x, x^{1/2}) + \pi(x^{1/2}),
\]

(8)

\[
s(x, x^{1/2}) = \frac{x(\lambda - 1)}{\ln \lambda} \sum_{n=1}^{a/2} \frac{1}{\lambda^n} \sum_{x\lambda^{-n} \leq p \leq x\lambda^{1-n}} \frac{1}{p}, \quad a = \lfloor x^{1/2} \rfloor, \quad \lambda = x^{1/a},
\]

Equation (8) is called the distribution of prime numbers Fundamental Theorem.

For example:
Let \( x = 256, a = 256^{1/2} = 16, \lambda = 256^{1/16} = 1.41421356, \) by (8):

\[
\frac{x(\lambda - 1)}{\ln \lambda} = 305.9629,
\]

\[
\sum_{n=1}^{8} \frac{1}{\lambda^n} \sum_{x\lambda^{-n} \leq p \leq x\lambda^{1-n}} \frac{1}{p} = \frac{1}{\lambda} \sum_{x/\lambda \leq p \leq x} \frac{1}{p} + \frac{1}{\lambda^2} \sum_{x/\lambda^2 \leq p \leq x/\lambda} \frac{1}{p} + \cdots + \frac{1}{\lambda^8} \sum_{x/\lambda^8 \leq p \leq x/\lambda^7} \frac{1}{p}
\]

= 0.0389781 + 0.0354226 + 0.0230385 + 0.0196627 + 0.0129908 + 0.0093341 + 0.0097421 + 0.0069659

= 0.1561
So that \( s(256, 16) + \pi(16) = 305.9629 \cdot 0.1561 + 6 = 54 - 0.2391931 \).
Ignore 0.2391931, the integer is 54, the actual \( \pi(256) = 54 \).

Set a large number \( x, a = x^{1/2}, \lambda = x^{1/\alpha} \), by (8):

\[
\begin{align*}
x & \quad \pi(x) \quad \quad s(x, x^{1/2}) + \pi(x^{1/2}) \\
10^8, & \quad 5761455, \quad 5761455 \\
10^9, & \quad 50847534, \quad 50847534 \\
10^{10}, & \quad 455052511, \quad 455052511 \\
10^{11}, & \quad 4118054813, \quad 4118054813 \\
10^{12}, & \quad 37607912018, \quad 37607912018 \\
10^{13}, & \quad 346065536839, \quad 346065536839 \\
10^{14}, & \quad 3204941750802, \quad 3204941750802 \\
10^{15}, & \quad 29844570422669, \quad 29844570422669 \\
10^{16}, & \quad 279238341033925, \quad 279238341033925
\end{align*}
\]

5. THE FUNDAMENTAL THEOREM OF TRANSFORMATION OF PRIME

From Equation (8), we can get:

\[
\pi(x) + \pi(x) - s(x^{1/2}) > s(x) - s(x^{1/2}) + \pi(x^{1/2}),
\]

so

\[
2\pi(x) > s(x),
\]

\[
2\pi(x^{1/2}) > s(x^{1/2}),
\]

From (8), we have

\[
s(x) - s(x^{1/2}) < \pi(x) < s(x) - s(x^{1/2}) + 2\pi(x^{1/2}),
\]

Combining (10) and (11), we can easily get

\[
s(x) - 2\pi(x^{1/2}) < \pi(x) < s(x) + 2\pi(x^{1/2}),
\]

and Equation (12) proves the prime number theorem.

6. MERTENS THEOREM

In 1874, mathematician Mertens had proven [5](p.11) [6](p.8)

\[
\sum_{p \leq x} \frac{1}{p} = \ln \ln x + A_1 + O \left( \frac{1}{\ln x} \right),
\]

where \( A_1 \) is constant. And Equation (13) is called Mertens Theorem.

Set \( \ln x \to \infty \), from (13) we can get

\[
\sum_{p \leq x} \frac{1}{p} = \ln \ln x + A_1,
\]

then

\[
\sum_{x^{\lambda - n} \leq p \leq x^{\lambda^{1-n}}} \frac{1}{p} = \ln \ln(x^{\lambda^{1-n}}) - \ln \ln(x^{\lambda^{-n}}) = \frac{\ln \lambda}{\ln x - n \ln \lambda},
\]
By using Equation (15), the following can be obtained from Equation (8):

\[ s(x) = x(\lambda - 1) \sum_{n=1}^{a} \frac{\lambda^{-n}}{\ln x - n \ln \lambda}, a = \left\lfloor \frac{\ln x - 1/2}{\ln \lambda} \right\rfloor, \lambda = x^{2/x}, \quad (16) \]

In the integral sense (16) and (2) are the same, so:

\[ \text{Li}(x) = s(x), \quad (17) \]

For example:
Let \( x = 10000000, \lambda = 1.0000003684136828, a = 48642829, \) then
By (16), Calculate: \( s(100000000) = 5762209, \)
By (3), Calculate: \( \text{Li}(100000000) = 5762209. \)

7. PROVED A PRIME NUMBER THEOREM

By (12) and (17), we can get

\[ \text{Li}(x) - 2\pi(x^{1/2}) < \pi(x) < \text{Li}(x) + 2\pi(x^{1/2}), \quad (x \to \infty). \]

\[ \pi(x) = \text{Li}(x) + o(x^{1/2}), \quad (x \to \infty). \quad (18) \]

This Equation (18) is a new prime number theorem.

8. DISCUSSION

From the course of distribution of primes, ever is not found with \( \pi(x) \) strictly equal theorem. Even with \( \pi(x) \) strictly equivalent to the conjecture has not. To discover and prove a \( \pi(x) \) equivalent theorem is not easy.

It can be said that this is a new way of prime number distribution. Along this way, not only many complicated problems such as the distribution of prime numbers, the Jie Bov conjecture, the Riemann conjecture has to improve, but also opens a wide field of number theory.

REFERENCES