CSCanada
Progress in Applied Mathematics
Vol. 5, No. 1, 2013, pp. [59–63]
DOI: 10.3968/j.pam.1925252820130501.1132

ISSN 1925-251X [Print]
ISSN 1925-2528 [Online]
www.cscanada.net
www.cscanada.org

Some Properties of a Class of Wiener Difference Processes and Wavelet Express

XIA Xuewen[a],[b],*

[a]Hunan Institute of Engineering, Xiangtan, China.
[b]Hunan Normal University, Changsha, China.

* Corresponding author.
Address: Hunan Institute of Engineering, Xiangtan 411104, China; Hunan Normal University, Changsha 410082, China; E-Mail: xxw1234567@163.com

Supported by Hunan Provinial Natural Science Foundation of China (12JJ9004); Scientific Research Fund of Hunan Provincial Education Department (12A033).

Received: February 6, 2012/ Accepted: July 23, 2012/ Published: January 31, 2013

Abstract: Recently, some researchers have studied wavelet problems of stochastic processes or stochastic system by using wavelet. In this paper, we take wavelet and use it in a series expansion of signals or functions. Wavelet has its energy concentration in time to give a tool for the analysis of transient and nonstationary and time-varying phenomena. Wavelets have contributed to this already intensely developed and rapidly advancing field. The study of Wiener difference processes stochastic system is very important in theory and application. In this paper, Wiener difference processes are studied by using wavelet analysis, and some properties and wavelet express are obtained.

Key words: Stochastics processes; Wiener difference processes; Wavelet analysis

Xia, X. (2013). Some Properties of a Class of Wiener Difference Processes and Wavelet Express. *Progress in Applied Mathematics, 5*(1), 59–63. Available from http://www.cscanada.net/index.php/pam/article/view/j.pam.1925252820130501.1132 DOI: 10.3968/j.pam.1925252820130501.1132

1. INTRODUCTION

Wavelet analysis is a remarkable tool for analyzing function of one or several variables that appear in mathematics or in signal and image processing. With hindsight

the wavelet transform can be viewed as diverse as mathematics, physics and electrical engineering. The basic idea is to use a family of building blocks to represent the object at hand in an efficient and insightful way, the building blocks come in different sizes, and are suitable for describing features with a resolution commensurate with their sizes. Recently some persons have studied wavelet problems of stochastic process or stochastic system [1–10]. In this paper, we study random processes by using wavelet analysis methods. Wiener difference processes are important processes.

Definition 1 Let $\{w(t), t \geq 0\}$ is one of σ^2-Wiener processes, $a > 0$.

$$x(t) = w(a+t) - w(t), \tag{1}$$

We call $x(t)$ is Wiener difference processes.
We have $E(x(t)) = 0$.

$$\begin{aligned} & E\left(x(s)x(t)\right) \\ =& E\left(w(a+s)w(a+t) - w(a+s)w(t) - w(s)w(a+t) + w(s)w(t)\right) \\ =& \left[\min(a+s, a+t) - \min(a+s, t) - \min(s, a+t) + \min(s,t)\right]\sigma^2 \\ =& \begin{cases} 0, & a \leq |t-s| \\ (a - |t-s|)\sigma^2, & |t-s| < a \end{cases} \end{aligned} \tag{2}$$

If $t > s$, then

$$E\left(x(s)x(t)\right) = \begin{cases} 0, & a \leq t-s \\ (a-t+s)\sigma^2, & a > t-s \end{cases} \tag{3}$$

Definition 2 Let $\{x(t), t \in R\}$ is a stochastic processes, $E(x(t))^2 < +\infty$, then

$$w(s,x) = \frac{1}{s}\int_R x(t)\psi\left(\frac{x-t}{s}\right)dt \tag{4}$$

is wavelet change of $x(t)$, where ψ is mather wavelet.

Haar wavelet is

$$\psi(x) = \begin{cases} 1, & 0 \leq x < 1/2 \\ -1, & 1/2 \leq x < 1 \\ 0, & \text{other} \end{cases} \tag{5}$$

2. SOME PROPERTIES

By Equation (4), we have

$$w(s, x+\tau) = \frac{1}{s}\int_R x(t)\psi\left(\frac{x+\tau-t}{s}\right)dt \tag{6}$$

then

$$\begin{aligned} & R(\tau) = E\left(w(s,x)w(s,x+\tau)\right) \\ =& E\left(\frac{1}{s}\int_R x(t)\psi\left(\frac{x-t}{s}\right)dt\frac{1}{s}\int_R x(t)\psi\left(\frac{x+\tau-t}{s}\right)dt_1\right) \\ =& \frac{1}{s^2}\iint E\left(x(t)x(t_1)\right)\psi\left(\frac{x-t}{s}\right)\psi\left(\frac{x+\tau-t}{s}\right)dtdt_1 \end{aligned} \tag{7}$$

We have the following by Equation (5):

$$\psi\left(\frac{x-t}{s}\right) = \begin{cases} 1, & x-s/2 \le t < x \\ -1, & x-s \le t < x-s/2 \end{cases}$$

$$\psi\left(\frac{x+\tau-t}{s}\right) = \begin{cases} 1, & x+\tau-s/2 \le t < x+\tau \\ -1, & x+\tau-s \le t < x+\tau-s/2 \end{cases}$$

then,

$$\begin{aligned} R(\tau) =& \frac{1}{s^2}\iint_{u-t>a}(a-t-u)\sigma^2\psi\left(\frac{x-t}{s}\right)\psi\left(\frac{x+\tau-u}{s}\right)dtdu \\ =& \frac{\sigma^2}{s^2}\left(\int_{x-s/2}^{x}(a-t+u)dt\int_{t+a}^{x+\tau}du - \int_{x-s/2}^{x}(a-t+u)dt\int_{t+a}^{x+\tau-s/2}du\right. \\ & \left. - \int_{x-s}^{x-s/2}(a-t+u)dt\int_{t+a}^{x+\tau}du + \int_{x-s}^{x-s/2}(a-t+u)dt\int_{t+a}^{x+\tau-s/2}du\right) \\ =& I1 + I2 + I3 + I4 \end{aligned}$$

where

$$\begin{aligned} I1 &= 1/s^2\left(\int_{x-s/2}^{x}dt\int_{t+a}^{x+\tau}(a-t+u)du\right) \\ &= \frac{1}{s^2}\int_{x-s/2}^{x}(a-t)(x+\tau-t-a) + \frac{1}{2}(x+\tau)^2 - \frac{1}{2}(t+a)^2dt \end{aligned}$$

$$\begin{aligned} I2 &= -\frac{1}{s^2}\int_{x-s/2}^{x}(a-t+u)dt\int_{t+a}^{x+\tau-s/2}du \\ &= -\frac{1}{s^2}\int_{x-s/2}^{x}(a-t)(x+\tau-\frac{s}{2}) + \frac{1}{2}(x+\tau-s/2)^2 - (a-t)(t+a) - \frac{1}{2}(t+a)^2dt \end{aligned}$$

$$\begin{aligned} I3 &= -\frac{1}{s^2}\int_{x-s}^{x-s/2}dt\int_{t+a}^{x+\tau}(a-t+u)du \\ &= -\frac{1}{s^2}\int_{x-s}^{x-s/2}(a-t)(x+\tau) + \frac{1}{2}(x+\tau)^2 - (a-t)(t+a) - \frac{1}{2}(t+a)^2dt \end{aligned}$$

$$\begin{aligned} I4 &= \frac{1}{s^2}\int_{x-s}^{x-s/2}dt\int_{t+a}^{x+\tau-s/2}(a-t+u)du \\ &= \frac{1}{s^2}\int_{x-s}^{x-s/2}(a-t)(x+\tau-s/2) + \frac{1}{2}(x+\tau-s/2) - (a-t)(t+a) - \frac{1}{2}(t+a)^2dt \end{aligned}$$

so that,

$$I1+I2=\frac{1}{s^2}\int_{x-s/2}^{x}(a-t)\frac{s}{2}+s(x+\tau)-\frac{s^2}{8}dt$$
$$I3+I4=\frac{1}{s^2}\int_{x-s}^{x-s/2}-\frac{s}{2}(a-t)-s(x+\tau)+\frac{s^2}{8}dt$$

and

$$\begin{aligned}R(\tau)&=\frac{1}{s^2}\int_{x-s}^{x}\left(-\frac{s}{2}(a-t)-s(x+\tau)+\frac{s^2}{8}\right)dt\\&=\frac{1}{s}\int_{x-s}^{x}\left(-\frac{1}{2}(a-t)-(x+\tau)+\frac{s}{8}\right)dt\\&=\frac{1}{s}\int_{x-s}^{x}\left(\frac{1}{2}t-\frac{1}{2}a-x-\tau+\frac{s}{8}\right)dt\\&=-8(4a+4x+s+8\tau)\end{aligned}$$

Apparently, $R'(\tau)=-64, R''(\tau)=0$.
So, the Zero density of $W(s,x)$ [10] is 0.

3. WAVELET EXPRESS

If let $\varphi(t)=\sqrt{2}\sum_k \varphi(2x-k)$, then wavelet express of $x(t)$ [2] can be written as

$$x(t)=2^{-\frac{J}{2}}\sum_{n\in Z}C_n^J\varphi(2^{-J}t-n)+\sum_{j\le J}2^{-\frac{j}{2}}\sum_{n\in Z}d_n^j\psi(2^{-j}t-n)$$

where

$$C_n^J=\int_R x(t)\varphi(2^{-J}t-n)dt,\quad d_n^j=\int_R x(t)\psi(2^{-j}t-n)dt.$$

Thus, the relational density of d_n^j is

$$\begin{aligned}E(d_n^j d_m^k)&=E\int_R x(t)\psi(2^{-j}t-n)dt\int_R x(s)\psi\left(2^{-k}s-m\right)ds\\&=\iint_{R^2}E\left(x(t)x(s)\right)\psi\left(2^{-j}t-n\right)\psi\left(2^{-k}s-m\right)dtds\\&=\iint_{|t-s|>a}\left(a-|t-s|\right)\sigma^2\psi\left(2^{-j}t-n\right)\psi\left(2^{-k}s-m\right)dtds\end{aligned}$$

REFERENCES

[1] Flandrin, P. (1992). Wavelet analysis and synthesis of fractional brownian motion. *IEEE Tran., On Information Theory, 38*(2), 910-916.

[2] Meyer, Y. (1990). *In Ondelettes et operatears.* Hermann.

[3] Bobrowski, A. (2007). *Functional analysis for probability and stochastic processes.* Cambridge University Press.

[4] Cambancs (1994). Wavelet approximation of deterministic and random signals. *IEEE Trans. on Information Theory, 40*(4), 1013-1029.

[5] Krim (1995). Multire solution analysis of a class nonstationary processes. *IEEE Trans. on Information Theory, 41*(4), 1010-1019.

[6] Ren, H. (2002). Wavelet estimation for jumps on a heterosedastic regression model. *Acta Mathematica Scientia, 22*(2), 269-277.

[7] Xia, X. (2005). Wavelet analysis of the stochastic system with coular stationary noise. *Engineering Science, 3*, 43-46.

[8] Xia, X. (2008). Wavelet density degree of continuous parameter AR model. *International Journal Nonlinear Science, 7*(4), 237-242.

[9] Xia, X. (2007). Wavelet analysis of browain motion. *World Journal of Modelling and Simulation*, (3), 106-112.

[10] Xia, X., & Dai, T. (2009). Wavelet density degree of a class of wiener processes. *International Journal of Nonlinear Science, 7*(3), 327-332.