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The Research on Correlations of Credit Risk and Moral Hazard for the EG

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Abstract
This paper, under the assumption of risk-neutral, analyzed the relationship between the probability of occurrence of moral hazard and the default probability of the EG (enterprise group), theoretically demonstrated the inner mechanism between the moral hazard and credit risk of the EG, and highlighted the role of bank loan interest played in the control of moral hazard and credit risk. The research shows: (1) There is nonlinear relationship between the probability of occurrence of moral hazard and the default probability of the EG; (2) There is an exogenous variable, loan interest rate, which makes the default probability extremum.

Key words: EG (Enterprise Group); Moral Hazard; Default Probability; Loan Interest

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INTRODUCTION
The EG (Enterprise group) is a combination of enterprises, with the large enterprise as the core, based on the equity relationships, economic and technical, or contact management. The moral hazard in this paper means the business risk that the EG make use of the low-risk project to apply for loans to banks and transfer the loans to the high-risk project to take high profits.

The EG usually has complicated inner structures and many affiliated companies that are either in different stages of their development or under different industry environment, so the levels of benefits and risks of their proposed investment projects are with great differences as well. Countless cases show that the direct reason for operating or financial crisis of the EG is that they used loans for other purposes without following the loan agreements. The commercial banks are usually at a disadvantage of asymmetric information when the loan is transferred within the EG. In this situation, the flow of loan is very difficult to be supervised. Thus, there is the driven force for the occurrence of moral hazard. As the moral hazard can lead to risk business of the EG, thus increase their credit risk. The changes in credit risk in turn would affect the decision-making behavior of the EG, which would also lead to further change in the likelihood of occurrence of moral hazard. Thus, it is with great academic value and practical significance that we reveal the intrinsic link and mechanism of interaction between the moral hazard and credit risk of the EG.

By far, researches on credit risk are primarily around the measurement and prediction of default probability, such as the classic structure model by Merton [1], the simplified model under the assumption that the default events are random[2], and various kinds of econometric models based on statistics or artificial intelligence[3]. There are some papers about quantification of credit risk of the EG. For example, Chen studied infection mechanism of credit risk in the EG [4,5], and he also discussed moral hazard problem of the EG in the principal-agent framework[6]. However, there are rare literatures on the interaction between the moral hazard and credit risk of the EG.

We discussed the relationship and interaction between the probability of occurrence of moral hazard and the

default probability of the EG, and depicted the underlying mechanism between the moral hazard and credit risk at the theoretical level. The rest of this paper is organized as follows. Section 2 of this paper is some relative assumptions; the influence of interest rate on moral hazard and credit risk of the EG was analyzed respectively in section 3. On this basis, the transfer of moral hazard to credit risk and reaction of credit risk on moral hazard of the EG were discussed.

1. RESEARCH BACKGROUND & ASSUMPTIONS

As we mentioned, the moral hazard in this paper means that the EG use loans for other purposes without following the loan agreements. To simplify our discussion, we make an assumption that one EG is composed of two companies, Company *A* and Company *B*. *A* has a high-risk, high-yield investment project, Project *A*, which does not have a direct qualification to apply for loans to banks, whereas Company *B* has low-risk project, Project *B*, which can get the loans from banks. Generally, we made the following hypotheses.

Assumption 1: For the high-risk Project *A*, the investment capital is *x*. The success probability of it is P_A, and it will get a positive earning $x\theta_A$[1], while the failure probability of it is 1-P_A and it will get no earning at all. The distribution function of θ_A is $\Phi(.)$ and the density function is $\varphi(.)$

Assumption 2: The low-risk Project *B* has the characteristics of all general projects, of which the return yield can be described by using the model of mean-shifting investment technologies (Koskela, 2000) [7] . In other words, assume that the return yield of the Project *B* is continuous. For investment capital *x,* the return of the Project *B*, *t,* is the random return in $[0, x\theta_B]$, and its conditional distribution function is $F(\tau \mid x)$ while conditional density function is $f(\tau \mid x)$. For the Project *B*, xq_B is upper bound of investment capital *x*. If the revenue of the Project *B* is not enough to pay back loan principal and interest to the bank, it will be considered as a failure of the Project *B*.

Assumption 3: Once there is a failure of any projects, the whole EG defaults.

Considering the reality, we also made other hypotheses as follows.

Assumption 4: The game between bank and EG is dominant, in other words, the bank does not know anything about the high-risk information of the Project *A*.

Assumption 5: If the interest rate is high enough, the probability of default of the EG will increase with the rise of interest rate.

2. INFLUENCE OF MORAL HAZARD OF EG ON CREDIT RISK

In this section, we first discussed how bank loan interest rate influences both the probability of occurrence of moral hazard and the probability of default. And on this basis we discussed the transfer mechanism from moral hazard of the EG to credit risk.

2.1 Influence of Interest Rate on EG's Moral Hazard

We suppose that the EG grant a loan x_B with a interest rate *r* and invest it into the Project *B*. According assumption 2, if the Project *B* successes, the expected return of the EG is

$$V_B(x_B) = \int_{x_B(1+r)}^{\beta} [\tau - x_B(1+r)] f(\tau \mid x_B) d\tau \tag{1}$$

Where $\beta = x_B\theta$ represents the upper bound of x_B for the Project *B*.

Integrate formula (1) by parts, we can get

$$V_B(x_B) = [\beta - (r+1)x_B]F(\beta \mid x_B) - \int_{x_B(1+r)}^{\beta} F(\tau \mid x_B) d\tau \tag{2}$$

According to the property of distribution function, we know that $F(\beta \mid x_B)=1$, so formula (2) can be written as

$$V_B(x_B) = \beta - (r+1)x_B - \int_{x_B(1+r)}^{\beta} F(\tau \mid x_B) d\tau \tag{3}$$

Similarly, suppose that the EG grant a loan x_B in the name of the Project *B* with the same interest rate *r* but invest it into the Project *A*. According assumption 1, the expected return of the Project *A* is

$$V_A(x_B) = P_A[x_B\theta_A - x_B(1+r)] \tag{4}$$

If there is moral hazard in the EG, and the expected return of the Project *A* is greater than that of the Project *B*, then the EG will transfer the loan x_B to the Project *A*, which means it will occur moral hazard in the EG when the following formulas are set up.

$$V_A(x_B) = P_A[x_B\theta_A - x_B(1+r)] > \beta - (r+1)x_B - \int_{x_B(1+r)}^{\beta} F(\tau \mid x_B) d\tau = V_B(x_B) \tag{5}$$

Or

$$\theta_A > \frac{\beta - x_B(1+r)(1-P_A) - \int_{x_B(1+r)}^{\beta} F(\tau \mid x_B) d\tau}{P_A x_B} \tag{6}$$

Furthermore,

$$\frac{\beta - x_B(1+r)(1-P_A) - \int_{x_B(1+r)}^{\beta} F(\tau \mid x_B) d\tau}{P_A x_B} = H(r) \tag{7}$$

Then the probability of the occurrence of moral hazard is

[1]To depict the high risk of Project A, assume that random variable A has a compound distribution, and the probability of getting a positive return is small.

$$P_M=1-\Phi(H(r)) \tag{8}$$

Where $\Phi(.)$is the distribution function of θ_A.

The derivative on r of formula (7) is

$$\frac{\partial H(r)}{\partial r}=F(x_B(1+r)\big|x_B)-(1-P_A) \tag{9}$$

As the default probability of the low-risk project B is lesser than that of the high-risk project A, which is

$$1-P_A>F(x_B(1+r)\big|x_B) \tag{10}$$

So there is $\frac{\partial H(r)}{\partial r}<0$, which means $H(r)$is monotonic decreasing as the increasing of interest rate r. According to the property of distribution function that P_M is monotonic increasing, we get the following conclusion.

Conclusion 1: If there is moral hazard in the EG, then the probability of the occurrence of moral hazard will be monotonic increasing as the increase of bank loan interest rate.

2.2 Influence of Interest Rate on Enterprise Group's Credit Risk

If the EG grant a loan x from a bank with interest rate r, the default probability that the bank will face is

$$P_D=P_M(1-P_A)+(1-P_M)\ F(x(1+r)\big|x) \tag{11}$$

Where the first part of this formula represents probability of failure of the Project A when moral hazard occurs; the second part represents probability of failure of the Project B when there is no moral hazard.

By formula (11), we know that P_D, the default probability of whole EG, is not monotonic increasing with the increase of probability of moral hazard. Indeed, it depends on the return distribution of both Project A and Project B. In particular, when the probability of the probability of occurrence of moral hazard is zero, the default probability of whole EG is equal to the probability that the Project B fails[2].

Next, we studied the relationship between the default probability P_D of the EG and bank loan interest rate r. The derivative of formula (11) is

$$\frac{dP_D}{dr}=[1-P_A-F(x\ (1+r)\big|x)]\frac{dP_M}{dr}-(1-P_M)xf(x(1+r)\big|x) \tag{12}$$

On the another hand, we know from the conclusion 1 that the interest rate that makes the probability of the occurrence of moral hazard minimum should be r_0=0. Thus, rewrite the formula (12), we have

$$\frac{dP_D}{dr}\Big|_{r=r_0}=[1-P_A-F(x\big|x)]\frac{dP_M}{dr}\Big|_{r=r_0}-(1-P_M)xf(x\big|x) \tag{13}$$

As $f(\tau\,|\,x)$ is the income density function of the Project B, the probability of which that the Project B obtain positive earnings should be greater than that has zero earnings. So there should be

$$f(x(1+r)\big|x)>f(x\big|x) \tag{14}$$

According to the property of distribution function and conclusion 1, we have $F(x(1+r)\big|x)>F(x\big|x)$ and $\frac{dP_M}{dr}$ 2 0[3]. Compare formula (12) and (13), there is

$$\frac{dP_D}{dr}\Big|_{r=r_0}>\frac{dP_D}{dr}\Big|_{r\neq r_0} \tag{15}$$

And from assumption 2, for any P_A ! (0,1), there is $r=\hat{r}$ which makes $1-P_A=F(x(1+\hat{r})\big|x)$. So according formula (12), when $r=\hat{r}$, there is $\frac{dP_D}{dr}$;$_{r=\hat{r}}$1 0; according assumption 5, there is $\tilde{r}>\hat{r}$,which makes $\frac{dP_D}{dr}\Big|_{r=\tilde{r}}>0$. Thus, there is a bank interest rate $r=\underline{r}$! ($\hat{r}$,$\tilde{r}$) that makes the default probability of the EG, P_D, the minimum value, which is $\frac{dP_D}{dr}$;$_{r=\underline{r}}$= 0. And from formula (15), we have

$$\frac{dP_D}{dr}\ ;_{r=\underline{r}}\ 2\ 0 \tag{16}$$

In other words, loan interest rate r_0 which can minimize the probability of moral hazard in the EG, cannot make default probability maximum. It was stated in formula (16) that P_D increased Monotonically in r_0. so interest rate $\bar{r}$ and $\underline{r}$ which maximize and minimize default probability of the EG existed, and $\bar{r}$ fell into the interval (r_0, $\underline{r}$).that was to say, with raising interest rate, default probability of the EG improved firstly, peaked when $r=\bar{r}$, and decreased gradually, bottomed when $r=\underline{r}$, as illustrated in figure 1 below.

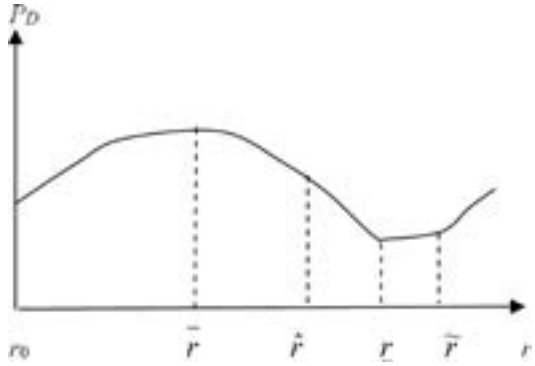

Figure 1
The Relationship Between Default Probability and Interest Rate

Inference 1: When there is moral hazard in the EG, the interest rate $\underline{r}$ which makes the default probability minimum is greater than r, which makes the default probability maximum.

[2] It's the same with the conclusion in literature 6.

[3] Generally, assume that there is no mutation effect between PM and bank interest rate.

Definition 1: If the EG's default probability increases with the loan interest rate (like $r\hat{I}(r_0, r)$), then r is defined as the interest rate based on rise of default probability; otherwise, r is defined as the interest rate based on decrease of default probability.

Inference 1 shows that P_D is not monotonic decreasing as the decrease of bank interest rate r when moral hazard exists in the EG. That means that cutting interest rate down will not necessarily reduce the credit risk of the EG. As there are both the interest rate based on rise of default probability and the interest rate based on decrease of default probability, the bank should choose the latter when making loan decision.

2.3 Transformation form Moral Hazard of the EG to Credit Risk

From the conclusion 1, we know that the probability of the occurrence of moral hazard will be monotonic increasing as the increase of bank loan interest rate. However, P_D, the default probability of the EG, is not monotonic increasing. So we get another important conclusion as follows.

Conclusion 2: The relationship between the probability of occurrence of moral hazard and default probability is not only non-monotonic, but also nonlinear. And there are the interest rates, r and $\underline{r}$, which are exogenous, respectively make the default probability maximum and minimum respectively.

Inference 2: As the probability of occurrence of moral hazard is monotonic increasing, the default probability get maximum at first, then get minimum.

Furthermore, according to formula (6) to (8), the smaller difference between $P_A\theta_A$ and θ_B is, the lager $H(r)$is, and the smaller the probability of the occurrence of moral hazard is. Now we can depict the transfer mechanism from the moral hazard of the EG to the credit risk, which is:

(1) If the probability of the occurrence of moral hazard is zero, the default probability of the EG, is equal to the probability with which the Project B failed.

(2) If the expected return of the Project A is only a little larger than the upper bound of that of the Project B, the probability of the occurrence of moral hazard is relatively small. Once the moral hazard occurs, the default probability of the EG will show a monotonic increasing trend as the rise of interest rate. (Shown in left part of Figure 1)

(3) If the expected return of the Project A is far more than the upper bound of that of the Project B, the probability of the occurrence of moral hazard is relatively large. In this situation, as the increase of interest rate, the default probability of the EG will show a trend that goes down at first and then increase.

In other words, the decrease of interest rate can reduce the moral hazard but cannot make the default probability of the EG minimum, and banks will not be satisfied with a very low interest rate, either. Thus, banks tend to find a best interest rate. From formula (12), the interest rate that makes default probability of the EG achieve extreme value depends on the value of P_A, which is the private information of the EG. So it is difficult for the banks to detect the change of credit risk when the moral hazard occurs in the EG. This is one root that why current the EG often occur credit crisis, which brings the banks huge loss.

CONCLUSION

We've discussed inner mechanism between the moral hazard and credit risk of the EG, and got some significant theoretical conclusions as follows. (1) There is nonlinear relationship between the probability of occurrence of moral hazard and the default probability of the EG; (2) There is an exogenous variable, loan interest rate, which makes the default probability extremum.

The study also shows that banks could use a low interest rate strategy to reduce the probability of moral hazard of the EG, which is monotonic decreasing with the decrease of banks' interest rate. On the other side, banks will face a dilemma when determining the interest rate, because the credit risk of the EG is actually not monotonic decreasing with the decrease of interest rate.

As the difficulty of data collection, we have described the interaction mechanism between moral hazard and credit risk of the EG only from the theoretical level. What's more, we have made some necessary simplifications and assumptions for the convenient of our discussion, so it is to collect real data to simulate the return distribution and spread these conclusions to more general cases that is our research work in the coming step.

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