

Estimation of Heteroscedasticity Effects in a Classical Linear Regression Model of a Cross-Sectional Data

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Abstract: This paper investigates the effects of heteroscedasticity in the Classical Linear Regression Model (CLRM) of auditor's remuneration. Several efforts of building a realistic econometric model for Auditor's Remuneration with regards to core banking activities have been undertaken. The work involves the use of White heteroscedasticity and Newey-West test techniques to examine the presence of heteroscedasticity, which shows that heteroscedasticity is an inherent feature of cross-sectional data. The superiority of Weighted Least Squares (WLS) on Ordinary Least Squares (OLS) was put to test in estimating the parameters of Auditors Remuneration model designed as:

$$AR_i = \theta_0 + \theta_1 TA_i + \theta_2 TE_i + \theta_3 CD_i + \theta_4 PBT_i + \varepsilon$$

And it was established that OLS is not appropriate for estimation if heteroscedasticity is present in research data, and that the model fitted using WLS is the most appropriate that is deemed fit for proper review of auditor's remuneration in banking industry.

Key words: Cross-sectional data; Weighted least squares; Auditor's remuneration; White heteroscedasticity; Newey-west

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1. INTRODUCTION

Heteroscedasticity is one of the associated problems with the Classical Linear Regression Model (CLRM), Gujarati and Porter (2009). By heteroscedasticity, we meant the existence of some non- constant variance function in a CLRM.

Long and Ervin (2000) confirmed that in the presence of heteroscedasticity, OLS estimates are unbiased, but the usual tests of significance are generally inappropriate and their use can lead to incorrect inferences. Among other things, they suggested that data analysts should correct for heteroscedasticity using Heteroscedasticity Consistent Covariance Matrix (HCCM) whenever there is reason to suspect its presence.

Xavier, Bernadino and Juan (2012) were also of the opinion that with regard to day-to-day imprecision, the phenomenon called heteroscedasticity should be taken into account: day-to-day meteorological variance depends on the value of the measured. In some cases of heteroscedasticity, in spite of variance differences with the measured value, the coefficient of determination remained constant; in these cases, the calculation of the variance due to day-to-day imprecision becomes easy to carry out.

In this research, we broaden the scope of heteroscedasticity by considering a $K - 1$ variable classical linear regression model where the relation between a response variable Y and regressors is given by

$$Y_t = \theta_0 + \theta_1 X_{1t} + \theta_2 X_{2t} + \dots \dots + \theta_k X_{kt} + \varepsilon_k \quad (1)$$

Where $t = 1, 2, \dots, n$ and ε_k denotes the error term.

This model identifies $K - 1$ explanatory variables (regressors) namely X_1, X_2, \dots, X_k and a constant term ε that assumed to influence the dependent variable (regressand). In the literature, model (1) has been thoroughly investigated for heteroscedasticity. It is well known that when the assumptions of the linear regression model are correct, Ordinary Least Squares (OLS) provides efficient and unbiased estimates of the parameters. When the errors are heteroscedastic, the OLS estimates remains unbiased, but becomes inefficient. More importantly the usual procedures for hypothesis testing are no longer appropriate and their use can lead to incorrect inferences. According to Phoebus (1978), this means that confidence intervals based on OLS will be unnecessarily larger and as a result, the t and F tests are likely to give us inaccurate results. Given that heteroscedasticity is common in cross-sectional data, methods that take care of heteroscedasticity are essential for prudent data analysis.

For the purpose of this paper, we used four (4) regressors namely Total Assets (TA), Total Equity (TE), Customers Deposit (CD) and Profit Before Tax (PBT) with Auditor's Remuneration (AR) as the regress and variable. All these information are obtained from a cross-sectional data of eleven (11) commercial banks in Nigeria. Figures related to them were extracted from the year 2008, 2009 and 2010 audited financial statements as published by all the eleven banks.

Thus, an Auditor's Remuneration model (AR) is designed as

$$AR = f(TA, TE, CD, PBT) + \varepsilon \quad (2)$$

Where $f(TA, TE, CD, PBT)$ denote the function of Total Assets (TA), Total Equity (TE), Customers Deposit (CD) and Profit Before Tax (PBT). Auditor's

Remuneration (AR) is the fees charged by auditors in carrying out their statutory role of examining the books of banks and companies alike.

Explicitly, we have

$$AR_i = \theta_0 + \theta_1 TA_i + \theta_2 TE_i + \theta_3 CD_i + \theta_4 PBT_i + \varepsilon_i \quad (3)$$

In the course of this research, we shall demonstrate with great dexterity that the conditional variance of AR_i increases as each of TA_i , TE_i , CD_i and PBT_i increases. That is, the variance of AR is not the same for each of the banks. Hence, there is presence of heteroscedasticity. i.e.,

$$E(u_i^2) = \sigma_i^2 \quad (4)$$

Where u_i and σ_i^2 denote the error terms and conditional variance of AR_i which is also equal to the conditional variance of u_i respectively.

“Ole-Kristian” *et al.* (2007) examined the relation between excess auditor remuneration and the implied required rate of return on equity capital in global markets, and they conjecture that when auditor remuneration is excessively large, investors may perceive the auditor to be economically bonded to the client, leading to lack of independence. Meanwhile, they failed to establish a scientific procedure for the appropriate fixing or review of this auditor remuneration, despite all the negative effects of its excess emphasized in their publication. To fill this gap, this research intends to build a scientific model that would have been fully examined for heteroscedasticity, and would suffice for the fixing and periodic review of auditor remuneration without prejudice.

Equation (2) also derives its justification from the board room negotiations that usually accompanied the review of Auditor’s remuneration. Meanwhile, Olutola (2003) gave a clearer picture of what an audit implied. He defined audit as “an independent examination of an expression of opinion on the financial statements of an enterprise by an appointed auditor in compliance with any relevant statutory obligation”. The definition suggested among other things that, the bank must have prepared its financial statements before the external auditor is invited to carry out an independent examination on them. Thus, it will only be fair enough for the external auditor to base his remuneration on the volume of operations as presented in the financial statements of the bank.

2. MATERIALS AND METHODS

The estimation of heteroscedasticity effect in classical linear regression model (CLR-M) enables us to see the cross-sectional nature of data collected from eleven Nigerian banks and provide answers to the following questions.

- (1). What are the consequences of heteroscedasticity presence in econometrics data?
- (2). How does one detect it?
- (3). What are the remedial measures?

To analyze the data collected and provide answers to the question listed above, the types of techniques adopted are:

- (1). Ordinary Least Squares (OLS) estimation in the presence of heteroscedasticity.
- (2). The method of Generalised Least Squares (GLS).

- (3). White heteroscedasticity-Consistent standard errors and covariance.
- (4). Newey-West HAC standard error and covariance.

2.1. OLS Estimation in the Presence of Heteroscedasticity

If we introduce heteroscedasticity by letting $E(u_i^2) = \sigma_i^2$ but retain all other assumptions of the classical model, the OLS estimator $\hat{\theta}$ is the same with that of the situation under the assumption of homoscedasticity but its variance is obviously different from the usual variance obtained under the assumption of homoscedasticity.

We consider a two-variable model given as:

$$Y_i = \theta_1 + \theta_2 X_i + u_i \tag{5}$$

Where θ_1 and θ_2 denote the regression constant and regression coefficient respectively, and u_i is the error term.

By minimizing the sum of square of error, the OLS estimator of θ_2 becomes

$$\hat{\theta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum x_i^2 - (\sum X_i)^2} \tag{6}$$

but its variance is now given by the following expression:

$$\begin{aligned} Var(\hat{\theta}_2) &= E[(\hat{\beta}_2 - \beta)^2] = E[(\sum k_i u_i)^2] \\ Var(\hat{\theta}_2) &= E(k_1^2 u_1^2 + k_2^2 u_2^2 + \dots + k_n^2 u_n^2 + 2 \text{ cross product terms}) \\ &= E(k_1^2 u_1^2 + k_2^2 u_2^2 + \dots + k_n^2 u_n^2) \end{aligned}$$

Since the expectation of the cross product terms are zero because of the assumption of no serial correlation.

$$Var(\hat{\theta}) = k_1^2 E(u_1^2) + k_2^2 E(u_2^2) + \dots + k_n^2 E(u_n^2)$$

Since, $E(u_i^2) = \sigma_i^2$, we have

$$Var(\hat{\theta}_2) = k_1^2 \sigma_1^2 + k_2^2 \sigma_2^2 + \dots + k_n^2 \sigma_n^2 = \sum k_i^2 \sigma_i^2$$

Since, $k_i = \frac{x_i}{\sum x_i^2}$ (from the linearity property of Gauss-Markov Theorem)

Therefore,

$$Var(\hat{\theta}) = \sum [(\frac{x_i}{\sum x_i^2})^2 \sigma_i^2] = \frac{\sum x_i^2 \sigma_i^2}{(\sum x_i^2)^2} \tag{7}$$

Equation (6) is obviously different from the usual variance formula obtained under the assumption of homoscedasticity, which is given as:

$$Var(\hat{\theta}_2) = \frac{\sigma_i^2}{\sum x_i^2} \tag{8}$$

If $\sigma_i^2 = \sigma^2$ for each i , the two variance formulas will be identical. This is because $\hat{\theta}^2$ is still linear and unbiased under heteroscedasticity assumption when all other assumptions of CLRM hold. Since the variance of u_i , homoscedastic or heteroscedastic plays no part in the determination of the unbiasedness property.

Also, $\hat{\theta}_2$ is a consistent estimator under the assumption of the CLRM despite heteroscedasticity; that is, as the sample size increases indefinitely (i.e., becomes asymptotically large) the estimated θ_2 converges to its true value. Furthermore, it can be shown that under regularity conditions, $\hat{\theta}_2$ is asymptotically normally distributed.

Granted that $\hat{\theta}_2$ is still linear, unbiased and consistent, it is pertinent to note that $\hat{\theta}_2$ is not efficient or best. That is, it does not have minimum variance in the class of unbiased estimators.

Thus, we can easily conclude that $\hat{\theta}_2$ is not BLUE in the presence of heteroscedasticity.

2.2. GLS Estimator

This is the procedure of transforming the original variables in such a way that the transformed variables satisfy the assumptions of classical model and then applying OLS to them. In short, GLS is OLS in the transformed variables that satisfy the standard least squares assumptions. The estimators thus obtained are known as GLS estimators and it is these estimators that are Best, Linear and Unbiased (BLUE).

Unlike the usual OLS method which does not make use of the information available in the unequal variability of the dependent variable Y , i.e., it assigns equal weight or importance to each observation. GLS takes such information into account explicitly and is therefore capable of producing estimators that are BLUE.

To illustrate this, we recall equation (5): $Y_i = \theta_1 + \theta_2 X_i + u_i$

Which for ease of algebraic manipulation, we write as:

$$Y_i = \theta_1 X_{oi} + \theta_2 X_i + u_i \quad (9)$$

Where $X_{oi} = 1$ for each i .

By assuming that the heteroscedastic variances σ_i^2 are known, and divide equation (9) through by σ_i to obtain:

$$\frac{Y_i}{\sigma_i} = \sigma_1 \left(\frac{X_{oi}}{\sigma_i} \right) + \theta_2 \left(\frac{X_i}{\sigma_i} \right) + \left(\frac{u_i}{\sigma_i} \right) \quad (10)$$

Which for ease of operation, we write as:

$$Y_i^* = \theta_1^* X_{oi}^* + \theta_2^* X_i^* + u_i^* \quad (11)$$

We used θ_1^* and θ_2^* the parameters of the transformed model, to distinguish them from the usual OLS parameters θ_1 and θ_2 .

Hence,

$$Var(u_i^*) = E(u_i^*)^2 = E\left(\frac{u_i}{\sigma_i}\right)^2 \quad (12)$$

Since, $E(u_i^*) = 0$

$$\begin{aligned} Var(u_i^*) &= \frac{1}{\sigma_i^2} E(u_i^2), \text{ since } \sigma_i^2 \text{ is known} \\ &= \frac{1}{\sigma_i^2} (\sigma_i^2), \text{ since } E(u_i^2) = \sigma_i^2 \\ &= 1, \text{ which is a constant.} \end{aligned}$$

That is, the variance of the transformed disturbance terms u_i^* is now homoscedastic. Since we are still retaining the other assumptions of the classical model, the finding that it is u^* that is homoscedastic suggest that if we apply OLS to the transformed model (11), it will produce estimators that are BLUE.

In short, the estimated σ_1^* and σ_2^* are now BLUE and not the OLS estimators, $\hat{\sigma}_1$ and $\hat{\sigma}_2$.

2.3. White Heteroscedasticity-Consistent Standard Errors and Covariance

White (1980) has derived a heteroscedasticity consistent covariance matrix estimator which provides estimates of the coefficient covariances in the presence of heteroscedasticity of unknown form. The white covariance matrix is given by:

$$\hat{\Sigma}_w = \frac{T}{T-K} (X'X)^{-1} \left(\sum_{t=1}^T u_t^2 x_t x_t' \right) (X'X)^{-1} \quad (13)$$

Where T is the number of observations, K is the number of regressors, X is the regressor and u_t is the least squares residual.

2.4. Newey-West HAC Standard Errors and Covariance

The white covariance matrix described above assumes that the residuals of the estimated equation are serially uncorrelated. Newey and West (1987) have proposed a more general covariance estimator that is consistent in the presence of both heteroscedasticity and autocorrelation of unknown form.

Newey and West (1987) give the Newey-West estimator as:

$$\hat{\Sigma}_{NW} = \frac{T}{T-K} (X'X)^{-1} \hat{\Omega} (X'X)^{-1} \quad (14)$$

Where

$$\hat{\Omega} = \frac{T}{T-K} \left\{ \sum_{t=1}^T U_t^2 x_t x_t' + \sum_{v=1}^q \left[\left(1 - \frac{v}{q+1}\right) \sum_{t=v+1}^T (x_t u_t u_{t-v} x_{t-v}' + x_{t-v} u_{t-v} x_t') \right] \right\} \quad (15)$$

In which q , the truncation lag, is a parameter representing the number of autocorrelations used in evaluating the dynamics of the OLS residuals u_t . Following the suggestion of Newey and West, E-views sets q to:

$$q = \text{floor} \left[4(T/100)^{2/9} \right] \quad (16)$$

It is pertinent to note that using the white heteroscedasticity or Newey-West does not change the point estimates of the parameters; only the estimated standard errors are different.

3. RESULTS AND DISCUSSION

The data collected is on the operational activities of eleven (11) commercial banks in Nigeria namely First Bank, United Bank for Africa, Zenith Bank, Stanbic ibtc, Skye bank, Union Bank, Access Bank, FCMB, Ecobank, GTBank and Diamond Bank for periods of year 2008, 2009 and 2010.

3.1. Fitting of OLS Model

The required OLS results from the analysis carried out on E-views package are presented as follows:

$$2008 : AR_i = 64,282,009 + 0.000103TA_i - 0.0000753TE_i - 0.0000687CD_i - 0.000404PBT_i \quad (17)$$

$$2009 : AR_i = 86,402,977 + 0.0000979TA_i + 0.0000879TE_i - 0.000125CD_i + 0.0000215PBT_i \quad (18)$$

$$2010 : AR_i = 87365584 - 0.000118TA_i + 0.000546TE_i + 0.0000572CD_i + 0.001012PBT_i \quad (19)$$

Table 1
Results of OLS Statistic

Year	2008	2009	2010
OLS	0.4962 0.1603	0.3538 -0.0770	0.5350 0.2250
F	1.4772 (with <i>P</i> value of 0.31826)	0.8213 (with <i>P</i> value of 0.556)	1.7258 (with <i>P</i> value of 0.2619)
AIC	36.7444	37.5447	37.9998
SWC	36.9249	37.7255	38.1806

The Classical Linear Regression Model (CLRM) built for Year 2008 is such that, if all other regressors are held constant, the auditor's remuneration goes up by one hundred and three naira only if there is one million naira worth of increase in the assets of the banks. All other regressors have a reducing effect on the auditor's remuneration for the year 2008.

The coefficient of determination (R^2) implies that only 49.6% of the variation in auditor's remuneration is explained by all the explanatory variables under consideration. The adjusted R^2 (0.160), akaike info criterion (36.74) and Schwarz criterion (36.92) further confirmed the position of our R^2 , which adjudged the model as not a "best goodness of fit".

The results of F -statistic shows that the regression coefficients are not statistically significant, which cast further aspersion on the integrity of the model for reasonable inference.

According to the results Year of 2009, the model is such that total assets, total equity and profit with customers deposit held constant, contribute approximately ninety eight naira, eighty eight naira and seventy two naira respectively to the auditor's remuneration if there is one million naira worth of increase in the three of them. The coefficient of determination (R^2) implies that only 35.4% of the variation in auditor's remuneration is accounted for by all the regressors under consideration. The adjusted R^2 (-7.7%) in fact shows that the model is a poorly fitted one. The duo of akaike info (37.54) and Schwarz (37.72) criterion further confirmed this position.

The result of F statistic shows that all the regression coefficients are statistically insignificant. Since the P -value is greater than 0.05.

The CLRM built for Year 2010 is such that, if total assets is held constant, the auditor's remuneration increases by approximately five hundred and forty six naira, fifty seven naira and one thousand and twelve naira only if there is one million naira worth of increase in total equity, customers deposit and profit respectively, provided that total assets is kept constant.

The coefficient of determination (R^2) implies that only 53.5% of the variation in auditor's remuneration is explained by the four explanatory variables. The adjusted R^2 (0.22), akaike info criterion (37.99) and Schwarz criterion (38.18) further confirmed the position of our R^2 , which adjudged the model to have been poorly fitted.

The result of F statistic shows that the regression coefficients are not statistically significant, since its P -value is above 0.05.

Meanwhile, the above results are expected for OLS, since we used cross-sectional data which speaks volume of the presence of heteroscedasticity.

3.2. Test for the Presence of Heteroscedasticity

The results of the two formal methods used to carry out test for the presence of heteroscedasticity in the cross-sectional data utilized for this research are presented as follows:

3.3. White Heteroscedasticity Results

$$2008 : AR_i = 64,282,009 + 0.000103TA_i - 0.0000753TE_i - 0.0000687CD_i - 0.000404PBT_i \quad (20)$$

$$2009 : AR_i = 86,402,977 + 0.0000979TA_i + 0.0000879TE_i - 0.000125CD_i + 0.0000215PBT_i \quad (21)$$

$$2010 : AR_i = 87365584 - 0.000118TA_i + 0.000546TE_i + 0.0000572CD_i + 0.001012PBT_i \quad (22)$$

3.4. Newey-West HAC Results

$$2008 : AR_i = 64,282,009 + 0.000103TA_i - 0.0000753TE_i - 0.0000687CD_i - 0.000404PBT_i \quad (23)$$

$$2009 : AR_i = 86,402,977 + 0.0000979TA_i + 0.0000879TE_i - 0.000125CD_i + 0.0000215PBT_i \quad (24)$$

$$2010 : AR_i = 87365584 - 0.000118TA_i + 0.000546TE_i + 0.0000572CD_i + 0.001012PBT_i \quad (25)$$

It is pertinent to note that the results of both the White and Newey heteroscedasticity test for the three years do not change the point estimates of the parameters from the ones obtained in the OLS analysis, which confirmed the presence of heteroscedasticity. Only the estimated standard errors of both test differed from that of the OLS estimates. Thus, the model arrived at by the two methods clearly show lack of goodness of fit. Hence, the computation of GLS (or WLS) estimates becomes highly necessary.

Table 2
Results of Newey-West and White Heteroscedasticity Statistic

Year	2008	2009	2010
NW	0.4962	0.3538	0.5350
	0.1603	-0.0770	0.2250
F	1.4772 (with P value of 0.31826)	0.8213 (with P value of 0.556)	1.7258 (with P value of 0.2619)
AIC	36.7444	37.5447	37.9998
SWC	36.9249	37.7255	38.1806
WH	0.4962	0.3538	0.5350
	0.1603	-0.0770	0.2250
F	1.4772 (with P value of 0.31826)	0.8213 (with P value of 0.556)	1.7258 (with P value of 0.2619)
AIC	36.7444	37.5447	37.9998
SWC	36.9249	37.7255	38.1806

3.5. Fitting of “GLS Model”

The required GLS results from our analysis are presented as follows:

$$2008 : AR_i = 81,594,678 + 0.0000463TA_i + 0.0000416TE_i - 0.0000232CD_i - 0.000847PBT_i \quad (26)$$

$$2009 : AR_i = 89,375,193 - 0.0000205TA_i + 0.000283TE_i - 0.00000126CD_i - 0.000229PBT_i \quad (27)$$

$$2010 : AR_i = 100,000,000 - 0.0000732TA_i + 0.000313TE_i + 0.00000274CD_i + 0.002445PBT_i \quad (28)$$

The GLS results presented for Year 2008 have taken care of all the deficiencies of OLS results presented/explained for the year 2008. Hence, it is adjudged to be a better model in the presence of heteroscedasticity. In fact, 95.11% variation in auditor’s remuneration accounted for by the explanatory variables is one of the best situations for the measure of goodness of fit.

Accordingly, the results presented in Equation (27) are also adjudged to be better than that of year 2009 OLS results. The coefficient of determination (R^2)

implies that 87.02% of the variation in auditor’s remunerations is explained by all the explanatory variables as against 35.38% presented by the OLS results.

Year 2010 equally gives a better result than that of the OLS which has a coefficient of determination (R^2) of 0.535. A 90.75% variation in auditor’s remuneration, as explained by the explanatory variables makes the GLS model to be valid enough for reasonable inference.

The adjusted R^2 , akaike info and Schwarz criterion results also pointed to the fact the GLS models are reasonably valid.

The result of F statistic shows that all the regression coefficients are statistically significant at both 5% and 1% levels of significance for the three years under consideration.

Table 3
Results of GLS Statistic

Year	2008	2009	2010
GLS	0.9511	0.8702	0.9075
	0.9187	0.7837	0.8459
F	29.23 (with P value of 0.000448)	10.0575 (with P value of 0.0079)	14.7223 (with P value of 0.002943)
AIC	35.2123	37.7103	38.1665
SWC	35.3932	37.8912	38.3473

4. CONCLUSION

Based on the results obtained by the empirical analysis of data collected, the following conclusions are therefore derived:

(i) Ordinary least squares (OLS) is not appropriate if heteroscedasticity is present in research data.

(ii) Generalized least squares (GLS) or weighted least squares (WLS) is the most appropriate method for estimation, in the presence of heteroscedasticity.

(iii) Cross-sectional data are usually heteroscedastic in nature.

(iv) Among the models presented in this paper, Equations (26), (27) and (28) are the only recommended models that satisfy the purpose for this research.

(v) Equation (28), being the CLRM of the most recent year in this research is the most reliable for the review of auditors remuneration in both domestic and foreign banks.

REFERENCES

- [1] Gujarati, D.N., & Porter, D.C. (2009). *Basic econometrics*(5th ed., p. 922). New York: Mc Graw-Hill.
- [2] Long, J.S., & Ervin, L.H. (2000). Using heteroscedasticity consistent standard errors in the linear regression model. *The American Statistician*, 54(3), 217-224. Retrieved April 5th, 2012 from <http://www.jstor.org/stable/2685594>
- [3] Newey, W., & West, K. (1987a). Hypothesis testing with efficient method of moments estimation. *International Economic Review*, 28, 777–787.

- [4] Newey, W., & West, K. (1987b). A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55, 703–708.
- [5] Ole-Kristian, H., Tony, K., Wayne, T., & Young, K. (2007). *Impact of excess auditor remuneration on cost of equity capital around the world*. Retrieved April 4th, 2012 from [http://leeds-faculty.Colorado.edu/gunny/Workshop series/Papers 0708/Thomas](http://leeds-faculty.Colorado.edu/gunny/Workshop%20series/Papers%200708/Thomas)
- [6] Olutola, O. (2003). *Principles and practice of auditing and investigations* (1st ed.). Lagos: Agbo Comm. Press.
- [7] Phoebus, J.D. (1978). *Introductory econometrics* (pp. 110–111). New York: Springer-Verlag.
- [8] Quantitative Micro Software (2002). *E-Views 4 user's guide*(Revised for E-Views 4.1). United States of America.
- [9] White, H. (1980). A heteroscedasticity consistent covariance matrix estimator and a direct test of heteroscedasticity. *Econometrica*, 48, 1180–1200.
- [10] Xavier, F.A., Bernadino, G., & Juan, C.M. (2012). Uncertainty of measurement and heteroscedasticity. *Journal of the International Federation of Clinical Chemistry and Laboratory Medicine*, *ejIFC*, 14(1). Retrieved April 4th, 2012 from [http://www.ifcc.org/ejifcc/vol 14 no1/140103200306n.htm](http://www.ifcc.org/ejifcc/vol%2014%20no1/140103200306n.htm)