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Magnetogravitational Vortex-Sheet Instability of Two Superposed Conducting Fluids in Porous Medium Under Strong Magnetic Field

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Abstract

The instability of two superposed homogeneous streaming fluids is discussed under gravitational force and uniform magnetic field in porous medium. The two streams are moving in opposite directions with equal velocities parallel to the horizontal plane. The solution has been obtained through the normal mode technique, and the most general dispersion relation has been obtained as 20th-order equation for the growth rate with quite complicated coefficients. Solving numerically the dispersion relation for appropriate boundary conditions with high Alfvén and sound velocities, it is found that fluid velocities and porosity of porous medium have stabilizing effects, and Alfvén and sound velocities have destabilizing effects, while medium permeability has a slightly stabilizing effect, and the dynamic viscosities have slightly destabilizing effect. The limiting cases of non-porous medium have also been studied for both streaming and stationary fluids.

Key words

Hydrodynamic stability; Conducting fluids; Flows through porous media; Gravitational force; Magnetohydrodynamics

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1. INTRODUCTION

The Kelvin-Helmholtz instability of the boundary layer between two fluids in relative motion is of great interest in many astrophysical and geophysical situations, ranging from the interaction of the solar wind with the magnetospheric boundary[1] and cometary tails[2], to the dynamics of jets in nuclei extragalactic radio sources[3,4] and young stellar objects[5]. Many studies have therefore been devoted to understanding the behavior of this instability under the influence of different physical ingredients, typical of the different environments[6]. Starting from the classical results for the incompressible case, which can be summarized in Chandrasekhar's book[7], the effects of compressibility have been introduced both in the pure hydrodynamical situation[8,9], and in the magnetohydrodynamic case[10,11]. The effect of a finite thickness of the shear layer has been discussed by Ray[12].

The gravitational instability problem is of central importance in understanding the process of formation of stars, planets, comets, asteroids and other astrophysical objects. Initially Jeans[13] considered the gravitational instability of non-viscous ideal fluid and showed that the system becomes unstable for all perturbations of wave numbers less than $k^* = [4\pi G\rho/C^2]^{1/2}$, where C is the velocity of sound in the medium of density ρ and G is the gravitational constant. Since then, several researchers have studied this problem under varying assumptions of hydrodynamics and hydromagnetics. A comprehensive account of these investigations has been given by Chandrasekhar[7] in his monograph on hydrodynamic and hydromagnetic stability. Chandrasekhar[14,15], Chandrasekhar and Fermi[16] considered the stability of an infinite homogeneous and static medium under uniform magnetic field and uniform rotation. He found the critical wave number k^* given by $k^* = [(4\pi G\rho - 4\Omega^2)/(M^2 + C^2)]^{1/2}$, where $M = [(\mu H^2/(4\pi\rho)]^{1/2}$ is the Alfvén velocity of the medium. Then, Segnar [17] or Radwan and Elazab[18] attempted this problem in different cases under uniform magnetic field and using linear perturbation they found that the magnetic field has a strong stabilizing effect on the system. Singh and Khare[19,20] discussed the instability of two semi-infinite homogeneous streams of infinite conductivity in absence and presence of uniform magnetic field, respectively. They found the critical wave number in some limiting cases of interest. The Jeans' instability problems have been studied by several researchers under the separate or simultaneous effects of different physical parameters[21−32]. In all such investigations, carried out separately under varying assumptions, it was found that the condition of instability has been determined by the Jeans criterion with some modifications, introduced by the inclusion of the various parameters.

Flows through porous media has been a subject of great interest for the last several decades. This interest was motivated by numerous engineering applications in various disciplines, such as geophysical, thermal and insulation engineering,the modelling of packed sphere bed, the cooling of electronic systems, groundwater hydrology, chemical catalytic reactors, ceramic processes, grain storage devices, fiber and granular insulation, petroleum reservoirs, coal combustors, ground water pollution and filtration processes, to name just a few of these applications[33]. Several other applications of the problems of flow throufh porous media in geophysics may be found in the the works of Dullien[34], Ingham and Pop[35], Nield and Bejan[36], and Vafai[37]. When considering flow in a porous medium, however, one must address some additional complexities, which are principally due to the interactions between the fluids and the porous material. Kelvin-Helmholtz instability for flow in porous media has attracted little attention in the scientific literature. Raghaven and Mardsen[38] have studied the Kelvin-Helmholtz instability for flow in porous media for Darcy-type flow. they used linear stability analysis to obtain a characteristic equation for the growth rate of the disturbance and then solved this equation numerically. They conclude that Kelvin-Helmholtz instability is possible only if the heavier fluid is overlying the light one (statically unstable situation). Sharma and Spanos[39] investigated the instability of the plane interface between two uniforms,superposed, and streaming fluids through porous media. A linear theory of Kelvin-Helmholtz instability for parallel flow in porous media was introduced by Bau[40] for Darcian and non-Darcian flows. In both cases, he found that the velocities should exceed some critical value for the instability to manifest itself.

On the other hand, magnetohydrodynamics (or MHD for short) is the macroscopic theory of electrically conducting fluids move in a magnetic field, providing a powerful and practical theoretical framework for describing both laboratory and astrophysical plasmas (a plasma is a hot ionized gas containing free electrons and ions). The simplest example of an electrically conducting fluid is a liquid metal, for example, mercury or liquid sodium[41]. However, the major use of MHD is in plasma physics. It is by no means obvious that plasmas can be regarded as fluids since the mean free paths for collisions between the electrons and ions are macroscopically long[42]. For the importance of studying magnetohydrodynamic flows through porous media see the excellent investigations of Geindreau and Auriault[43,44], and Zakaria et al.[45].

In this paper, we have studied the hydromagnetic instability of two superposed homogeneous conducting fluids streaming in opposite directions with equal horizontal velocities under gravitational force and high Alfvén and sound velocities in porous medium. The perturbation propagation is taken simultaneously along and perpendicular to streaming motion in the horizontal interface. The most general dispersion relation has been obtained as 20th-order equation for the growth rate with quite complicated coefficients. Solving numericaly the dispersion relation for appropriate boundary conditions with high Alfvén and sound velocities.

The stability criteria are obtained for the problem and discussed analytically and numerically. The obtained results are listed in the conclusions section with the main findings of various physical parameters including in the analysis.

2. FORMULATION OF THE PROBLEM

We consider two infinitely conducting semi infinite homogeneous streams separated by the plane $z = 0$. The upper region $z > 0$ is of density ρ_1, dynamic viscosity μ_1, pressure P_1, and the lower region $z < 0$ is of density ρ_2, dynamic viscosity μ_2, pressure P_2. The streams are moving along the positive direction of x-axis with velocities V_1 and V_2 in the upper and lower regions, respectively. A uniform magnetic field $\mathbf{H} = (H, 0, 0)$ acts in the direction of streaming motion, i.e. positive direction of x-axis. C_1 and C_2 are the velocities of sound in the regions $z > 0$ and $z < 0$, respectively, ε is the porosity of the medium, and k_1 is the medium permeability. The analysis is carried out by taking wave propagation in the horizontal plane $z = 0$. By retaining the first-order terms in the perturbed quantities, we can obtain the linearized perturbation equations from the governing equations of motion.

Following Chandrasekhar[7], the linearized perturbation equation of the problem in rectangular components form are

$$\frac{\rho_r}{\varepsilon}\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)u_r=-\frac{\partial\delta P_r}{\partial x}+\rho_r\frac{\partial\delta\phi}{\partial x}-\frac{\mu_r u_r}{k_1} \tag{1}$$

$$\frac{\rho_r}{\varepsilon}\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)v_r=-\frac{\partial\delta P_r}{\partial y}+\rho_r\frac{\partial\delta\phi}{\partial y}+\frac{\mu_e H}{4\pi}\left[\frac{\partial\left(h_y\right)_r}{\partial x}-\frac{\partial\left(h_x\right)_r}{\partial y}\right]-\frac{\mu_r v_r}{k_1} \tag{2}$$

$$\frac{\rho_r}{\varepsilon}\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)w_r=-\frac{\partial\delta P_r}{\partial z}+\rho_r\frac{\partial\delta\phi}{\partial z}-\frac{\mu_e H}{4\pi}\left[\frac{\partial\left(h_x\right)_r}{\partial z}-\frac{\partial\left(h_z\right)_r}{\partial x}\right]-\frac{\mu_r w_r}{k_1} \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)\delta\rho_r=-\frac{\rho_r}{\varepsilon}\left(\frac{\partial u_r}{\partial x}+\frac{\partial v_r}{\partial y}+\frac{\partial w_r}{\partial z}\right) \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)(h_x)_r=-\frac{H}{\varepsilon}\left(\frac{\partial v_r}{\partial y}+\frac{\partial w_r}{\partial z}\right) \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)\left(h_y\right)_r=\frac{H}{\varepsilon}\left(\frac{\partial v_r}{\partial x}\right) \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)(h_z)_r=\frac{H}{\varepsilon}\left(\frac{\partial w_r}{\partial x}\right) \tag{7}$$

$$\frac{\partial\left(h_x\right)_r}{\partial x}+\frac{\partial\left(h_y\right)_r}{\partial y}+\frac{\partial\left(h_z\right)_r}{\partial z}=0 \tag{8}$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\delta\phi_r=-4\pi G\delta\rho_r \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)\delta P_r=C_r^2\left(\frac{\partial}{\partial t}+\frac{V_r}{\varepsilon}\frac{\partial}{\partial x}\right)\delta\rho_r \tag{10}$$

where $\mathbf{U}_r = (u_r, v_r, w_r)$, $\delta\rho_r$, δP_r, $\delta\varphi_r$ and $\mathbf{h}_r = (h_x, h_y, h_z)_r$ denote, respectively, the perturbations in velocity vector, density, pressure, gravitational potential and magnetic field, there the subscript $r = 1, 2$

indicate the equations in the two regions $z > 0$ and $z < 0$, respectively. The equations in both the streams are similar.

Let us discuss the stability of the streams for horizontal wave propagation of the perturbation. Thus we take the perturbation of the form

$$\psi(z)\exp[i\left(k_x x + k_y y\right) + nt] \tag{11}$$

where k_x, k_y are the real numbers denoting horizontal wave numbers of perturbation propagation along and perpendicular to streaming motion, respectively, n is the growth rate of the disturbance and $\psi(z)$ is some function of z ; and k given by $k = \sqrt{k_x^2 + k_y^2}$ being the wavenumber of perturbation propagation.

Using Eq. (11), then the linearized perturbation Eqs. (1)-(10) can be written in the form

$$\left(\frac{\rho_r\sigma_r}{\varepsilon^2} + \frac{\mu_r}{k_1}\right)u_r = -ik_x\delta P_r + ik_x\rho_r\delta\phi \tag{12}$$

$$\left(\frac{\rho_r\sigma_r}{\varepsilon^2} + \frac{\mu_r}{k_1}\right)v_r = -ik_y\delta P_r + ik_y\rho_r\delta\phi + \frac{\mu_e H}{4\pi}\left[ik_x\left(h_y\right)_r - ik_y\left(h_x\right)_r\right] \tag{13}$$

$$\left(\frac{\rho_r\sigma_r}{\varepsilon^2} + \frac{\mu_r}{k_1}\right)w_r = -D\delta P_r + \rho_r\, D\delta\phi - \frac{\mu_e H}{4\pi}\left[D\left(h_x\right)_r - ik_x\left(h_z\right)_r\right] \tag{14}$$

$$\sigma_r\delta\rho_r = -\rho_r\left(ik_x u_r + ik_y v_r + Dw_r\right) \tag{15}$$

$$\sigma_r\left(h_x\right)_r = -H\left(ik_y v_r + Dw_r\right) \tag{16}$$

$$\sigma_r\left(h_y\right)_r = H\left(ik_x v_r\right) \tag{17}$$

$$\sigma_r\left(h_z\right)_r = H\left(ik_x w_r\right) \tag{18}$$

$$ik_x\left(h_x\right)_r + ik_y\left(h_y\right)_r + D\left(h_z\right)_r = 0 \tag{19}$$

$$\left(D^2 - k^2\right)\delta\phi = -4\pi G\delta\rho_r \tag{20}$$

$$\delta P_r = C_r^2\delta\rho_r \tag{21}$$

where $D = d/dz$ and $\sigma_r = \varepsilon n + ik_x V_r$. Solving these equations, we obtain

$$\left(\frac{\rho_r\sigma_r}{\varepsilon^2} + \frac{\mu_r}{k_1}\right)\frac{\sigma_r}{4\pi G\rho_r}\left(D^2 - k^2\right)\delta\phi_r = \frac{C_r^2}{4\pi G}\left(D^2 - k^2\right)^2\delta\phi_r + \rho_r\left(D^2 - k^2\right)\delta\phi_r - \frac{\mu_e H}{4\pi}\left(D^2 - k^2\right)\left(h_x\right)_r \tag{22}$$

and

$$\left(h_x\right)_r = -\frac{H}{\sigma_r}\left[\frac{\sigma_r}{4\pi G\rho_r}\left(D^2 - k^2\right)\delta\phi_r + k_x^2\left(\frac{\rho_r\sigma_r}{\varepsilon^2} + \frac{\mu_r}{k_1}\right)^{-1}\left\{\frac{C_r^2}{4\pi G}\left(D^2 - k^2\right)\delta\phi_r + \rho_r\delta\phi_r\right\}\right] \tag{23}$$

Substituting from Eq. (23) into Eq. (22), and simplifying the resulting equation, we have a fourth-order differential equation in $\delta\phi_r$ as

$$\left(D^2 - k^2\right)\left(D^2 - \alpha_r^2\right)\delta\phi_r = 0 \tag{24}$$

where

$$\alpha_r^2 = k^2 + \frac{\sigma_r^2 L_r^2 - 4\pi G\rho_r\left(\sigma_r L_r + k_x^2 M_r^2\right)}{\left(M_r^2 + C_r^2\right)\sigma_r L_r + k_x^2 M_r^2 C_r^2} \tag{25}$$

in which

$$L_r = \sigma_r + \frac{\mu_r}{k_1}\frac{\varepsilon}{\rho_r}, \;\; M_r^2 = \frac{\mu_e H^2}{4\pi\rho_r}, \quad r = 1, 2 \tag{26}$$

and M_r are the Alfvén velocities in the two regions. On further simplification by considering the case of high values for M_r^2 and C_r^2, we can write α_r (using the binomial theorem) in the following form

$$\alpha_r = k\left\{1 + \frac{\sigma_r^2 L_r^2 - 4\pi G\rho_r\left(\sigma_r L_r + k_x^2 M_r^2\right)}{2k^2\left[\left(M_r^2 + C_r^2\right)\sigma_r L_r + k_x^2 M_r^2 C_r^2\right]}\right\} \tag{27}$$

3. BOUNDARY CONDITIONS AND SOLUTIONS

The solution of the differential equation (24) are to be bounded in the two regions. This leads to the solutions $\delta\phi_1$ in the region $z > 0$, and $\delta\phi_2$ in the region $z < 0$ as

$$\delta\phi_1 = A_1\exp(-kz) + B_1\exp(-\alpha_1 z), \;\; z > 0 \tag{28}$$

$$\delta\phi_2 = A_2\exp(kz) + B_2\exp(\alpha_2 z), \;\; z < 0 \tag{29}$$

where α_1 and α_2 are non-negative quantities, A_1, A_2, B_1 and B_2 are the arbitrary constants, to be determined by the following boundary conditions of the problem.

(1) The perturbed gravitational potential $\delta\phi$ is continuous at the interface $z = 0$, i.e.

$$\delta\phi_1 = \delta\phi_2 \quad \text{at} \quad z = 0$$

this gives

$$A_1 + B_1 - A_2 - B_2 = 0 \tag{30}$$

(2) The normal derivative of the perturbed potential is continuous at the interface $z = 0$, i.e.

$$D\left(\delta\phi_1\right) = D\left(\delta\phi_2\right) \quad \text{at} \quad z = 0$$

this gives

$$kA_1 + \alpha_1 B_1 + kA_2 + \alpha_2 B_2 = 0 \tag{31}$$

(3) The total perturbed pressure is continuous at the interface $z = 0$, i.e.

$$\delta P_1 + \frac{\mu_e H}{4\pi}\left(h_x\right)_1 = \delta P_2 + \frac{\mu_e H}{4\pi}\left(h_x\right)_2 \quad \text{at} \quad z = 0$$

Consequently

$$\begin{aligned}\frac{4\pi G\rho_1 k_x^2 M_1^2}{\sigma_1 L_1}A_1 &+ \left[\frac{\left\{\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2\right\}}{\sigma_1 L_1}\left(\alpha_1^2 - k^2\right) + \frac{4\pi G\rho_1 k_x^2 M_1^2}{\sigma_1 L_1}\right]B_1 - \frac{4\pi G\rho_2 k_x^2 M_2^2}{\sigma_2 L_2}A_2 \\ &- \left[\frac{\left\{\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2\right\}}{\sigma_2 L_2}\left(\alpha_2^2 - k^2\right) + \frac{4\pi G\rho_2 k_x^2 M_2^2}{\sigma_2 L_2}\right]B_2 = 0\end{aligned} \tag{32}$$

(4) Normal displacement of any point is unique at the interface $z = 0$; i.e.

$$\frac{w_1}{\sigma_1} = \frac{w_2}{\sigma_2} \quad \text{at} \quad z = 0$$

Consequently, we get

$$\frac{k}{\sigma_1 L_1}A_1 + \left[\frac{\alpha_1}{\sigma_1 L_1} + \frac{\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2}{4\pi G\rho_1\sigma_1 L_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)}\alpha_1\left(\alpha_1^2 - k^2\right)\right]B_1 + \frac{k}{\sigma_2 L_2}A_2 + \left[\frac{\alpha_2}{\sigma_2 L_2} + \frac{\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2}{4\pi G\rho_2\sigma_2 L_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)}\alpha_2\left(\alpha_2^2 - k^2\right)\right]B_2 = 0 \tag{33}$$

Note that, in the limiting case of non-porous medium, i.e. when $k_1 \to \infty$ or $L_r = \sigma_r$ $(r = 1, 2)$, Eqs. (24)-(26), and the boundary conditions (30)-(33), reduce to the same equations obtained earlier by Singh and Khare[20], and their results are, therefore, recovered. They obtained their dispersion relation, and they not discussed the stability analysis for this general dispersion relation, but discussed the stability conditions only for some of its limiting cases. Here, in the present work, we shall obtain the general dispersion relation for the considered system including the effect of porous medium, and discuss the effects of various parameters on the stability of the system due to the obtained dispersion relation in its general form in presence (or absence) of porous medium and fluid velocities.

4. DISPERSION RELATION

Writing the above linear equations (30)-(33) in matrix form, we have

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}\begin{bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{34}$$

Symbolically as $\left[a_{ij}\right]\left[X_j\right] = [0]$, $i = j = 1, 2, 3, 4$, where

$$X_1 = A_1,\ X_2 = B_1,\ X_3 = A_2,\ X_4 = B_2$$

$$a_{11} = 1,\ a_{12} = 1,\ a_{13} = -1,\ a_{14} = -1,$$

$$a_{21} = k,\ a_{22} = \alpha_1,\ a_{23} = k,\ a_{24} = \alpha_2,$$

$$a_{31} = \frac{4\pi G\rho_1 k_x^2 M_1^2}{\sigma_1 L_1}$$

$$a_{32} = \frac{\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2}{\sigma_1 L_1}\left(\alpha_1^2 - k^2\right) + \frac{4\pi G\rho_1 k_x^2 M_1^2}{\sigma_1 L_1}$$

$$a_{33} = -\frac{4\pi G\rho_2 k_x^2 M_2^2}{\sigma_2 L_2}$$

$$a_{34} = -\left[\frac{\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2}{\sigma_2 L_2}\left(\alpha_2^2 - k^2\right) + \frac{4\pi G\rho_2 k_x^2 M_2^2}{\sigma_2 L_2}\right]$$

$$a_{41} = \frac{k}{\sigma_1 L_1}$$

$$a_{42} = \frac{\alpha_1}{\sigma_1 L_1} + \frac{\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2}{4\pi G\rho_1\sigma_1 L_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)}\alpha_1\left(\alpha_1^2 - k^2\right)$$

$$a_{43} = \frac{k}{\sigma_2 L_2}$$

$$a_{44} = \frac{\alpha_2}{\sigma_2 L_2} + \frac{\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2}{4\pi G\rho_2\sigma_2 L_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)}\alpha_2\left(\alpha_2^2 - k^2\right)$$

For the non-trivial solution of Eq. (34), the determinant of the matrix $\left[a_{ij}\right]$ must vanish, i.e.

$$\left|a_{ij}\right| = 0 \tag{35}$$

This determinant can be reduced to the form

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{vmatrix} = 0 \tag{36}$$

where

$$\begin{aligned}
b_{11} &= \alpha_1 - k = \frac{\sigma_1^2 L_1^2 - 4\pi G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)}{2k\left[\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2\right]} \\
b_{12} &= 2k \\
b_{13} &= \alpha_2 + k = 2k + \frac{\sigma_2^2 L_2^2 - 4\pi G\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)}{2k\left[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2\right]} \\
b_{21} &= \sigma_1 L_1 - 4\pi G\rho_1 \\
b_{22} &= \frac{4\pi G k_x^2(\rho_1 M_1^2 \sigma_2 L_2 + \rho_2 M_2^2 \sigma_1 L_1)}{\sigma_1\sigma_2 L_1 L_2} \\
b_{23} &= \frac{4\pi G\rho_1 k_x^2 M_1^2}{\sigma_1 L_1} - \sigma_1 L_1 + 4\pi G\rho_1 \\
b_{31} &= \frac{\sigma_1 L_1\left[\sigma_1^2 L_1^2 - 4\pi G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\right]}{8\pi k G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\left[\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2\right]} \\
b_{32} &= \frac{k(\sigma_2 L_2 - \sigma_1 L_1)}{\sigma_1\sigma_2 L_1 L_2} \\
b_{33} &= \frac{k}{\sigma_1 L_1} + \frac{k\sigma_2 L_2}{4\pi G\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)} \\
&\quad + \frac{\sigma_2 L_2\left[\sigma_2^2 L_2^2 - 4\pi G\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)\right]}{8\pi k G\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)\left[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2\right]}
\end{aligned}$$

By expanding the determinant in Eq. (36), we obtain the most general case of dispersion relation described by

$$\begin{aligned}
&2\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\Big\{8\pi G k^2\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 \\
&+k_x^2 M_2^2 C_2^2] + 2k^2\sigma_1\sigma_2 L_1 L_2\left[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2\right] \\
&+\sigma_1\sigma_2 L_1 L_2\left[\sigma_2^2 L_2^2 - 4\pi G\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)\right]\Big\} \\
&\times\Big\{\pi G k_x^2\left(\rho_1 M_1^2\sigma_2 L_2 + \rho_2 M_2^2\sigma_1 L_1\right)\left[\sigma_1^2 L_1^2 - 4\pi G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\right] \\
&-k^2\sigma_1\sigma_2 L_1 L_2\left(\sigma_1 L_1 - 4\pi G\rho_1\right)\left[\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2\right]\Big\} \\
&+2k^2\rho_2\left(\sigma_2 L_2 + k_x^2 M_2^2\right)[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2][4\pi G\rho_1 k_x^2 M_1^2
\end{aligned}$$

$$
\begin{aligned}
&-\sigma_1\sigma_2 L_1 L_2 + 4\pi G\rho_2\sigma_1 L_1]\left\{\sigma_1^2\sigma_2 L_1^2 L_2[\sigma_1^2 L_1^2 - 4\pi G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)]\right.\\
&-2\pi G\rho_1\left(\sigma_2 L_2 - \sigma_1 L_1\right)\left(\sigma_1 L_1 + k_x^2 M_1^2\right)[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2]\}\\
&+2\pi G\rho_2\sigma_1 L_1\left(\sigma_2 L_2 + k_x^2 M_2^2\right)\left\{2k^2\rho_1\left(\sigma_2 L_2 - \sigma_1 L_1\right)\left(\sigma_1 L_1 - 4\pi G\rho_1\right)\right.\\
&\times\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\left[\left(M_1^2 + C_1^2\right)\sigma_1 L_1 + k_x^2 M_1^2 C_1^2\right]\\
&\left.-k_x^2\sigma_1 L_1\left(\rho_1 M_1^2\sigma_2 L_2 + \rho_2 M_2^2\sigma_1 L_1\right) + \sigma_1^2 L_1^2 - 4\pi G\rho_1\left(\sigma_1 L_1 + k_x^2 M_1^2\right)\right\}\\
&\times\left\{4k^2\left[\left(M_2^2 + C_2^2\right)\sigma_2 L_2 + k_x^2 M_2^2 C_2^2\right] + \sigma_2^2 L_2^2 - 4\pi G\rho_2(\sigma_2 L_2\right.\\
&\left.+k_x^2 M_2^2)\right\} = 0
\end{aligned}
\tag{37}
$$

Suppose that the two streams be moving in opposite directions with equal velocities parallel to the x-axis, i.e. $\mathbf{V}_1 = V\mathbf{i}$ and $\mathbf{V}_2 = -V\mathbf{i}$. In this case

$$\sigma_1 = n + ik_x V \quad \text{and} \quad \sigma_2 = n - ik_x V \tag{38}$$

Substituting for L_r, σ_r ($r = 1, 2$) from Eqs. (26) and (38), respectively, into Eq. (37) we get the following 20th-order equation for the growth rate n,

$$
\begin{aligned}
&\beta_1 n^{20} + \beta_2 n^{19} + \beta_3 n^{18} + \beta_4 n^{17} + \beta_5 n^{16} + \beta_6 n^{15} + \beta_7 n^{14} + \beta_8 n^{13}\\
&+\beta_9 n^{12} + \beta_{10} n^{11} + \beta_{11} n^{10} + \beta_{12} n^9 + \beta_{13} n^8 + \beta_{14} n^7 + \beta_{15} n^6 + \beta_{16} n^5\\
&+\beta_{17} n^4 + \beta_{18} n^3 + \beta_{19} n^2 + \beta_{20} n + \beta_{21} = 0
\end{aligned}
\tag{39}
$$

where the coefficients $\beta_1 - \beta_{21}$ are quite complicated. These coefficients are not given here as they are quite lengthy expressions involving the wave number, and the parameters characterizing the effects of the porosity of porous medium, medium permeability, streaming velocity Alfvén and sound velocities, fluid densities, fluid viscosities, and gravitational constant.

5. STABILITY DISCUSSION

If we simplify equation (39) and setting $n = 0$, we can get the critical wave number k^* when the instability sets in. However this equation is very complicated to be handled in its most general form for k^*, and so it is not possible to obtain an analytical expression. Let us firstly consider the case of non-porous medium in which the perturbation propagates along the streaming motion (positive direction of x-axis) with wave number k_x, and perpendicular to the streaming motion (positive direction of y-axis) with wave number k_y, In this case, we have

$$\sigma_1^2 = \sigma_2^2 = \sigma^2(= -k_x^2 V^{\,2}), \quad n = 0 \tag{40}$$

If we put the above values given by equation (40) into the dispersion relation (39), and using the values $\varepsilon = 1$ and $k_1 \to \infty$, and simplify the resulting equation, we get

$$\frac{\alpha_1}{\rho_1(\sigma^2 + k_x^2 M_1^2)} + \frac{\alpha_2}{\rho_2(\sigma^2 + k_x^2 M_2^2)} = 0 \tag{41}$$

where α_r^2 can be obtained from equation (25) as

$$\alpha_r^2 = k^2 + \frac{\sigma^4 - 4\pi G\rho_r\left(\sigma^2 + k_x^2 M_r^2\right)}{(M_r^2 + C_r^2)\,\sigma^2 + k_x^2 M_r^2 C_r^2}, \quad r = 1, 2 \tag{42}$$

On further simplification of equation (41) with the help of equation (42), we get

$$\rho_2^2\left(\sigma^2 + k_x^2 M_2^2\right)^2\left[\left(M_2^2 + C_2^2\right)\sigma^2 + k_x^2 M_2^2 C_2^2\right][\left(\sigma^2 + k^2 M_1^2\right)\sigma^2 + \left(\sigma^2 + k_x^2 M_1^2\right)\left(C_r^2 k^2 - 4\pi G\rho_1\right)]$$

$$= \rho_1^2\left(\sigma^2 + k_x^2 M_1^2\right)^2\left[\left(M_1^2 + C_1^2\right)\sigma^2 + k_x^2 M_1^2 C_1^2\right][\left(\sigma^2 + k^2 M_1^2\right)\sigma^2 + \left(\sigma^2 + k_x^2 M_2^2\right)\left(C_2^2 k^2 - 4\pi G\rho_2\right)] \quad (43)$$

which reduces to the form

$$\frac{k_x^2}{\Lambda_1^2} + \frac{k_y^2}{\Lambda_2^2} = 1 \quad (44)$$

where

$$\Lambda_1^2 = \frac{4\pi G\rho_1\rho_2\left[\rho_2 A_2^2(V^2 - M_2^2) - \rho_1 A_1^2(V^2 - M_1^2)\right]}{\left[\rho_2^2 A_2^2(V^2 - M_2^2)(C_1^2 - V^2) - \rho_1^2 A_1^2(V^2 - M_1^2)(C_2^2 - V^2)\right]} \quad (45)$$

$$\Lambda_2^2 = \frac{4\pi G\rho_1\rho_2(V^2 - M_1^2)(V^2 - M_2^2)\left[\rho_2 A_2^2(V^2 - M_2^2) - \rho_1 A_1^2(V^2 - M_1^2)\right]}{A_1^2 A_2^2\left[\rho_2^2(V^2 - M_2^2)^2 - \rho_1^2(V^2 - M_1^2)^2\right]} \quad (46)$$

in which

$$A_r^2 = \left(M_r^2 + C_r^2\right)V^2 - M_r^2 C_r^2 \quad (47)$$

We observe that the perturbation propagating along both the axes x and y with wave numbers k_x and k_y, respectively, and magnetic field acting along the x-axis, leads to the value of the critical wave number k^* lying on elliptic orbit in the first quadrant given by equation (44), whose axes are $k_x = 0$ and $k_y = 0$. This result agrees with the result found by Singh and Khare[20]. The stability of the system depends on streaming velocity, magnetic field, densities and velocity of sound of the two medium.

Let us secondly consider the instability of two superposed medium of densities ρ_1 and ρ_2 in the case of non-porous medium in which streaming motion is absent, i.e. when $V = 0$, the perturbation propagates along the streaming motion (positive direction of x-axis) with wave number k_x, and perpendicular to the streaming motion (positive direction of y-axis) with wave number k_y, In this Rayleigh-Taylor case, we have

$$\sigma_1^2 = \sigma_2^2 = 0, \quad n = 0 \quad (48)$$

Putting this value in the dispersion relation (41), and in view of the fact

$$\rho_1 M_1^2 = \rho_2 M_2^2 = \mu H^2/(4\pi) \quad (49)$$

Therefore, using equations (48) and (49), then equation (41) reduces to

$$\alpha_1 + \alpha_2 = 0 \quad (50)$$

since α_1, α_2 are non-negative, hence, this gives

$$\alpha_1 = \alpha_2 = 0 \quad (51)$$

From equation (42) in case of marginal state, the critical wave number k^* is given by

$$C_r^2 k^{*2} - 4\pi G\rho_r = 0 \quad (52)$$

Thus, we have

$$k^* = \sqrt{k_x^2 + k_x^2} = \left[\frac{4\pi G\rho_r}{C_r^2}\right]^{1/2}, \quad r = 1, 2 \quad (53)$$

This shows that the critical wave number lies on the circular path in the first quadrant given by equation (53), and the stability is unaffected by the magnetic field. The stability criteria here disentangles the two-media systen, The two fluids become independent of each other.

Numerical solutions, in the presence of porous medium, may lead to the values which are related to the instability criterion for physical problem. In order to study the effects of various physical parameters on the growth rate instability, we have performed numerical calculations of the dispersion relation (39), using Mathematica 9, to locate the roots of the growth rate *n* againt the wave number k for various values of the parameters included in the analysis. These calculations are presented in Figs. (1)-(6) to show the variation of the growth rate with wave number of the considered system for different values of Alfvén velocities, fluid viscosities, sound velocities, medium permeability, streaming velocity, and the porosity of porous medium, respectively.

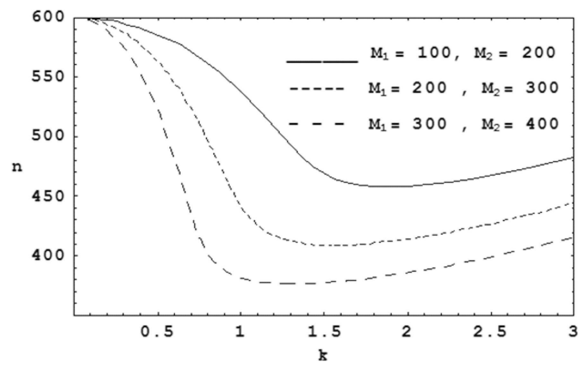

Figure 1

Variation of growth rate n with wave number k for various values of the Alfvén velocities M_1 and M_2 in the system $\rho_1 = 0.01$, $\rho_2 = 0.02$, $k_1 = 0.5$, $G = 6.6 \times 10^{-11}$, $V = 100$, $\varepsilon = 0.3$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $C_1 = 100$, $C_2 = 150$, for the cases ($M_1 = 100$, $M_2 = 200$), ($M_1 = 200$, $M_2 = 300$) and ($M_1 = 300$, $M_2 = 400$)

Fig. (1) shows the variation of the negative real part of growth rate *n* with the wave number *k* for various values of Alfvén velocities M_1 and M_2. It is clear from this figure that, for any wave number value, the negative Re(*n*) increases by increasing the Alfvén velocities, which indicates that the Alfvén velocities for the medium M_1 and M_2 have stabilizing effects. It is seen also from Fig. (1) that, for fixed values of M_1 and M_2, the negative Re(*n*) decreases by increasing the wave nymber *k* till a fixed critical wave number value after which the negative Re(*n*) increases for higher wave number values, which indicates that the system is unstable for small wave number values and then it is stable afterwards. Note that this critical wave number values decrease by increasing the Alfvén velocities M_1 and M_2 values.

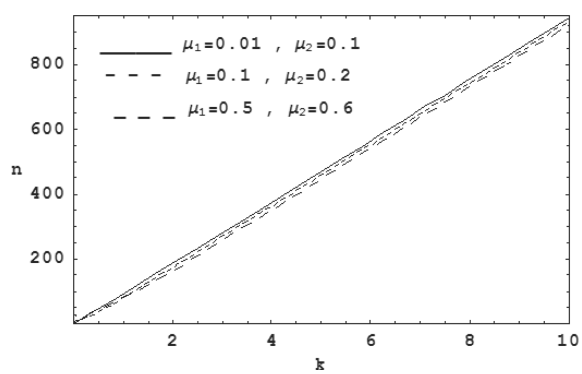

Figure 2

Variation of growth rate n with wave number k for various values of the fluid viscosities μ_1 and μ_2 in the system$\rho_1 = 0.01$, $\rho_2 = 0.02$, $k_1 = 0.5$, $M_1 = 100$, $M_2 = 150$, $G = 6.6 \times 10^{-11}$, $V = 100$, $\varepsilon = 0.3$, $C_1 = 200$, $C_2 = 250$, for the cases ($\mu_1 = 0.01$, $\mu_2 = 0.1$), ($\mu_1 = 0.1$, $\mu_2 = 0.2$) and ($\mu_1 = 0.5$, $\mu_2 = 0.6$)

Fig. (2) shows the variation of the negative real part of growth rate n with the wave number k for various values of dynamic viscosities μ_1 and μ_2. It is clear from this figure that, for any wave number value, the negative Re(n) decreases by increasing the dynamic viscosities, which indicates that the dynamic viscosities μ_1 and μ_2 have destabilizing effects. It is seen also from Fig. (2) that, for fixed values of μ_1 and μ_2, the negative Re(n) increases linearly by increasing the wave number k, which indicates that the system is stable for all wave number values.

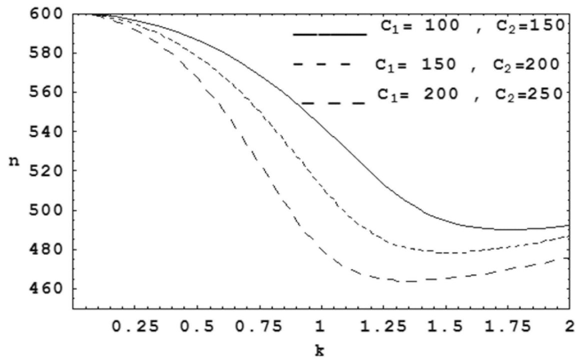

Figure 3
Variation of growth rate n with wave number k for various values of the sound velocities C_1 and C_2 in the system $\rho_1 = 0.01$, $\rho_2 = 0.02$, $k_1 = 0.5$, $M_1 = 100$, $M_2 = 150$, $G = 6.6 \times 10^{-11}$, $V = 100$, $\varepsilon = 0.3$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, for the cases ($C_1 = 100$, $C_2 = 150$), ($C_1 = 150$, $C_2 = 200$) and ($C_1 = 200$, $C_2 = 250$)

Fig. (3) shows the variation of the negative real part of growth rate n with the wave number k for various values of sound velocities C_1 and C_2. It is clear from this figure that, for any wave number value, the negative Re(n) decreases by increasing the sound velocities, which indicates that the sound velocities C_1 and C_2 have destabilizing effects. It is seen also from Fig. (3) that, for fixed values of C_1 and C_2, the negative Re(n) decreases by increasing the wave nymber k till a fixed critical wave number value after which the negative Re(n) increases for higher wave number values, which indicates that the system is unstable for small wave number values and then it is stable afterwards. Note that this critical wave number values decreases by increasing the sound velocities C_1 and C_2 values.

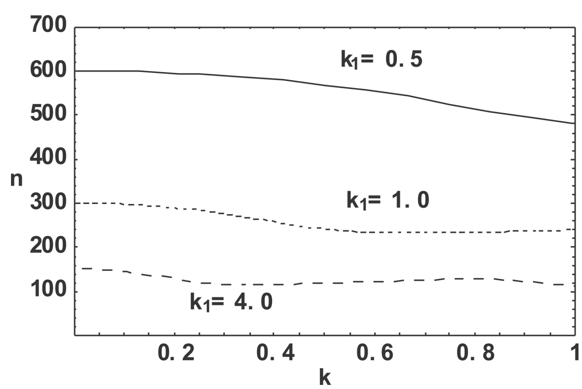

Figure 4
Variation of growth rate n with wave number k for various values of the medium permeability k_1 in the system$\rho_1 = 0.01$, $\rho_2 = 0.02$, $M_1 = 100$, $M_2 = 150$, $G = 6.6 \times 10^{-11}$, $V = 100$, $\varepsilon = 0.3$, $\mu_1 = 0.8$, $\mu_2 = 0.9$, $C_1 = 200$, $C_2 = 250$, for the cases $k_1 = 0.5, 1$ and 4

Fig. (4) shows the variation of the negative real part of growth rate n with the wave number k for various values of medium permeability k_1. It is clear from this figure that, for any wave number value, the negative

Re(n) increases by increasing the medium permeability, which indicates that the medium permeability k_1 has a stabilizing effect. It is seen also from Fig. (4) that, for fixed value of k_1, the negative Re(n) increases linearly by increasing the wave number k which indicates that the system is stable for all wave number values.

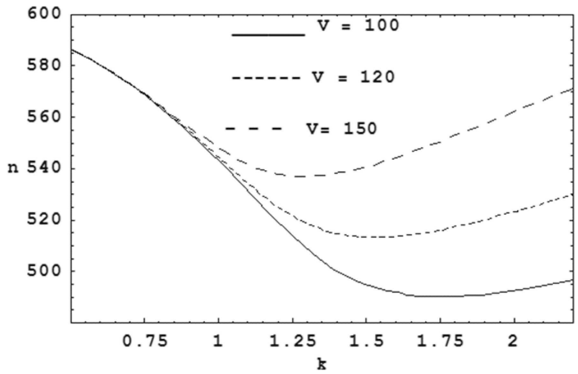

Figure 5

Variation of growth rate n with wave number k for various values of the stream velocity V in the system$\rho_1 = 0.01$, $\rho_2 = 0.02$, $k_1 = 0.5$, $M_1 = 100$, $M_2 = 150$, $G = 6.6 \times 10^{-11}$, $\varepsilon = 0.3$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $C_1 = 100$, $C_2 = 150$, for the cases $V = 100, 120$ and 150

Fig. (5) shows the variation of the negative real part of growth rate n with the wave number k for various values of streaming velocity V. It is clear from this figure that, for wave number values $k \geq 0.75$, the negative Re(n) increases by increasing the streaming velocity, which indicates that the streaming velocity for the medium V has a stabilizing effect. Note that, for very small wave number values $0 < k < 0.75$, the obtained curves are coincide, and this means that the streaming velocity has no effect on the stability of the considered system in this wave numbers range. It is seen also from Fig. (5) that, for fixed value of V, the negative Re(n) decreases by increasing the wave nymber k till a fixed critical wave number value after which the negative Re(n) increases for higher wave number values, which indicates that the system is unstable for small wave number values and then it is stable afterwards. Note that this critical wave number values increases by increasing the streaming velocity values.

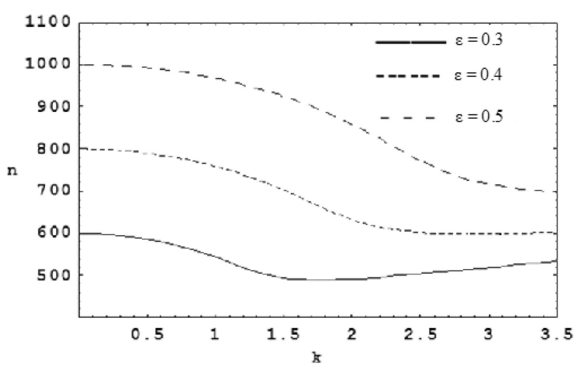

Figure 6

Variation of growth rate n with wave number k for various values of the medium porosity ε in the system$\rho_1 = 0.01$, $\rho_2 = 0.02$, $k_1 = 0.5$, $M_1 = 100$, $M_2 = 150$, $G = 6.6 \times 10^{-11}$, $V = 100$, $\mu_1 = 0.1$, $\mu_2 = 0.2$, $C_1 = 100$, $C_2 = 150$, for the cases $\varepsilon = 0.3, 0.4$ and 0.5

Fig. (6) shows the variation of the negative real part of growth rate n with the wave number k for various values of the porosity of porous medium ε. It is clear from this figure that, for any wave number value, the negative Re(n) increases by increasing the porosity of porous medium, which indicates that the

the porosity of porous medium ε has a stabilizing effect. It is seen also from Fig. (6) that, for fixed values of ε, the negative Re(n) decreases by increasing the wave number k till a fixed critical wave number value after which the negative Re(n) increases for higher wave number values, which indicates that the system is unstable for small wave number values and then it is stable afterwards. Note that this critical wave number values decreases by increasing the porosity of porous medium ε values.

6. CONCLUSIONS

In this paper, we have studied the hydromagnetic instability of two superposed homogeneous conducting fluids streaming in opposite directions with equal horizontal velocities under gravitational force and high Alfvén and sound velocities in porous medium. The perturbation propagation is taken simultaneously along and perpendicular to streaming motion in the horizontal interface. The obtained results can be summarized as follows:

(1) The Alfvén and sound velocities have destabilizing effects.

(2) The fluid velocities and porosity of porous medium have stabilizing effects.

(3) The medium permeability has a slightly stabilizing effect, while the dynamic viscosities have slightly destabilizing effect.

(4) The growth rate varies linearly with wave number for differents values of medium permeability or dynamic viscosities.

(5) Finally, the limiting cases of non-porous medium have also been studied for both streaming and stationary fluids, and show that the critical wave numbers lying on elliptic orbit and a circular path in the first quadrant, respectively.

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REFERENCES

[1] Boller, B. R., & Stolov, H. L. (1970). Explorer 18 Study of the Stability of the Magnetopause Using a Kelvin-Helmholtz Instability Criterion. *J. Geophys. Res., 78*, 8078-8086.

[2] Ray, T. P. (1982). Kelvin-Helmholtz Instabilities in Cometary Ion Tails. *Planet. Space Sci., 30*, 245-250.

[3] Ferrari, A., Trussoni, E., & Zaninetti, L. (1978). Relativistic Kelvin-Helmholtz Instabilities in Extragalactic Radio Sources. Astron. Astrophys., 64, 43-52.

[4] Ray, T. P. (1981). Kelvin-Helmholtz Instabilities in Radio Jets. *Mon. Notes R. Astron. Soc., 196*, 195-207.

[5] Blondin, J. M., Fryxell, B. A., & Könrgl, A. (1990). The Structure and Evolution of Radiatively Cooling Jets. *Astrophys. J., 360*, 370-386.

[6] Bodo, G., Massaglia, S., Rossi, P., Trussoni, E., & Ferrari, A. (1993). Kelvin-Helmholtz Instabilities in Radiating Flows. *Phys. Fluids, 5*, 405-411.

[7] S. Chandrasekhar, S. (1961). *Hydrodynamic and Hydromagnetic Stability*. Oxford: Clarendon.

[8] Sen, A. K. (1964). Effect of Compressibility on Kelvin-Helmholtz Instability in a Plasma. *Phys. Fluids, 7*, 1293-1298.

[9] Gerwin, R. A. (1968). Stability of the Interface Between Two Fluids in Relative Motion. *Rev. Mod. Phys.*, *40*, 652-658.

[10] Sen, A. K. (1963). Stability of Hydromagnetic Kelvin-Helmholtz Discontinuity. *Phys. Fluids*, *6*, 1154-1163.

[11] Pu, Z. Y., & Kivelson, M. G. (1983). Kelvin-Helmholtz Instability at the Magnetopause: Solution for Compressible Plasmas. *J. Geophys. Res.*, *88*, 841-852.

[12] Ray, T. P. (1982). The Effect of Simple Shear Layer on the Growth of the Kelvin-Helmholtz Instability. *Mon. Notes R. Astron. Soc.*, *198*, 617-625.

[13] Jeans, J. H. (1902). The Stability of Spherical Nebula. *Phil Trans. Roy. Soc. London A*, *199*, 1-53.

[14] Chandrasekhar, S. (1954). Gravitational Instability of a Non-Linear Magnetic Fluid. *Astrophys. J.*, *119*, 7-9.

[15] Chandrasekhar, S. (1955). Wave Propagation and Instabilities in a Rotating Anisotropic Plasma. *Vistas in Astronomy I, Pergamon Press, Oxford, Vol. I, Ed. A. Beer*, 344-347.

[16] Chandrasekhar, S. & Fermi, E. (1953). Gravitational Instability Problem of Nonuniform Media. *Astrophys. J.*, *118*, 116–141.

[17] Sengar, R. S. (1984). Stability of Two Superposed Gravitating Streams in a Uniform Vertical Magnetic Field. *Proc. Nat. Acad. Sci. India*, *54*, 438-443.

[18] Radwan, A. E., & Elazab, S. S. (1988). Magnetogravitational Stability of Variable Streams. *J. Phys. Soc. Japan*, *57*, 461-463.

[19] Singh, R. P., & Khare, H. C. (1992). Stability of Two Superposed Homogeneous Fluids. *Acta Phys. Hungarica*, *72*, 13-22.

[20] Singh, R. P., & Khare, H. C. (1991). Stability of Two Superposed Homogeneous Fluids Under Uniform Magnetic Field. *Astrophys. Space Sci.*, *186*, 125-135.

[21] Shaikh, S., & Khan, A. (2010). Instability of Thermally Conducting Self-Gravitating Systems. *J. Mod. Phys.*, *1*, 77-82.

[22] Shaikh, S., Khan, A., & Bhatia, P. K. (2007). Stability of Thermally Conducting Plasma in a Variable Magnetic Field. *Astrophys. Space Sci.*, *312*, 35-40.

[23] Khan, A., Tak, S. S., & Sharma, N. (2009). Jeans' Gravitational Instability of Rotating Plasma. *African J. Math. Phys.*, *7*, 61-70.

[24] Khan, A., Tak, S. S., & Patni, P. (2011). Effects of Hall Currents on Jeans' Gravitational Instability of Rotating Unbounded Plasma. *Rom. J. Phys.*, *56*, 209-223.

[25] El-Sayed, M. F., & Mohamed, R. A. (2010). Magnetogravitational Instability of Thermally Conducting Rotating Viscoelastic Fluid with Hall Current in Brinkman Porous Medium. *J. Porous Media*, *13*, 779-798.

[26] El-Sayed, M. F., & Mohamed, R. A. (2011). Magnetogravitational Instability of Rotating Viscoelastic Partially Ionized Plasma in the Presence of an Oblique Magnetic Field and Hall Current. *ISRN Mech. Eng.*, *2011*, 597172.

[27] Janaki, M. S., Chakrabarti, N., & Banerjee, D. (2011). Jeans Instability in a Viscoelastic Fluid. *Phys. Plasmas*, *18*, 012901.

[28] Prajapati, R. P., Parihar, A. K., & Chhajlani, R. K. (2008). Self-Gravitational Instability of Rotating Anisotropic Heat-Conducting Plasma. *Phys. Plasmas*, *15*, 012107.

[29] Prajapati, R. P., Pensia, R. K., Kaothekar, S., & Chhajlani, R. K. (2010). Self-Gravitational Instability of Rotating Viscous Hall Plasma with Arbitrary Radiative Heat-Loss Functions and Electron Inertia. *Astrophys. Space Sci.*, *327*, 139-154.

[30] Pandey, B. P., Van Der Holst, B., Vranjes, J., & Poedts, S. (2003). Jeans' Instability of an Inhomoge-

neous Streaming Dusty Plasma. *Pramana J. Phys., 61*, 109-120.

[31] Shukla, P. K., & Stenflo, L. (2006). Jeans Instability in a Self-Gravitating Dusty Plasma. *Proc. Roy. Soc. London A, 462*, 403-407.

[32] Yang, X.-S., Liu, S.-Q., & Li, X.-Q. (2010). Gravito-Magneto-Modulational Instability of Self- Gravitating System. *Gravitation and Cosmology, 16*, 316-322.

[33] Pop, I., & Ingham, D. B. (2001). *Convective Heat Transfer: Mathematical and Computational Modeling of Viscous Fluid and Porous Medium*, Oxford: Pergamon.

[34] Dullien, F. A. I. (1992). *Porous Media*, New York: Academic Press.

[35] Ingham, D. B., & I. Pop, I. (1998). *Transport Phenomena in Porous Media*, New York: Springer-Verlag.

[36] Nield, D. A., & A. Bejan, A. (2005). *Convection in Porous Media*, New York: Springer-Verlag.

[37] Vafai, K. (2005). *Handbook of Porous Media*, 2nd Ed., New York: Taylor and Francis.

[38] Raghavan, R., & Marsden, S. S. (1973). A Theoretical Study of the Instability in the Parallel Flow of Immiscible Liquids in a Porous Medium, *Q. J. Mech. Appl. Math., 26*, 205-216.

[39] Sharma, R. C., & Spanos, J. T. (1982). The Instability of Streaming Fluids in a Porous Medium. *Can. J. Phys., 60*, 1391-1395.

[40] Bau, H. H. (1982). Kelvin-Helmholtz Instability for Parallel Flow in Porous Media: A Linear Theory. *Phys. Fluids, 25*, 1719-1722.

[41] Melcher, J. R. (1981). *Cintinuum Electromechanics*, Mass.: MIT Press.

[42] Blokhin, A., & Trakhinin, Yu. (2003). *Stability of Strong Discontinuities in Magnetohydrodynamics and Electrohydrodynamics*. New York: Nova Sience Publishers.

[43] Geindreau, C., & Auriault, J. L. (2001). Magnetohydrodynamic Flow Through Porous Media. *Comltes Rendus Acad. Sci. Ser. II B, 329*, 445-450.

[44] Geindreau, C., & Auriault, J. L. (2002). Magnetohydrodynamic Flow Through Porous Media. *J. Fluid Mech., 466*, 343-363.

[45] Zakaria, Z., Sirwah, M. A., & Alkharashi, S. (2009). Instability Through Porous Media of Three Layers Superposed Conducting Fluids. *Eur. J. Mech. B/Fluids, 28*, 259-270.