CSCanada
Progress in Applied Mathematics
Vol. 5, No. 2, 2013, pp. [6–10]
DOI: 10.3968/j.pam.1925252820130502.1894

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
www.cscanada.org

Elementary Discussion of the Distribution of Prime Numbers

LIU Dan[a],*

[a] Department of Mathematics, Neijiang Normal University, Neijiang, Sichuan, China.

* Corresponding author.
Address: Department of Mathematics, Neijiang Normal University, Neijiang 641003, China; E-Mail: zxc_576672568@qq.com

Received: August 24, 2012/ Accepted: November 8, 2012/ Published: April 30, 2013

Abstract: Propose The interval number distributed theorem. And Proof that the interval number distributed theorem.Then get the fundamental theorem of prime number distribution. This proved a new theorem of prime number. Given the detailed empirical calculation.

Key words: Prime number; Density; Theorem

Liu, D. (2013). Elementary Discussion of the Distribution of Prime Numbers. *Progress in Applied Mathematics, 5*(2), 6–10. Available from http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820130502.1894 DOI: 10.3968/j.pam.1925252820130502.1894

1. INTRODUCTION

In 1849, in the letter to Encke the German mathematician Gauss wrote that, he found a large x by examining the number of prime numbers in the period of 1000 adjacent integers from 1792 to 1793, primes average distribution density should be as follows [1–3]

$$\frac{1}{\ln x}, \tag{1}$$

which points [1–3,5,6]:

$$Li(x) = \int_2^x \frac{1}{\ln u} du, \tag{2}$$

The following equation can be obtained from (2):

$$Li(x)=\frac{x}{\ln x}+\frac{x}{\ln^2 x}+\frac{2!x}{\ln^3 x}+\cdots\cdots+\frac{(k-1)!x}{\ln^k x},\quad k\le \ln x, \tag{3}$$

Here (3) has different forms.

Mathematicians have proven [4]

$$\pi(x)\sim Li(x),\quad (x\to\infty). \tag{4}$$

Equation (4) is called *the prime number theorem*. $\pi(x)$ means the number of prime numbers less than x.

In this paper, the interval prime number theorem is proposed and proved in order to prove the prime number theorem. Thus a new prime number theorem is proved.

2. THE REGIONAL DISTRIBUTION OF PRIME NUMBER-S THEOREM

Set a large number x, the prime number p, the number of prime numbers is $\pi(x,x/\lambda)$, by ignoring the remainder, table as an integer, get [7](p.112) [8](p.538):

$$\pi(x,x/\lambda)=s(x,x/\lambda),\quad s(x,x/\lambda)=\frac{x(\lambda-1)}{\lambda\ln\lambda}\sum_{x/\lambda\le p\le x}\frac{1}{p}, \tag{5}$$

Equation (5) is called *regional distribution of prime numbers theorem*.

For example:

Let $x=100000000,\ \lambda=1.000048$, from (5) we can get

$$\pi(x,x/\lambda)=259+0.00028,$$

Ignore 0.00028, the integer is 259, so $\pi(x,x/\lambda)=259$.

3. PROOF REGIONAL DISTRIBUTION OF PRIME NUMBERS THEOREM

Proof. Set a large number x, there is only one prime number p from x to the interval x/λ, label the integer by ignoring the remainder. apparently:

$$\frac{x-x/\lambda}{p\ln\lambda}=1,$$

For example:

Let $x=108,\ \lambda=1.018,\ x/\lambda=106.09037328,\ p=107$. Calculate:

$$1+0.0003957,$$

Ignore 0.0003957, the integer is 1.

Generally speaking, get a prime number p from x to the interval x/λ, ignore the remainder, table as an integer, get:

$$\frac{x-x/\lambda}{\pi(x,x/\lambda)\ln\lambda}\sum_{x/\lambda\leqslant p\leqslant x}\frac{1}{p}=1, \tag{6}$$

For example:
Let $x = 100000000$, $\lambda = 1.000048$, by (6) Calculate:

$$1 + 0.0000010810530266,$$

Ignore 0.0000010810530266, the integer is 1.
By (6), the following can be obtained:

$$\pi(x, x/\lambda) = \frac{x(\lambda-1)}{\lambda\ln\lambda}\sum_{x/\lambda\leqslant p\leqslant x}\frac{1}{p},$$

Then (5) is proved. □

4. THE DISTRIBUTION OF PRIME NUMBERS FUNDAMENTAL THEOREM

Set prime number $\pi(x, x^{1/2})$ in the range from x to $x^{1/2}$

$$\pi(x, x^{1/2}) = \sum_{n=1}^{a/2}\pi(x\lambda^{1-n}, x\lambda^{-n}),$$

So that:

$$\pi(x) = \pi(x, x^{1/2}) + \pi(x^{1/2}) = \sum_{n=1}^{a/2}\pi(x\lambda^{1-n}, x\lambda^{-n}) + \pi(x^{1/2}), \tag{7}$$

By setting the real numbers from (5), we can get:

$$\pi(x) = s(x, x^{1/2}) + \pi(x^{1/2}), \tag{8}$$

$$s(x, x^{1/2}) = \frac{x(\lambda-1)}{\ln\lambda}\sum_{n=1}^{a/2}\frac{1}{\lambda^n}\sum_{x\lambda^{-n}\leq p\leq x\lambda^{1-n}}\frac{1}{p},\quad a=[x^{1/2}],\quad \lambda = x^{1/a},$$

Equation (8) is called *the distribution of prime numbers Fundamental Theorem.*
For example:
Let $x = 256$, $a = 256^{1/2} = 16$, $\lambda = 256^{1/16} = 1.41421356$, by (8):

$$\frac{x(\lambda-1)}{\ln\lambda} = 305.9629,$$

$$\begin{aligned}
&\sum_{n=1}^{8}\frac{1}{\lambda^n}\sum_{x\lambda^{-n}\leqslant p\leqslant x\lambda^{1-n}}\frac{1}{p}\\
=&\frac{1}{\lambda}\sum_{x/\lambda\leqslant p\leqslant x}\frac{1}{p} + \frac{1}{\lambda^2}\sum_{x/\lambda^2\leqslant p\leqslant x/\lambda}\frac{1}{p} + \cdots\cdots + \frac{1}{\lambda^8}\sum_{x/\lambda^8\leqslant p\leqslant x/\lambda^7}\frac{1}{p}\\
=&0.0389781 + 0.0354226 + 0.0230385 + 0.0196627\\
&+ 0.0129908 + 0.0093341 + 0.0097421 + 0.0069659\\
=&0.1561
\end{aligned}$$

So that $s(256, 16) + \pi(16) = 305.9629 \cdot 0.1561 + 6 = 54 - 0.23919131$.
Ignore 0.23919131, the integer is 54, the actual $\pi(256) = 54$.
Set a large number x, $a = x^{1/2}$, $\lambda = x^{1/a}$, by (8):

x	$\pi(x)$	$s(x, x^{1/2}) + \pi(x^{1/2})$
10^8,	5761455,	5761455,
10^9,	50847534,	50847534,
10^{10},	455052511,	455052511,
10^{11},	4118054813,	4118054813,
10^{12},	37607912018,	37607912018,
10^{13},	346065536839,	346065536839,
10^{14},	3204941750802,	3204941750802,
10^{15},	29844570422669,	29844570422669,
10^{16},	279238341033925,	279238341033925.

5. THE FUNDAMENTAL THEOREM OF TRANSFORMATION OF PRIME

From Equation (8), we can get:

$$\pi(x) + \pi(x) - s(x^{1/2}) > s(x) - s(x^{1/2}) + \pi(x^{1/2}), \tag{9}$$

so

$$2\pi(x) > s(x),$$

$$2\pi(x^{1/2}) > s(x^{1/2}), \tag{10}$$

From (8), we have

$$s(x) - s(x^{1/2}) < \pi(x) < s(x) - s(x^{1/2}) + 2\pi(x^{1/2}), \tag{11}$$

Combining (10) and (11), we can easily get

$$s(x) - 2\pi(x^{1/2}) < \pi(x) < s(x) + 2\pi(x^{1/2}), \tag{12}$$

and Equation (12) proves the prime number theorem.

6. MERTENS THEOREM

In 1874, mathematician Mertens had proven [5](p.11) [6](p.8)

$$\sum_{p \leqslant x} \frac{1}{p} = \ln \ln x + A_1 + O\left(\frac{1}{\ln x}\right), \tag{13}$$

where A_1 is constant. And Equation (13) is called *Mertens Theorem*.

Set $\ln x \to \infty$, from (13) we can get

$$\sum_{p \leq x} \frac{1}{p} = \ln \ln x + A_1, \tag{14}$$

then

$$\sum_{x\lambda^{-n} \leq p \leqslant x\lambda^{1-n}} \frac{1}{p} = \ln \ln(x\lambda^{1-n}) - \ln \ln(x\lambda^{-n}) = \frac{\ln \lambda}{\ln x - n \ln \lambda}, \tag{15}$$

By using Equation (15), the following can be obtained from Equation (8):

$$s(x) = x(\lambda - 1)\sum_{n=1}^{a} \frac{\lambda^{-n}}{\ln x - n\ln\lambda}, a = [\frac{\ln x - 1/2}{\ln\lambda}], \lambda = x^{2/x}, \tag{16}$$

In the integral sense (16) and (2) are the same, so:

$$Li(x) = s(x), \tag{17}$$

For example:
Let $x = 100000000$, $\lambda = 1.0000003684136828$, $a = 48642829$, then
By (16), Calculate: $s(100000000) = 5762209$,
By (3), Calculate: $Li(100000000) = 5762209$.

7. PROVED A PRIME NUMBER THEOREM

By (12) and (17), we can get

$$\begin{aligned} &Li(x) - 2\pi(x^{1/2}) < \pi(x) < Li(x) + 2\pi(x^{1/2}), \quad (x \to \infty). \\ &\pi(x) = Li(x) + o(x^{1/2}), \quad (x \to \infty). \end{aligned} \tag{18}$$

This Equation (18) is a new prime number theorem.

8. DISCUSSION

From the course of distribution of primes, ever is not found with $\pi(x)$ strictly equal theorem. Even with $\pi(x)$ strictly equivalent to the conjecture has not. To discover and prove a $\pi(x)$ equivalent theorem is not easy.

It can be said that this is a new way of prime number distribution. Along this way, not only many complicated problems such as the distribution of prime numbers, the Jie Bov conjecture, the Riemann conjecture has to improve, but also opens a wide field of number theory.

REFERENCES

[1] Manin, Yu. I., *et al.* (2006). *Introduction to modern number theory.* Beijing: Science Press.

[2] Hua, Loo-Keng. (1979). *An introduction to number theory* (In Chinese). Beijing: Science Press.

[3] Neukirch, Jurgen. (2007). *Algebraic number theory.* Beijing: Science Press.

[4] Hua, Loo-Keng, & Wang, Yuan. (1963). *Numerical integration and its application.* Beijing: Science Press.

[5] Pan, Cheng-Dong, & Pan, Cheng-Biao. (1988). *The elementary proof of prime number theorem* (In Chinese). Shanghai: Shanghai Science and Technology Press.

[6] Lu, Changhai. (2004). *The Riemann hypothesis* (In Chinese). Beijing: Tsinghua University Press.

[7] Liu, Dan. (2011). Distribution of prime numbers iterated function. *Journal of Mathematics Research, 3*(4), 112-117.

[8] Liu, Dan. (2012). Conversion of the Riemann prime number formula. *South Asian Journal of Mathematics, 2*(5), 535-541.