Comparable Analysis of Long-term Memory of EUR/USD Based on Non-parametrical Statistics

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Abstract: Non-parameter statistical methods of classical R/S, revised R/S and V/S was proposed in long-term Memory of EUR/USD. It was concluded through comparable analysis that: (1) Through Normality Test of Return Sequences of EUR/USD Daily Closing Prices, the experimental results imply that bias of daily return sequences of EUR/USD is not equal to zero, and the curve appears to high peak and fat tail. (2) Jarque-Bera test is adopted. The estimated values reject the null hypothesis of normal distribution. (3) The paper estimates daily return series of EUR/USD using classical R/S method. The results show that Hurst exponent is equal to 0.612425; the statistical cycle is 160 days; the correlative scale is close to 1.3432. This study's conclusion was that long-term memory exists in daily return time series of EUR/USD is proved.

Key words: rescaled range (R/S) analysis; rescaled variance (V/S) analysis; long-term memory

1. INTRODUCTION

1.1 Analysis Based on Classical R/S Tests

Greene and Fielitz (1977) firstly introduced classical R/S Test into capital market, and studied the behavioral characteristics of ordinary American stocks which concluded that the return series have long term dependence and are abnormal distributed (Greene & Fielitz, 1977). Peters (1991) studied daily exchange rates data of US dollars; Japanese yen; UK pounds; Euros; and Singapore round, and concluded that the exchange market turned out Hurst Statistical Features through R/S Test. The Hurst Indexes of the formal three exchanges are similar to 0.60 and with high level of constancy; while the Hurst Index of US dollars to Singapore round is 0.5, which is a real random sequence (Edgar, 1994).

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Although the stability of R/S would not be affected and could be applied in Non-cycle Prediction in Classical R/S Test, no matter it is with normality or not. When the time length is too short or the sample is too significantly related, the classical R/S tests have shortages such as estimating errors, and rescaled range’s sensibility to Short Memory Effect. Lo and the colleagues put forward a modified R/S method according to the shortage of classical R/S rescaled range’s sensibility to Short Memory Effect.

1.2 Analysis Based on Modified R/S Tests
Lo modified the V statistic based on classical R/S method (Lo, 1991). He then applied the method in studies on American market, and concluded that neither daily return rate series nor monthly return rate series have significant long memory. Menggen Chen (2003) used a modified R/S method to study between Shanghai Stock Index and Shenzhen Stock Index, and concluded that they were not significantly long term related (CHEN, 2003). Renhai Hua and Baizhu Chen (2004) used modified R/S method to analyze long memory between future price return and fluctuation variance of cooper, aluminum, soybean, rubber and maize. They concluded that the future price return series of cooper and aluminum were with no long memory, the fluctuation variance of which were with long memory as well; the future price return series and fluctuation variance of rubber and maize were with long memory; the future price return series and fluctuation variance of soybean were neither with long memory (HUA & CHEN, 2004). Xingqiang He (2005) used modified R/S method to research the closing price index series of Shanghai A and B shares between 1st March, 1993 and 24th September, 2004, and Shenzhen A and B shares between 1st January, 1996 and 24th September, 2004. He concluded that on the market of Shanghai and Shenzhen Stocks, daily return rates and weekly return rates were with no significant memory (HE, 2005). Assaf (2006) used modified R/S method to research the daily return rate series between 1st April, 1997 and 26th April, 2002 among the stock markets of Egypt, Jordan, Morocco, and Turkey. He concluded that the stock index series of Egypt were with memory; while the other three stock indexes were not (Assaf, 2006). Serletis and Rosenberg (2007) used modified R/S method to research daily statistics of Down Jones Industrial Average, Standard & Poor's 500 index, and NASDAQ Index between 5th February, 1971 and 1st December, 2006. They concluded that daily return rates on US stock market were with no long memory (Serletis & Rosenberg, 2007). Batten、Ellis and Fethertson (2008) used modified R/S method to check Down Jones Index between 1st January, 1970 and 17th March, 2004 and the sub-sample, and concluded that they did not accept the null hypothesis of the existence of long memory (Battena et al., 2008).

Modified R/S method has the advantages of classical one; besides, it eliminates the influence of short term negative correlation, which could test long term correlations between series more effectively. On the other side, the built statistics have obvious statistical distribution forms, which could make the test more stable and steady. However, modified R/S method has its own shortcomings that the significance tests are apt to reject the null hypothesis with no long memory. On the other side, when choosing lagged orders, the methodology is ambiguous over the academic circles, which might lead to inconvincible conclusions.

1.3 Analysis Based on Rescaled Variance (V/S) Tests
Giraitis and the colleagues’ (2003) centralized the KPSS statistics (Kwiatkowski, Phillips, Schmidt, and Shin) and put forward a V/S method to test long memory of series. They compared the statistics of modified R/S, KPSS, and V/S standing on the views of a variety of theories and Monte Carlo Emulation, and concluded that the method of V/S was more stable and steady in testing long memory of series (Giraitis et al., 2003). Jun Yu, Aili Fang, and the colleagues (2008) used modified R/S method and V/S method to study long memory of stock indexes among 31 countries and areas in the world, and concluded that series of developed countries in Europe and America, such like the US and the Britain, had no long memory; series of the four little dragons in Asia, including China Hong Kong, China Taiwan, Singapore, and Korea, together with Japan, Australia, and New Zealand were with no long memory; while series of certain emerging countries, such like China, India, Indonesia, and Egypt, are with significant long memory (YU et al., 2008). Xingqiang He, Zhongfei Li (2006) studied samples of Shanghai A shares, and they conclude that the wholesome sample of Shanghai A and B shares are neither with significant long memory, while B shares had a relatively more significant long memory; returns of A and B share markets had separately twice and four times of significant variance shift breaks; returns of A shares had no long memory in all
stages and returns of B shares had significant long memory in certain stages; according to the randomly selected 10 stocks, there was only one stock of which return series were with significant long memory, and return series of B shares had relatively more significant long memory than that of A shares (HE & LI, 2006).

As extreme differences are replaced by variances in the method of V/S test, Hurst Index Estimation of V/S is more stable and steady than that of R/S. Meanwhile, the curves of V/S are sharper which could be another advantage. However, as V/S is a new method, it still would still need further tests in practice.

Though Autocorrelation function method is very simple and rough, it could use some graph and figures to directly reflect whether series have long memory or not.

Above all, there are lack of studies between daily return rates of Euros and US dollars on Euro foreign exchange market. As EUR and USD are two most important currencies in the current world, this paper tries to start with autocorrelation method, which integrated advantages of three non-parametrical statistics methods (classical R/S, modified R/S, V/S). This paper covers all three methods above and makes a comparative analysis the long memory between EUR and USD.

2. METHOD

2.1 Histogram Test on Distribution of Returns

This is a kind of direct test of normality, which could directly check the differences between histograms and normal distribution curves. If return histograms and normal distribution curves are obviously different, then we could not consider the return series on the focused exchange market are normally distributed (CAO, 2008).

2.2 Jarque-Bera (JB) Test

As to exchange return series \( \{R_t\} (1 \leq t \leq N) \), sample mean value is \( \hat{\mu} \), sample standard deviation is \( \hat{\sigma} \), while the sample skewness and sample kurtosis are:

\[
\hat{S} = \frac{1}{\hat{\sigma}^3} \frac{1}{N} \sum_{i=1}^{N} (R_i - \hat{\mu})^3
\]

(1)

\[
\hat{K} = \frac{1}{\hat{\sigma}^4} \frac{1}{N} \sum_{i=1}^{N} (R_i - \hat{\mu})^4
\]

(2)

If \( \{R_t\} (1 \leq t \leq N) \) is part of normal population, then \( \hat{S} / \sqrt{6/N} \) and \( (\hat{K} - 3) / \sqrt{24/N} \) are approximate standard normal distributions. If tested sample skewness and sample kurtosis are far more than 3 times of standard deviation, the return series distribution is far beyond normal distribution, which is one of the important characteristics of nonlinearity.

JB statistics are:

\[
JB = N[\hat{S}^2 + (\hat{K} - 3)^2 / 4] / 6
\]

(3)

\( H_0 : R_t \sim N(0, \sigma^2), \hat{S} = 0, \hat{K} = 3, JR \sim \chi^2(2) \). JB test is a joint test of sample skewness and sample kurtosis.

2.3 Classical R/S Fractal Method

If a time series are with self similarity, it could be considered as long term dependent or long memory characterized, which means changes of recent prices would influence future price fluctuation. Hurst (1951) put forward a statistic as Hurst Exponent to test long memory, with which he used R/S method to analyze
the fractal time series. In order to avoid heteroscedasticity this paper uses a logged return rate as in function (4).

\[ S_t = \ln(P_t / P_{t-1}) = \ln P_t - \ln P_{t-1} \]  \hspace{1cm} (4)

S is logged return rate to time t; P is price of time t (logged return rate are more effective than the widely used price percentage in R/S analysis).

General form of R/S method is (Hurst, 1951),

\[ (R / S)_n = C n^H \]  \hspace{1cm} (5)

R is rescaled extreme difference; S is standard deviation; n is increased time length; C is constant; H is Hurst Exponent. Specific calculation method and steps (Peters, 1994) are as below.

Let time series \( \{N_i\}, i = 1, 2, 3 \ldots N \).

(1) Equalized time series by a set of sub-series of n. Thus, \( A^* n = N \). Mark every sub-series with \( * \). In the sub-series \( I_a, I_a, a = 1, 2, 3 \ldots A \), mark every element with \( * \).

\[ N_{k,a}, k = 1, 2, 3 \ldots n \]

(2) Calculate the average values of every sub-series \( I_a \).

\[ e_a = \frac{1}{n} \sum_{k=1}^{n} N_{k,a} \]  \hspace{1cm} (6)

(3) Calculate accumulated deviation of every element in sub-series \( I_a \).

\[ X_{k,a} = \sum_{i=1}^{k} (N_{i,a} - e_a) \quad K = 1, 2, \ldots n \] \hspace{1cm} (7)

(4) Calculate extreme deviation of every sub-series \( I_a \), when \( k = 1, 2, \ldots n \).

\[ R_a = \max_{1 \leq k \leq n} (X_{k,a}) - \min_{1 \leq k \leq n} (X_{k,a}) \] \hspace{1cm} (8)

(5) Calculate standard deviation of every sub-series \( I_a \).

\[ S_a = \left[ \frac{1}{n} \sum_{k=1}^{n} (N_{k,a} - e_a)^2 \right]^{0.5} \] \hspace{1cm} (9)

(6) Calculate rescaled range of every sub-series \( I_a \).

\[ (R / S)_a = \frac{R_a}{S_a} \] \hspace{1cm} (10)

(7) Restart from step (2) to step (6), and one could get a series of scaled range \( (R / S)_a \). Calculate mean value of this series.

\[ (R / S)_n = \frac{1}{A} \sum_{a=1}^{A} \frac{R_a}{S_a} \] \hspace{1cm} (11)
(8) Increase \( n \) and restart from (2) to (7) until \( n \) reaches \( \frac{N}{2} \), one gets a series \( (n, (R/S)_n) \).

(9) According to function

\[
\log (R/S)_n = \log C + \log n
\]  

(12)

Using least square method to do the linear regression and get an intercept as \( a \), and a slope as \( H \).

Hurst Index \( H \) is used to measure correlations between time series. Take associated scale function as below.

\[
C(t) = 2^{2H} - 1
\]  

(13)

If \( H = 0.5 \), time series increments have a 0 related coefficient in between, which means they are not correlated;

If \( 0 \leq H < 0.5 \), time series increments have a related coefficient smaller than 0, which means they are negatively correlated;

If \( 0.5 < H < 1 \), time series increments have a related coefficient bigger than 0, which means they are positively correlated.

After figuring out the relationship between \( \log(R/S) \) and \( \log(R/S) \) in a single figure, it could be clearly checked out on which point the break point has occurred. Further more; one can estimates recycle statistic \( V_n \). In the year of 1951, \( V_n \) was first used by Hurst to test stability, and then it was improved by Peters and could be efficiently used to estimate the length of recycle. When noises exist, statistic \( V_n = (R/S)_n / \sqrt{n} \) is more efficient. As to independent random process, statistic \( V_n \) is plain to \( \log n \). While as to continuous process, R/S has a square coefficient larger than time span \( (H > 0.5) \), \( V_n \) leans upwards to \( \log n \); on the opposite, as to anti-continuous process \( (H < 0.5) \), \( V_n \) leans downwards to \( \log n \). When the figure of \( V_n \) changes, break occurs, and long memory disappears.

2.4 Modified R/S Fractal Method

Lo (1991) pointed out that classical R/S method had its limits. Although it could check out the dependence of short term and long term, it could not check one of the two out of the other. Therefore, when a time series shows out stronger short term correlation, classical R/S analysis would occur bias, which is apt to conclude with existing of correlation. As a result, Lo (1991) modified the classical R/S method, mainly according to function (9), and the modified function is (14).

\[
\left(\frac{R}{S}\right)_n = \frac{1}{A} \sum_{a=1}^{A} \frac{R_a}{\sigma_a(q)}
\]  

(14)

\( R_a \) is the same as in (8).

\[
\sigma^2_a(q) = \frac{1}{n} \sum_{k=1}^{n} \left( N_{k,a} - e_a \right)^2 + \frac{2}{n} \sum_{j=1}^{q} \omega_j(q) 
\]

\[
\times \left[ \sum_{k=j+1}^{n} (N_{k,a} - e_a)(N_{k-j,a} - e_a) \right]
\]

\[
= \sigma^2_a + 2 \sum_{j=1}^{q} \omega_j(q) \gamma_j
\]  

(15)
When \( q < n \),
\[
\omega_j(q) = 1 - \frac{k}{q+1}, \quad \sigma^2_j \text{ and } \gamma_j \text{ are sample variance and sample j-order auto covariance}
\]
of \( \{N_{k,a}\} \). Modified \( (R/S)_n \) has a characteristic as Asymptotic distribution.
\[
V_n(q) = \frac{1}{\sqrt{n}}(R/S)_n \rightarrow V
\]
The distribution function of the \( V \) is as (17).
\[
F(v) = 1 + 2 \sum_{k=1}^{n} (1 - 4k^2 v^2) e^{-2(kv)^2}
\]
Lo (1991) offered a common threshold of \( V_n(q) \). Through testing the significance of \( V_n(q) \), one can decide whether or not long term correlations exist.

### 2.5 V/S Analysis

Cajueiro and the colleagues (2005) put forward a new non-parametrical statistical method to estimate Hurst index as V/S test. They used Monte Carlo imitation to compare between V/S and R/S statistics. They concluded that within the estimation of Hurst index, V/S was more stable and efficient.

In the analysis of V/S, they use accumulated deviation to replace extreme difference, and define V/S statistic as below.
\[
(V/S)_n = \frac{1}{n} \left[ \sum_{k=1}^{n} \left( x_k - x_{n-k} \right)^2 - \frac{1}{n} \sum_{k=1}^{n} \sum_{t=1}^{k} \left( x_t - x_{n+t} \right)^2 \right]
\]
It is clear from (3) that,
\[
(V/S)_n \sim \left( \frac{n}{2} \right)^{2H}
\]
According to that, they use least square regression to calculate Hurst index \( H \) through figures of \( (V/S)_n \) and \( n \).

### 3. EXPERIMENTAL PROCEDURES AND THE RESULTS

#### 3.1 Data Sources and Preprocessing

This paper uses daily exchange closing price series of EUR to USD since the begging when euro was put into use. The sample covers from 4th Jan. 1999 to 29th Nov. 2008. The data source is from Shihua Financial Report. In the experimental analysis, original data forms a time series as \( \{P_t\} \). First, take the series of logged returns as below.
\[
R_t = \log(P_t / P_{t-1})
\]
\( R_t \) represents logged returns a the moment of \( t \); \( P_t \) represents closing exchange prices at the moment of \( t \).
3.2 Normality Test

Some basic statistics of daily return rate of EUR to USD and JB test results are shown in Figure 1. The bias of daily exchange return rate distribution is not 0; kurtosis of the three are larger than 3, showing a peak situation. The estimated value of JB test statistics is far larger than threshold values by 1%, 5%, thus null hypothesis of normal distribution is rejected.

![Fig.1: Histogram of daily return of EUR/USD](image)

Table 1 and Figure 1 separately show that there are obvious shortages using normal distribution method to picture the distribution characters of exchange prices. And that shows that:

First, modern capital market theory and the analysis, which are based on the hypothesis of normal distribution, have some shortages.

Second, fractal market theory, which is based on non-normal distribution, has its theoretical foundation.

3.3 Comparison among Results from 5 Analysis Methods of Long Memory

3.3.1 Classical R/S Analysis

Test the sample with classical R/S method, using software such as matlab6.5, and Eview5.1. This paper gets Fig.2 and Table 2, which represent recycle length of return rates of EUR to USD, and classical R/S test result of return rates of EUR to USD. $E\left(\frac{R}{S}\right)$ was calculated by the modified function (Peters, 1989) as below.

$$y = \frac{n - 0.5}{n} \left( \frac{n \pi^2}{2} \right)^{\frac{1}{2}} \sum_{r=1}^{n-1} \left( \frac{n - r}{r} \right)^{\frac{1}{2}}$$

(21)

![Fig. 2: Statistical cycle of daily return of EUR/USD](image)
We can see from Fig. 2 that the recycle length of EUR/USD return rates is about 160 days. In between 0 and 160 days, V is stable to log (n); when n>160, V is very sensitive to the increase of log (n), and shows obvious random characteristics. Therefore, this paper could primarily judge that EUR/USD daily return rates have a statistical recycle length of 160 days.

**Table 1: Statistical characterization and JB statistic of daily returns of EUR/USD data**

<table>
<thead>
<tr>
<th>Type</th>
<th>Mean</th>
<th>Median</th>
<th>Max.</th>
<th>Min.</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>0.0000167</td>
<td>0.0000426</td>
<td>0.012666</td>
<td>-0.01142</td>
<td>0.000279</td>
<td>-0.02323</td>
<td>4.05935</td>
<td>114.687</td>
</tr>
</tbody>
</table>

**Table 2: Estimated result of daily return of EUR/USD based on classical R/S**

<table>
<thead>
<tr>
<th>n</th>
<th>Log 10n</th>
<th>V 计算</th>
<th>log(R/S)</th>
<th>E(R/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>0.88289</td>
<td>0.44591</td>
<td>0.42329</td>
</tr>
<tr>
<td>20</td>
<td>1.301</td>
<td>0.98813</td>
<td>0.64533</td>
<td>0.63596</td>
</tr>
<tr>
<td>30</td>
<td>1.4771</td>
<td>1.0156</td>
<td>0.7453</td>
<td>0.74833</td>
</tr>
<tr>
<td>50</td>
<td>1.699</td>
<td>1.0371</td>
<td>0.86531</td>
<td>0.88187</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
<td>1.0891</td>
<td>1.0371</td>
<td>1.0535</td>
</tr>
<tr>
<td>150</td>
<td>2.1761</td>
<td>1.1573</td>
<td>1.1515</td>
<td>1.1504</td>
</tr>
<tr>
<td>160</td>
<td>2.2041</td>
<td>1.1742</td>
<td>1.1718</td>
<td>1.1656</td>
</tr>
<tr>
<td>170</td>
<td>2.2304</td>
<td>1.1502</td>
<td>1.176</td>
<td>1.1199</td>
</tr>
<tr>
<td>200</td>
<td>2.301</td>
<td>1.2104</td>
<td>1.2334</td>
<td>1.2179</td>
</tr>
<tr>
<td>250</td>
<td>2.3979</td>
<td>1.1665</td>
<td>1.2658</td>
<td>1.2698</td>
</tr>
<tr>
<td>300</td>
<td>2.4771</td>
<td>1.2154</td>
<td>1.3233</td>
<td>1.3119</td>
</tr>
<tr>
<td>500</td>
<td>2.699</td>
<td>1.0876</td>
<td>1.386</td>
<td>1.4291</td>
</tr>
<tr>
<td>600</td>
<td>2.7782</td>
<td>1.2304</td>
<td>1.4791</td>
<td>1.47</td>
</tr>
<tr>
<td>1000</td>
<td>3</td>
<td>1.2454</td>
<td>1.5953</td>
<td>1.5849</td>
</tr>
<tr>
<td>1500</td>
<td>3.1761</td>
<td>1.5828</td>
<td>1.785</td>
<td>1.6754</td>
</tr>
<tr>
<td>2000</td>
<td>3.301</td>
<td>1.6457</td>
<td>1.8669</td>
<td>1.7393</td>
</tr>
<tr>
<td>2400</td>
<td>3.3874</td>
<td>1.6058</td>
<td>1.8958</td>
<td>1.7798</td>
</tr>
</tbody>
</table>

**Table 3: Hurst exponent of daily return of EUR/USD**

<table>
<thead>
<tr>
<th>Type</th>
<th>Sample Interval</th>
<th>C(t)</th>
<th>H Index (t)</th>
<th>R²</th>
<th>F Test</th>
<th>Correlation Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USD</td>
<td>15≤n≤160</td>
<td>-0.0587</td>
<td>0.6124(164.1)</td>
<td>0.9942</td>
<td>26923.5</td>
<td>1.3432</td>
</tr>
<tr>
<td>EUR/USD</td>
<td>160≤n≤2445</td>
<td>0.5703(143.8)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4: V and V (q0) estimate result of daily returns of EUR/USD**

<table>
<thead>
<tr>
<th>Type</th>
<th>H Index (t)</th>
<th>p</th>
<th>q</th>
<th>V(q0)</th>
<th>V</th>
<th>Bias (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EUR/USA</td>
<td>0.612(164)</td>
<td>0.056</td>
<td>4</td>
<td>1.1745</td>
<td>1.1742</td>
<td>0.2</td>
</tr>
</tbody>
</table>

**Table 5: Critical value of V statistic in different significance level**

<table>
<thead>
<tr>
<th>P(V&lt;X)</th>
<th>0.005</th>
<th>0.025</th>
<th>0.05</th>
<th>0.100</th>
<th>0.200</th>
<th>0.300</th>
<th>0.400</th>
<th>0.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.721</td>
<td>0.809</td>
<td>0.861</td>
<td>0.927</td>
<td>1.018</td>
<td>1.090</td>
<td>1.157</td>
<td>1.223</td>
</tr>
<tr>
<td>P(V&lt;X)</td>
<td>0.600</td>
<td>0.700</td>
<td>0.800</td>
<td>0.900</td>
<td>0.950</td>
<td>0.975</td>
<td>0.990</td>
<td>0.995</td>
</tr>
<tr>
<td>X</td>
<td>1.294</td>
<td>1.374</td>
<td>1.473</td>
<td>1.620</td>
<td>1.747</td>
<td>1.862</td>
<td>2.001</td>
<td>2.098</td>
</tr>
</tbody>
</table>

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Table 2 and Table 3 show by statistics that EUR/USD daily return rate has a recycle length about 160 days. In Table 3, when 1<n<160, H index is 0.6124 and has passed T test, which means EUR/USD daily return rates have long memory; when n>160, H index turns to 0.57 and has passed T test, which means the characteristics of long memory fades out, the begging exchange rate changes’ influences have faded out as well.

### 3.3.2 Modified R/S Analysis

During the process modifying R/S method, one needs to test the significance of $V_n(q_0)$ through every n and q, so that to judge whether the series have long memory. However, during the process of calculation and test, there is a problem about how to decide the value of q. $V_n(q_0)$ would be different at every q value; hence the results would change simultaneously. In order to solve this problem, Lo (1991) carried out a best q value rule as below.

$$q_0 = \text{int}\left\{\left(\frac{3N}{2}\right)^{1/3} \times \left[2 \rho / (1 - \rho)^2\right]^{2/3}\right\}$$

$\text{int}[\cdot]$ means to take the integer value of $\cdot$; $\rho$ is the first-order autocorrelation coefficient; $N$ is the sum of samples.

$V$ is EUR/USD daily return rate. Estimated results of $V(q_0)$ is shown in Table 4. Threshold values of $V$ under different significant level is shown in Table 5.

Through analysis of Table 4 and Table 5, the research concludes that both $V(q_0)$ and $V$ have passed threshold test about V distribution by 10% and 20%, which means the series of EUR/USD daily return rate have long memory.

![Fig. 3: $V$ statistic and $V_n(q_0)$ of EUR/USD](image)

Although $V$ statistics and modified $V(q_0)$ statistics in R/S test have some differences. The current short term $V(q_0)$ is smaller than $V$, while in a relatively long time span, the two are apt to become similar. That means classical R/S test could be affected by short term correlations and leads to bias in estimating H indexes.

### 3.3.3 V/S Analysis

According to Log(n), we put V curves onto the coordinate graph. When V/S series changes simultaneously with time at the same scale (H=0.5), the process is independent and random. At this time, V is plain to Log (n). If series have anti-continuous structures (0≤H<0.5), V would slope downward to Log(n).
Analyzing the samples with V/S method by matlab6.5 and Eviews5.1, this paper concludes that V obviously slopes upward. That could be explained that V/S series has a scale change ratio faster than time. EUR/USD daily return rate series do have obvious long memory, and the relevant Hurst index is about 0.7, between 0.5 and 1. Thus, V/S method is more stable in estimating Hurst indexes. This information can be delivered by the fig4.

![V/S analysis of daily return of EUR/USD](image)

**4. CONCLUSION**

This paper chooses closing price return rate series of EUR and USD to study. Sample interval covers from 4th Jan. 1999 to 29th Nov. 2008. The research is based on long memory theories and takes the method of non-parametrical statistics, including classical R/S method, modified R/S method, and V/S method. The paper concludes as below.

1. Normality test of daily return rate series between EUR and USD shows that the skewness of daily return rate distribution is not 0; kurtosis of which is greater than 3, showing peak and fat tail postures.

   There is quantity of values concentrated around mean value. On each side of the histogram, there are some points that could not be ignored. That means the daily return rate series of EUR to USD is not like that in the traditional effective markets, in which time series is random.

2. Estimated value of JB test is far larger than the threshold values at 1%, and 5%, which means the null hypothesis of normal distribution is rejected.

   This step reveals that daily return rate series of EUR to USD is not under normal distribution, which is the foundation of the Fractal Market Hypothesis, and the foundation of this paper in researching the long memory of daily return rate series of EUR to USD.

3. Taking non-parametrical statistical method, using classical R/S test to estimate daily return series of EUR to USD, this paper concludes that H indexes is 0.6124; statistical recycle length is 160 days; and correlation length is 1.3432.

   This reveals that one event's influence might last for 160 days as long, and after 160 days the influences could be omitted. The degree of influence could be measured by correlation scale, the larger the stronger.

4. Using modified R/S method and classical R/S method to compare, this paper concludes that V has a 2% bias statistically.

   That means classical R/S methods might overestimate Hurst index, as it is influenced by short term correlations, compared to modified R/S method. Modified R/S analysis has no common standard in
defining lagged stages of \( q \), which leads to worse long memory testing convinces. This paper uses V/S analysis verify the conclusion in a deeper degree. It concludes that daily return rate series of EUR to USD does have long memory.

REFERENCES


