Multifractality in the Philippine Foreign Exchange Market

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Abstract
This paper investigates the multifractality of the daily exchange rate between the Philippine Peso and the US Dollar from January 2, 1998 to July 31, 2013 using the multifractal detrended fluctuation analysis. The behavior of the generalized Hurst exponent detects the presence of multifractality in the peso-dollar exchange rate. Moreover, the small fluctuations in the exchange rate show persistence. By quantifying the contribution of long-range correlations and broad fat-tail distributions to multifractality, the paper shows that the multifractility of daily peso-dollar exchange rate is mainly due to the broad fat-tail distributions.

Key words: Multifractality; Hurst exponents; Financial markets; Efficiency

INTRODUCTION
Fractal as introduced by Mandelbrot (1977; 1982) describes a geometric patterns with large degree of self-similarities at all scales. The smaller piece of a pattern can be said to be a reduced-form image of a larger piece. This characteristic is used to measure fractal dimensions as a fraction rather than an integer. Some examples of fractal shapes are rugged coastlines, mountain heights, cloud outlines, river tributaries, tree branches, blood vessels, cracks, wave turbulences and chaotic motions.

However, there are self-similar patterns that involve multiple scaling rules which are not sufficiently described by a single fractal dimension but by a spectrum of fractal dimensions instead. Generalizing this single dimension into multiple dimension differentiates multifractal from fractal discussed earlier. To distinguish multifractal from single fractal, the term monofractal is used for single fractal in this paper. Among the natural systems that have been observed to have a multifractal property are earthquakes (Parisi & Frisch, 1985), heart rate variability (Goldberger et al., 2002) and neural activities (Zheng, Gao, Sanchez, Principe, & Okun, 2005).

Mandelbrot (1997) introduced multifractal models to study economic and financial time series in order to address the shortcomings of traditional models such as fractional Brownian motion and GARCH processes which are not appropriate with the stylized facts of the said time series such as long-memory and fat-tails in volatility. Further studies confirmed multifractility in stock market indices (Katsuragi, 2000; Jiang & Zhou, 2008; Sun, Chen, Wu, & Yuan, 2001; Oswiecimka, Kwapien, Drozdz, Gorski, & Rak, 2006; Zunino et al., 2009; Zunino et al., 2008; Lye & Hooy, 2012; Lyo, Tian, Zhou, & Li, 2012; Yuan, Zhuang, & Jin, 2009; Wang, Zhou, & Xiang, 2012), foreign exchange rates (Vandewalle & Ausloos, 1998; Norouzzadeh & Rahmani, 2006; Oh et al., 2012; Ioan, Anita, & Razvan, 2012) and interest rates (Cajueiro & Tabak, 2007), to name a few.

This paper investigates the presence of multifractality of the exchange rate between the Philippine Peso and the US Dollar. The period after the 1997 Asian financial crisis is chosen since the pre-crisis exchange rate regime was a “de facto” peg (Aquino, 2005). This implies that the peso-dollar exchange rate prior to the 1997 crisis did not fully reflect the heterogeneous interactions in the Philippine foreign exchange market. The paper shows that the small fluctuations in the exchange rate are persistence. Furthermore, since multifractality can be due to long-
range correlations or due to broad fat-tail distributions, this paper identifies which of the two factors dominates the multifractality in peso-dollar exchange rate.

The paper is arranged as follows. Methodology is discussed in Section 2. Data is described in Section 3. Presentation of results is in Section 4. Finally, the paper concludes in Section 5.

1. METHODOLOGY

In measuring multifractality, the paper uses the method of Multifractal Detrended Fluctuation Analysis (MFDFA) as outlined by Kantelhardt et al. (2002). Matlab codes used are based on Ihlen (2012). The procedure is summarized in the following steps.

1. Given a time series $u_i, i = 1, \ldots, N$, where $N$ is the length, create a profile $Y(k) = \sum_{i=1}^k u_i - \bar{u}$, $k = 1, \ldots, N$, where $\bar{u}$ is the mean of $u$.

2. The profile $Y(k)$ is divided into $N_s = N/S$ non-overlapping segment of length $s$.

3. For each segment $v = 1, \ldots, N_s$, the detrended time series for length $s$ is denoted by $Y_s(i) = Y(i) - Y_s(i)$, where $Y_s(i)$ is the $m^{th}$ order fitting polynomial in the $v^{th}$ segment.

4. For each of the $N_s$ segments, the variance of $Y_s(i)$ is computed as $\hat{F}_s^2(v) = \frac{1}{s} \sum_{i=1}^s [Y_s(i+1)-Y_s(i)]^2$.

5. The $q^{th}$ order fluctuation function is obtained by $F_q(s) = \{\hat{F}_s^2(v)\}^{1/2}$. If the time series are long-range correlated then $F_q(s)$ is distributed as power laws, $F_q(s) \sim s^{h(q)}$. The exponent $h(q)$ is called as the generalized Hurst exponent. The degree of multifractality can be quantified as $\Delta h(h_{\text{max}} - h_{\text{min}})$.

To identify whether the multifractality is due to long-range correlations or is due to broad fat-tail distributions, shuffled data and surrogated data are generated. Shuffling the data will remove the long-range correlation in the time series. It is done by randomizing the order of the original data. The multifractality due to long-range correlation can be computed as $h_c = \Delta h - \Delta h_r$ where the index $c$ refers to shuffled data.

Surrogated data is produced by randomizing the phases of original data in Fourier space. This will make the data to have normal distribution. The multifractality due to broad fat-tail distributions can be measured as $h_d = \Delta h - \Delta h_r$ where the index $d$ refers to surrogated data.

2. DATA

The log daily returns of exchange rate between the Philippine Peso and the US Dollar from January 2, 1998 to July 31, 2013 is used for a total of 3847 observations. The exchange rate is quoted as the price of a dollar in terms of peso. The exchange rate data is downloaded from the online statistics page of the Bangko Sentral ng Pilipinas website: http://www.bsp.gov.ph/statistics/statistics_online.asp.

![Figure 1](image_url)

**Figure 1**

(a) Plots of the Peso-dollar Exchange Rate (b) Its Daily Returns (c) Shuffled Time Series (d) Surrogated Time Series
3. RESULTS

Figure 1 shows the plots of the peso-dollar exchange rate, its daily returns, and the associated shuffled and surrogated time series of its daily returns. The original daily returns and the shuffled time series show some extreme fluctuations which is a sign of having fat-tail distribution. The surrogated time series do not have extreme fluctuation, a characteristic of a normal distribution.

In doing the MF DFA procedure, \(m=3\) is used as the order of polynomial fit in Step 3. The length \(s\) varies from 20 to \(\frac{N}{4}\) with a step of 4 as suggested by Kantelhardt et al. (2002). Finally, \(q\) runs from \(-10\) to 10 with a step of 0.5. Figure 2 presents the generalized Hurst exponents for the original returns, shuffled returns and surrogated returns. For monofractals, the Hurst exponent is independent of \(q\) which is also equal to the generalized Hurst exponents of multifractals at, \(q=2\), that is, \(h(2)\). In other words, monofractals have only one single Hurst exponent which is \(h(2)\) regardless of the value of \(q\). In contrast, multifractals have a spectrum of generalized Hurst exponents which vary depending upon the value of \(q\). It is noted in Figure 2 that for the daily returns time series, \(h(q)\) is dependent upon \(q\). As \(q\) increases, \(h(q)\) decreases. This is a confirmation that the peso-dollar exchange rate series is indeed a multifractal which means monofractal models are not appropriate for this time series.

![Figure 2](Image)

**Generalized Hurst Exponent, \(h(q)\), as a Function of \(q\) for the Original, Shuffled and Surrogated Daily Returns**

The value of \(h(q)\) tells something about the behavior of the fluctuations in the time series. When \(h(q)>0.5\), the fluctuations are persistent. This means that an increase (decrease) in the previous period is followed by another increase (decrease) in succeeding period. When \(h(q)<0.5\), the fluctuations are anti-persistent. This implies that an increase (decrease) in the previous is followed by a decrease (increase) in succeeding period. The last case of \(h(q)=0.5\) implies that the fluctuations are just random walks.

Table 1 presents the generalized Hurst exponents, \(h(q)\) with values of \(q\) ranging from \(-10\) to 10 for the original return time series, shuffled and surrogated time series. For \(q>0\), \(h(q)\) describes the behavior of large fluctuations, while for \(q<0\), \(h(q)\) describes the behavior of small fluctuations. Since in Table 1, for \(q<0\), \(h(q)>0.5\). This suggests that small fluctuations in the peso-dollar exchange rate are persistent.

Since, \(h_1=\Delta h_1=0.5521 - 0.4498 = 0.1023\). While, \(h_2=\Delta h_2=0.5521 -0.0594 =0.4927\). We have \(h_1<h_2\). This means that the multifractality is mainly due to broad fat-tail distributions. This is consistent with the findings by Ioan et al. (2012).

### Table 1
**Generalized Hurst exponents, \(h(q)\) with \(-10\) to \(10\)**

<table>
<thead>
<tr>
<th>(q)</th>
<th>(-10)</th>
<th>(-9)</th>
<th>(-8)</th>
<th>(-7)</th>
<th>(-6)</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Original</strong></td>
<td>0.8505</td>
<td>0.8396</td>
<td>0.8264</td>
<td>0.8103</td>
<td>0.7908</td>
<td>0.7670</td>
<td>0.7390</td>
<td>0.7081</td>
<td>0.6768</td>
<td>0.6466</td>
<td>0.6156</td>
</tr>
<tr>
<td><strong>Shuffled</strong></td>
<td>0.6876</td>
<td>0.6807</td>
<td>0.6726</td>
<td>0.6633</td>
<td>0.6525</td>
<td>0.6400</td>
<td>0.6258</td>
<td>0.6096</td>
<td>0.5913</td>
<td>0.5704</td>
<td>0.5463</td>
</tr>
<tr>
<td><strong>Surrogated</strong></td>
<td>0.5376</td>
<td>0.5348</td>
<td>0.5318</td>
<td>0.5287</td>
<td>0.5256</td>
<td>0.5224</td>
<td>0.5192</td>
<td>0.5160</td>
<td>0.5128</td>
<td>0.5096</td>
<td>0.5064</td>
</tr>
</tbody>
</table>

**CONCLUSION**

By applying MF DFA to the daily time series data of peso-dollar exchange rate, the paper provides an empirical evidence of multifractality in the time series. The plot of the generalized Hurst exponents is an unmistakable fingerprint of the presence of multifractality. This finding suggest that monofractal models like fractional Brownian motion and GARCH processes are not sufficient to capture the inherent richness of the time series properties of the peso-dollar exchange rate.

Moreover, the values of the generalized Hurst exponents suggest that small fluctuations in the exchange rate time series are persistent. This means that an increase...
in the exchange rate in the current period will be followed by another increase in the next period; and in the same manner, a decrease in the exchange rate in the current period will be followed by another decrease in the next period.

Finally, using shuffled data and surrogated data, it confirms that the multifractality is dominated by the broad fat-tail distributions.

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**REFERENCES**


