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# Statistical Analysis of MOBVE Distribution with TFR Model Under Step-Stress Accelerated Life Test

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**Abstract:** We obtain the maximum likelihood estimates of parameters of MOBVE distribution with tampered failure rate model under step-stress accelerated life test. Thereafter we show the feasibility of this method by using the Monte-Carlo simulation.

**Key words:** MOBVE; Step-stress accelerated life test; Tampered failure rate model; Maximum likelihood estimate

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# 1. INTRODUCTION

Marshall and Olkin proposed a bivariate exponential distribution in 1967, which has the joint survival function

$$\overline{F}(x,y) = \exp\{-\lambda_1 x - \lambda_2 y - \lambda_{12}\max(x,y)\},\ x>0,\ y>0$$

And the corresponding joint density function is

$$f(x,y) = \begin{cases} \lambda_1(\lambda_2+\lambda_{12})\exp\{-\lambda_1 x - (\lambda_2+\lambda_{12})y\}, y>x>0 \\ \lambda_2(\lambda_1+\lambda_{12})\exp\{-(\lambda_1+\lambda_{12})x - \lambda_2 y\}, x>y>0 \\ \lambda_{12}\exp\{-(\lambda_1+\lambda_2+\lambda_{12})x\}, x=y>0 \end{cases}$$

where the parameters are $\lambda_1,\ \lambda_2,\ \lambda_{12} > 0$. This bivariate exponential distribution is usually called MOBVE distribution, which is denoted by $(X,Y) \sim MOBVE$ $(\lambda_1, \lambda_2, \lambda_{12})$. Assumed that the system consists of two components and is impacted by three independent impact sources controlled by Poisson process with the strength $\lambda_1,\ \lambda_2,\ \lambda_{12}$, the first two impact sources cause a component to fail and the third impact source causes two components to fail at the same time. So MOBVE distribution can be derived by this fatal shock model.

Previous researches mainly focused on the properties of MOBVE distribution [2–4] and parameter estimates under the constant stress situation [5]. However, there are few researches about the parameter estimates of MOBVE distribution in the changeable stress situation. In this paper, we discuss the step-stress accelerated life test of MOBVE distribution based on TFR model and obtain the maximum likelihood estimates of parameters. Thereafter, the example is illustrated to show the feasibility of this method by Monte-Carlo simulations.

# 2. PARAMETER ESTIMATES OF MOBVE DISTRIBUTION UNDER TFR MODEL

$n$ systems are randomly selected from a large number of systems with two components and put on the simple step-stress accelerated life test. The life of two components is $(X,Y) \sim MOBVE(\lambda_1, \lambda_2, \lambda_{12})$. Supposed that the stress is $S_1$ at the beginning of test and the stress is increasing from $S_1$ to $S_2$ at the time $t_1$, the failure rate function of TFR model [7] is

$$\lambda^*(t) = \begin{cases} \lambda(t) & t \le t_1 \\ \alpha\lambda(t) & t > t_1 \end{cases}$$

Supposed that the failure rate is increasing from $\lambda_1,\ \lambda_2,\ \lambda_{12}$ to $\alpha_1\lambda_1,\ \alpha_2\lambda_2,\ \alpha_{12}\lambda_{12}$, $\alpha_1,\ \alpha_2,\ \alpha_{12} > 1$ at the time $t_1$, where $\alpha_1,\ \alpha_2,\ \alpha_{12} > 1$ depend on $S_1$, $S_2$ and maybe depend on the time $t_1$, which is usually called tamper factor, we have

$$\overline{F}^*(t) = \begin{cases} \overline{F}(t) & t \le t_1 \\ \overline{F}(t_1)\left[\dfrac{\overline{F}(t)}{\overline{F}(t_1)}\right]^{\alpha} & t > t_1 \end{cases}$$

## *2.1. The Lives $X$, $Y$ of Two Components Are Both Observed*

Assumed that component $X$ has $r_1$ failure, component $Y$ has $r_2$ failure and system $Z = \min(X,Y)$ has $r$ failure at the time $t \le t_1$, the test is continuous under stress

$S_2$ until all units are failed. Under stress $S_1$, $r_1$ failure time of component $X$ are $X_1 \le X_2 \le ... \le X_{r_1}$; $r_2$ failure time of component $Y$ are $Y_1 \le Y_2 \le ... \le Y_{r_2}$; $r$ failure time of system are $Z_1 \le Z_2 \le ... \le Z_r$. Under stress $S_2$, $n - r_1$ failure time of component $X$ are $X_{r_1+1} \le X_{r_1+2} \le \cdots \le X_n$; $n - r_2$ failure time of component $Y$ are $Y_{r_2+1} \le Y_{r_2+2} \le \cdots \le Y_n$; $n - r$ failure time of system are $Z_{r+1} \le Z_{r+2} \le \cdots \le Z_n$.

Under stress $S_1$, we have $\overline{F_X}(x) = e^{-(\lambda_1+\lambda_{12})x}$, $\overline{F_Y}(y) = e^{-(\lambda_2+\lambda_{12})y}$, $\overline{F_Z}(z) = e^{-(\lambda_1+\lambda_2+\lambda_{12})z}$.

Under stress $S_2$, we have $\overline{F_X}(x) = e^{-(\alpha_1\lambda_1+\alpha_{12}\lambda_{12})x}$, $\overline{F_Y}(y) = e^{-(\alpha_2\lambda_2+\alpha_{12}\lambda_{12})y}$, $\overline{F}_Z(z) = e^{-(\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_{12}\lambda_{12})z}$.

The log-likelihood function of life of component $X$ is

$$\begin{aligned}\ln L_X &= r_1 \ln(\lambda_1+\lambda_{12}) + (n-r_1)\ln(\alpha_1\lambda_1+\alpha_{12}\lambda_{12}) \\ &- (\lambda_1+\lambda_{12})\sum_{j=1}^{r_1} x_j - (\alpha_1\lambda_1+\alpha_{12}\lambda_{12})\sum_{j=r_1+1}^{n} x_j \\ &- (\lambda_1+\lambda_{12}-\alpha_1\lambda_1-\alpha_{12}\lambda_{12})(n-r_1)t_1\end{aligned}$$

Setting $\lambda_1+\lambda_{12} = a_X$, $\alpha_1\lambda_1+\alpha_{12}\lambda_{12} = b_X$, $\frac{\partial \ln L_X}{\partial a_X} = 0$, $\frac{\partial \ln L_X}{\partial b_X} = 0$, the likelihood equation set is

$$\begin{cases} \dfrac{r_1}{a_X} - \sum\limits_{j=1}^{r_1} x_j - (n-r_1)t_1 = 0 \\ \dfrac{n-r_1}{b_X} - \sum\limits_{j=r_1+1}^{n} x_j + (n-r_1)t_1 = 0 \end{cases}$$

It can be solved that the maximum likelihood estimates of $a_X$, $b_X$ are

$$\begin{cases} \widehat{a_X} = \dfrac{r_1}{\sum\limits_{j=1}^{r_1} x_j + (n-r_1)t_1} \\ \widehat{b_X} = \dfrac{n-r_1}{\sum\limits_{j=r_1+1}^{n} x_j - (n-r_1)t_1} \end{cases}$$

The log-likelihood function of life of component $Y$ is

$$\begin{aligned}\ln L_Y &= r_2 \ln(\lambda_2+\lambda_{12}) + (n-r_2)\ln(\alpha_2\lambda_2+\alpha_{12}\lambda_{12}) \\ &- (\lambda_2+\lambda_{12})\sum_{j=1}^{r_2} y_j - (\alpha_2\lambda_2+\alpha_{12}\lambda_{12})\sum_{j=r_2+1}^{n} x_j \\ &- (\lambda_2+\lambda_{12}-\alpha_2\lambda_2-\alpha_{12}\lambda_{12})(n-r_2)t_1\end{aligned}$$

Setting $\lambda_2+\lambda_{12} = a_Y$, $\alpha_2\lambda_2+\alpha_{12}\lambda_{12} = b_Y$, $\frac{\partial \ln L_Y}{\partial a_Y} = 0$, $\frac{\partial \ln L_Y}{\partial b_Y} = 0$, the likelihood equation set is

$$\begin{cases} \dfrac{r_2}{a_Y} - \sum\limits_{j=1}^{r_2} y_j - (n-r_2)t_1 = 0 \\ \dfrac{n-r_2}{b_Y} - \sum\limits_{j=r_2+1}^{n} y_j + (n-r_2)t_1 = 0 \end{cases}$$

It can be solved that the maximum likelihood estimates of $a_Y,\ b_Y$ are

$$\begin{cases} \widehat{a_Y} = \dfrac{r_2}{\sum\limits_{j=1}^{r_2} y_j + (n-r_2)\,t_1} \\ \widehat{b_Y} = \dfrac{n-r_2}{\sum\limits_{j=r_2+1}^{n} y_j - (n-r_2)\,t_1} \end{cases}$$

The log-likelihood function of system life $Z$ is

$$\begin{aligned} &\ln L_Z = r\ln(\lambda_1+\lambda_2+\lambda_{12}) + (n-r)\ln(\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_{12}\lambda_{12}) \\ &-(\lambda_1+\lambda_2+\lambda_{12})\sum_{j=1}^{r} z_j - (\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_{12}\lambda_{12})\sum_{j=r+1}^{n} z_j \\ &-(\lambda_1+\lambda_2+\lambda_{12}-\alpha_1\lambda_1-\alpha_2\lambda_2-\alpha_{12}\lambda_{12})(n-r)\,t_1 \end{aligned}$$

Setting $\lambda_1+\lambda_2+\lambda_{12}=a_Z$, $\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_{12}\lambda_{12}=b_Z$, $\dfrac{\partial \ln L_Z}{\partial a_Z}=0$, $\dfrac{\partial \ln L_Z}{\partial b_Z}=0$, the likelihood equation set is

$$\begin{cases} \dfrac{r}{a_Z} - \sum\limits_{j=1}^{r} z_j - (n-r)\,t_1 = 0 \\ \dfrac{n-r}{b_Z} - \sum\limits_{j=r+1}^{n} z_j + (n-r)\,t_1 = 0 \end{cases}$$

It can be solved that the maximum likelihood estimates of $a_Z,\ b_Z$ are

$$\begin{cases} \widehat{a_Z} = \dfrac{r}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1} \\ \widehat{b_Z} = \dfrac{n-r}{\sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1} \end{cases}$$

Therefore, the maximum likelihood estimates of parameters $\lambda_1,\ \lambda_2,\ \lambda_{12},\ \alpha_1,\ \alpha_2,\ \alpha_{12}$ are

$$\widehat{\lambda_1} = \widehat{a_Z} - \widehat{a_Y} = \frac{r}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1} - \frac{r_2}{\sum\limits_{j=1}^{r_2} y_j + (n-r_2)\,t_1}$$

$$\widehat{\lambda_2} = \widehat{a_Z} - \widehat{a_X} = \frac{r}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1} - \frac{r_1}{\sum\limits_{j=1}^{r_1} x_j + (n-r_1)\,t_1}$$

$$\widehat{\lambda_{12}} = \widehat{a_X} + \widehat{a_Y} - \widehat{a_Z} = \frac{r_1}{\sum\limits_{j=1}^{r_1} x_j + (n-r_1)\,t_1} + \frac{r_2}{\sum\limits_{j=1}^{r_2} y_j + (n-r_2)\,t_1} - \frac{r}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}$$

$$\widehat{\alpha_1} = \frac{\widehat{b_Z} - \widehat{b_Y}}{\widehat{\lambda_1}} = \frac{\frac{n-r}{\sum_{j=r+1}^{n} z_j - (n-r)t_1} - \frac{n-r_2}{\sum_{j=r_2+1}^{n} y_j - (n-r_2)t_1}}{\frac{r}{\sum_{j=1}^{r} z_j + (n-r)t_1} - \frac{r_2}{\sum_{j=1}^{r_2} y_j + (n-r_2)t_1}}$$

$$\widehat{\alpha_2} = \frac{\widehat{b_Z} - \widehat{b_X}}{\widehat{\lambda_2}} = \frac{\frac{n-r}{\sum\limits_{j=r+1}^{n} z_j-(n-r)t_1} - \frac{n-r_1}{\sum\limits_{j=r_1+1}^{n} x_j-(n-r_1)t_1}}{\frac{r}{\sum\limits_{j=1}^{r} z_j+(n-r)t_1} - \frac{r_1}{\sum\limits_{j=1}^{r_1} x_j+(n-r_1)t_1}}$$

$$\widehat{\alpha_{12}} = \frac{\widehat{b_X} + \widehat{b_Y} - \widehat{b_Z}}{\widehat{\lambda_{12}}} = \frac{\frac{n-r_1}{\sum\limits_{j=r_1+1}^{n} x_j-(n-r_1)t_1} + \frac{n-r_2}{\sum\limits_{j=r_2+1}^{n} y_j-(n-r_2)t_1} - \frac{n-r}{\sum\limits_{j=r+1}^{n} z_j-(n-r)t_1}}{\frac{r_1}{\sum\limits_{j=1}^{r_1} x_j+(n-r_1)t_1} + \frac{r_2}{\sum\limits_{j=1}^{r_2} y_j+(n-r_2)t_1} - \frac{r}{\sum\limits_{j=1}^{r} z_j+(n-r)t_1}}$$

### *2.2. $Z = \min(X,\ Y)$ and Which Component Is Failed Are Observed*

Assumed that system $Z = \min(X, Y)$ has $r$ failure at the time $t \leq t_1$, the test is continuous under stress $S_2$ until all systems are failed. Under stress $S_1$, $r$ failure time of system are $Z_1 \leq Z_2 \leq ... \leq Z_r$; Under stress $S_2$, $n-r$ failure time of system are $Z_{r+1} \leq Z_{r+2} \leq \cdots \leq Z_n$. The lives of two components are $(X_j, Y_j)$ corresponding to $Z_j$, and it is denoted that

$$I_j = \begin{cases} 1, & Z_j = X_j < Y_j \\ 2, & Z_j = Y_j < X_j \\ 3, & Z_j = X_j = Y_j \end{cases}, \quad j = 1, 2, ..., n.$$
$$P(I = i) = p_i, \quad i = 1, 2, 3,$$

$p_i f_i(z)$ is the joint density of $(Z, I)$.

It is also denoted that

$$W_1 = \begin{cases} 1, & X < Y \\ 0, & \text{others} \end{cases}, \quad W_2 = \begin{cases} 1, & X > Y \\ 0, & \text{others} \end{cases}, \quad W_3 = \begin{cases} 1, & X = Y \\ 0, & \text{others} \end{cases},$$
$$W_{1j} = \begin{cases} 1, & X_j < Y_j \\ 0, & \text{others} \end{cases}, \quad W_{2j} = \begin{cases} 1, & X_j > Y_j \\ 0, & \text{others} \end{cases}, \quad W_{3j} = \begin{cases} 1, & X_j = Y_j \\ 0, & \text{others} \end{cases},$$
$$j = 1, 2, \cdots, n.$$

Under stress $S_1$, the joint density of $(Z, I)$ [5] is

$$p_i f_i(z) = \begin{cases} \lambda_1 e^{-\lambda z}, & i = 1, z > 0 \\ \lambda_2 e^{-\lambda z}, & i = 2, z > 0 \\ \lambda_{12} e^{-\lambda z}, & i = 3, z > 0 \end{cases},$$

where $\lambda = \lambda_1 + \lambda_2 + \lambda_{12}$.

Under stress $S_2$, the joint density of $(Z, I)$ is

$$p_i f_i(z) = \begin{cases} \gamma_1 e^{-\gamma z}, & i = 1, z > 0 \\ \gamma_2 e^{-\gamma z}, & i = 2, z > 0 \\ \gamma_{12} e^{-\gamma z}, & i = 3, z > 0 \end{cases},$$

where $\gamma_1 = \alpha_1\lambda_1,\ \gamma_2 = \alpha_2\lambda_2,\ \gamma_{12} = \alpha_{12}\lambda_{12},\ \gamma = \gamma_1 + \gamma_2 + \gamma_{12}$.

The log-likelihood function is

$$\begin{aligned} \ln L = & \sum_{j=1}^{r} W_{1j} \bullet \ln\lambda_1 + \sum_{j=1}^{r} W_{2j} \bullet \ln\lambda_2 + \sum_{j=1}^{r} W_{3j} \bullet \ln\lambda_{12} \\ & -\lambda \sum_{j=1}^{r} z_j - (n-r)\lambda t_1 + \sum_{j=r+1}^{n} W_{1j} \bullet \ln\gamma_1 + \sum_{j=r+1}^{n} W_{2j} \bullet \ln\gamma_2 \\ & + \sum_{j=r+1}^{n} W_{3j} \bullet \ln\gamma_{12} - \gamma \sum_{j=r+1}^{n} z_j + (n-r)\gamma t_1 \end{aligned}$$

Setting $\dfrac{\partial \ln L}{\partial \lambda_1} = 0$, $\dfrac{\partial \ln L}{\partial \lambda_2} = 0$, $\dfrac{\partial \ln L}{\partial \lambda_{12}} = 0$, $\dfrac{\partial \ln L}{\partial \gamma_1} = 0$, $\dfrac{\partial \ln L}{\partial \gamma_2} = 0$, $\dfrac{\partial \ln L}{\partial \gamma_{12}} = 0$, the likelihood equation set is

$$\begin{cases} \dfrac{\sum\limits_{j=1}^{r} W_{1j}}{\lambda_1} - \sum\limits_{j=1}^{r} z_j - (n-r)\,t_1 = 0; \; \dfrac{\sum\limits_{j=1}^{r} W_{2j}}{\lambda_2} - \sum\limits_{j=1}^{r} z_j - (n-r)\,t_1 = 0; \\ \dfrac{\sum\limits_{j=1}^{r} W_{3j}}{\lambda_{12}} - \sum\limits_{j=1}^{r} z_j - (n-r)\,t_1 = 0; \; \dfrac{\sum\limits_{j=r+1}^{n} W_{1j}}{\gamma_1} - \sum\limits_{j=r+1}^{n} z_j + (n-r)\,t_1 = 0; \\ \dfrac{\sum\limits_{j=r+1}^{n} W_{2j}}{\gamma_2} - \sum\limits_{j=r+1}^{n} z_j + (n-r)\,t_1 = 0; \; \dfrac{\sum\limits_{j=r+1}^{n} W_{3j}}{\gamma_{12}} - \sum\limits_{j=r+1}^{n} z_j + (n-r)\,t_1 = 0. \end{cases}$$

It can be solved that the maximum likelihood estimates of $\lambda_1,\ \lambda_2,\ \lambda_{12}, \alpha_1,\ \alpha_2,\ \alpha_{12}$ are

$$\widehat{\lambda_1} = \frac{\sum\limits_{j=1}^{r} W_{1j}}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}; \quad \widehat{\gamma_1} = \frac{\sum\limits_{j=r+1}^{n} W_{1j}}{\sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1};$$

$$\widehat{\alpha_1} = \frac{\sum\limits_{j=r+1}^{n} W_{1j} \bullet \sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}{\sum\limits_{j=1}^{r} W_{1j} \bullet \sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1};$$

$$\widehat{\lambda_2} = \frac{\sum\limits_{j=1}^{r} W_{2j}}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}; \quad \widehat{\gamma_2} = \frac{\sum\limits_{j=r+1}^{n} W_{2j}}{\sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1};$$

$$\widehat{\alpha_2} = \frac{\sum\limits_{j=r+1}^{n} W_{2j} \bullet \sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}{\sum\limits_{j=1}^{r} W_{2j} \bullet \sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1};$$

$$\widehat{\lambda_{12}} = \frac{\sum\limits_{j=1}^{r} W_{3j}}{\sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}; \quad \widehat{\gamma_{12}} = \frac{\sum\limits_{j=r+1}^{n} W_{3j}}{\sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1};$$

$$\widehat{\alpha_{12}} = \frac{\sum\limits_{j=r+1}^{n} W_{3j} \bullet \sum\limits_{j=1}^{r} z_j + (n-r)\,t_1}{\sum\limits_{j=1}^{r} W_{3j} \bullet \sum\limits_{j=r+1}^{n} z_j - (n-r)\,t_1}$$

## 3. SIMULATION CALCULATION

Under simple step-stress accelerated life test, the stress is increasing from $S_1$ to $S_2$ at the time $t_1 = 5$. Let tampered factors be $\alpha_1 = 10$, $\alpha_2 = 15$, $\alpha_{12} = 5$ and sample sizes be $n = 15, 20, 25, 30, 40, 50$. The mean and mean square error (MSE) of maximum likelihood estimates are calculated under two situations by 10000 Monte-Carlo simulations. The results are list on Table 1 and Table 2. From the calculation results, we find that the mean is close to the true value of parameter and the mean square error is under the acceptable range, which shows that the given estimate method is meaningful.

**Table 1**
**The Lives of Two Components Are Both Observed**

| $n$ | $\lambda_1$ | $\lambda_2$ | $\lambda_{12}$ | $\widehat{\lambda_1}$ mean | $\widehat{\lambda_1}$ MSE | $\widehat{\lambda_2}$ mean | $\widehat{\lambda_2}$ MSE | $\widehat{\lambda_{12}}$ mean | $\widehat{\lambda_{12}}$ MSE |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 0.06 | 0.08 | 0.1 | 0.0642 | 0.0023 | 0.0852 | 0.003 | 0.1038 | 0.0026 |
| | 0.05 | 0.07 | 0.09 | 0.0531 | 0.0017 | 0.0739 | 0.0023 | 0.0933 | 0.0022 |
| | 0.04 | 0.06 | 0.08 | 0.0427 | 0.0013 | 0.0623 | 0.0018 | 0.0829 | 0.0017 |
| | 0.03 | 0.05 | 0.07 | 0.0311 | 0.0008 | 0.0525 | 0.0013 | 0.072 | 0.0014 |
| 20 | 0.06 | 0.08 | 0.1 | 0.0621 | 0.0016 | 0.0827 | 0.0021 | 0.1031 | 0.0018 |
| | 0.05 | 0.07 | 0.09 | 0.0524 | 0.0012 | 0.0728 | 0.0017 | 0.0922 | 0.0015 |
| | 0.04 | 0.06 | 0.08 | 0.0412 | 0.0009 | 0.0621 | 0.0013 | 0.0817 | 0.0012 |
| | 0.03 | 0.05 | 0.07 | 0.0314 | 0.0006 | 0.0524 | 0.001 | 0.0721 | 0.001 |
| 25 | 0.06 | 0.08 | 0.1 | 0.0622 | 0.0013 | 0.083 | 0.0016 | 0.102 | 0.0014 |
| | 0.05 | 0.07 | 0.09 | 0.0515 | 0.0009 | 0.0725 | 0.0013 | 0.0919 | 0.0012 |
| | 0.04 | 0.06 | 0.08 | 0.0417 | 0.0007 | 0.0616 | 0.001 | 0.0814 | 0.001 |
| | 0.03 | 0.05 | 0.07 | 0.0314 | 0.0005 | 0.0515 | 0.0008 | 0.0713 | 0.0008 |
| 30 | 0.06 | 0.08 | 0.1 | 0.0616 | 0.001 | 0.0821 | 0.0013 | 0.1015 | 0.0012 |
| | 0.05 | 0.07 | 0.09 | 0.0511 | 0.0008 | 0.0722 | 0.001 | 0.0919 | 0.0009 |
| | 0.04 | 0.06 | 0.08 | 0.0408 | 0.0006 | 0.0611 | 0.0008 | 0.0816 | 0.0008 |
| | 0.03 | 0.05 | 0.07 | 0.0307 | 0.0004 | 0.0511 | 0.0006 | 0.0713 | 0.0006 |
| 40 | 0.06 | 0.08 | 0.1 | 0.0617 | 0.0007 | 0.0821 | 0.001 | 0.1005 | 0.0009 |
| | 0.05 | 0.07 | 0.09 | 0.0512 | 0.0006 | 0.0714 | 0.0008 | 0.0914 | 0.0007 |
| | 0.04 | 0.06 | 0.08 | 0.041 | 0.0004 | 0.0609 | 0.0006 | 0.0814 | 0.0006 |
| | 0.03 | 0.05 | 0.07 | 0.0305 | 0.0003 | 0.0511 | 0.0005 | 0.0712 | 0.0005 |
| 50 | 0.06 | 0.08 | 0.1 | 0.0612 | 0.0006 | 0.0809 | 0.0008 | 0.1009 | 0.0007 |
| | 0.05 | 0.07 | 0.09 | 0.0509 | 0.0005 | 0.071 | 0.0006 | 0.0908 | 0.0006 |
| | 0.04 | 0.06 | 0.08 | 0.0404 | 0.0003 | 0.0609 | 0.0005 | 0.0808 | 0.0005 |
| | 0.03 | 0.05 | 0.07 | 0.0304 | 0.0002 | 0.051 | 0.0004 | 0.0705 | 0.0004 |

Example: The stress is increasing from $S_1$ to $S_2$ at the time $t_1 = 5$. Let the true value of failure rate be $\lambda_1 = 0.01$, $\lambda_2 = 0.03$, $\lambda_{12} = 0.05$, and the true value of tampered factors be $\alpha_1 = 10$, $\alpha_2 = 15$, $\alpha_{12} = 5$. Under the sample size $n = 20$, the random data are generated and calculated by Monte-Carlo simulations. When the lives of two components are both observed, the maximum likelihood estimates of failure rate $\lambda_1,\ \lambda_2,\ \lambda_{12}$ are $\widehat{\lambda_1} = 0.0125$, $\widehat{\lambda_2} = 0.0268$, $\widehat{\lambda_{12}} = 0.0434$. When the life of the first failed component is observed, the maximum likelihood estimates of failure rate $\lambda_1,\ \lambda_2,\ \lambda_{12}$ are $\widehat{\lambda_1} = 0.0127$, $\widehat{\lambda_2} = 0.0270$, $\widehat{\lambda_{12}} = 0.0429$.

**Table 2**
**The Life of First Failed Component Is Observed**

| $n$ | $\lambda_1$ | $\lambda_2$ | $\lambda_{12}$ | $\widehat{\lambda_1}$ mean | $\widehat{\lambda_1}$ MSE | $\widehat{\lambda_2}$ mean | $\widehat{\lambda_2}$ MSE | $\widehat{\lambda_{12}}$ mean | $\widehat{\lambda_{12}}$ MSE |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 0.06 | 0.08 | 0.1 | 0.063 | 0.0016 | 0.0844 | 0.0021 | 0.1053 | 0.0026 |
| | 0.05 | 0.07 | 0.09 | 0.052 | 0.0012 | 0.0722 | 0.0017 | 0.0955 | 0.0023 |
| | 0.04 | 0.06 | 0.08 | 0.0425 | 9.39e-04 | 0.0625 | 0.0014 | 0.0848 | 0.002 |
| | 0.03 | 0.05 | 0.07 | 0.0309 | 6.25e-04 | 0.0521 | 0.001 | 0.0732 | 0.0016 |
| 20 | 0.06 | 0.08 | 0.1 | 0.0618 | 0.0011 | 0.0828 | 0.0015 | 0.1031 | 0.0019 |
| | 0.05 | 0.07 | 0.09 | 0.0515 | 8.96e-04 | 0.073 | 0.0013 | 0.094 | 0.0016 |
| | 0.04 | 0.06 | 0.08 | 0.0414 | 6.69e-04 | 0.0618 | 9.99e-04 | 0.0833 | 0.0014 |
| | 0.03 | 0.05 | 0.07 | 0.0308 | 4.59e-04 | 0.0518 | 7.85e-04 | 0.0729 | 0.0011 |
| 25 | 0.06 | 0.08 | 0.1 | 0.062 | 8.99e-04 | 0.0827 | 0.0012 | 0.1028 | 0.0015 |
| | 0.05 | 0.07 | 0.09 | 0.0514 | 7.10e-04 | 0.0719 | 9.75e-04 | 0.0924 | 0.0012 |
| | 0.04 | 0.06 | 0.08 | 0.0411 | 5.24e-04 | 0.0612 | 7.70e-04 | 0.0823 | 0.001 |
| | 0.03 | 0.05 | 0.07 | 0.0307 | 3.55e-04 | 0.0512 | 6.02e-04 | 0.0721 | 8.45e-04 |
| 30 | 0.06 | 0.08 | 0.1 | 0.0608 | 7.15e-04 | 0.0815 | 9.80e-04 | 0.1021 | 0.0012 |
| | 0.05 | 0.07 | 0.09 | 0.0511 | 5.78e-04 | 0.0718 | 8.02e-04 | 0.0927 | 0.001 |
| | 0.04 | 0.06 | 0.08 | 0.0409 | 4.32e-04 | 0.0614 | 6.31e-04 | 0.0826 | 8.69e-04 |
| | 0.03 | 0.05 | 0.07 | 0.0308 | 3.00e-04 | 0.0514 | 5.11e-04 | 0.0719 | 7.22e-04 |
| 40 | 0.06 | 0.08 | 0.1 | 0.0609 | 5.23e-04 | 0.0815 | 7.22e-04 | 0.1017 | 9.07e-04 |
| | 0.05 | 0.07 | 0.09 | 0.0507 | 4.37e-04 | 0.0706 | 5.92e-04 | 0.0914 | 7.76e-04 |
| | 0.04 | 0.06 | 0.08 | 0.0406 | 3.18e-04 | 0.0612 | 4.73e-04 | 0.0819 | 6.59e-04 |
| | 0.03 | 0.05 | 0.07 | 0.0304 | 2.18e-04 | 0.0507 | 3.56e-04 | 0.0714 | 5.15e-04 |
| 50 | 0.06 | 0.08 | 0.1 | 0.0608 | 4.24e-04 | 0.0811 | 5.70e-04 | 0.101 | 7.25e-04 |
| | 0.05 | 0.07 | 0.09 | 0.0508 | 3.36e-04 | 0.0711 | 4.69e-04 | 0.091 | 6.00e-04 |
| | 0.04 | 0.06 | 0.08 | 0.0403 | 2.43e-04 | 0.0606 | 3.79e-04 | 0.0809 | 5.03e-04 |
| | 0.03 | 0.05 | 0.07 | 0.0303 | 1.72e-04 | 0.0505 | 2.93e-04 | 0.0708 | 4.16e-04 |

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