METHOD BASED DEA FOR MULTIPLE ATTRIBUTE DECISION MAKING

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Abstract: Using DEA, A new concept for multiple attribute decision making and analysis, relative efficiency and weak relative efficiency is proposed with its relations to the Pareto efficiency being discussed, its economic interpretation and geometrical meaning and existence theorem being given. On the basis of this concept, a new method of unified evaluations and decision making and analysis is designed. Finally, some useful suggestion are given for the applications in R&D project selection and science & technology achievement evaluations.
Key words: DEA, multiple attribute decision making, project selection and evaluation

1. INTRODUCTION

The evaluation and decision system has n alternatives to be selected and m attributes to be taken into account. Such kind of decision making problems often exist in value engineering and systems engineering and industrial engineering analysis. To solve such problems, two kind of decision making information about the importance degree of the attributes, and one kind of information about the utility or value of the alternatives under the attributes. These two properties of information can be generally expressed in Table 1.

Table 1. Decision Making Information

	O_1	O_2	......	O_j	......	O_m
	W_1	W_2	......	W_j	......	W_m
A_1	U_{11}	U_{12}	......	U_{1j}	......	U_{1m}
A_2	U_{21}	U_{22}	......	U_{2j}	......	U_{2m}
......	......	......	......	......	......	......
A_i	U_{i1}	U_{i2}	......	U_{ij}	......	U_{im}
......	......	......	......	......	......	......
A_n	U_{n1}	U_{n2}	......	U_{nj}	......	U_{nm}

where O_j (j=1,2,...,m) is used to denote the j^{th} attribute, A_i (i=1,2,...,n) i^{th} alternative, W_j (j=1,2,...,m) the information about the importance degree for attribute O_j , U_{ij} the information about the utility for the

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alternative A_i with respect to the attribute O_j. The above two properties of decision making information are generally expressed in the following two manners: cardinal manner, that is, the weights or relative values of the attributes, and ordinal manner, that is, the ranking for the alternatives. Generally speaking, the utility information for the attributes is easily obtained or can be supplied by the decision maker, but the weight information is difficult to get, often adopting the AHP method or MONTE-CARLO simulation technique to determine. Even if the weights are got, such kind of the weight scoring method for the alternatives unified evaluation commonly has great subjectivity, fuzzibility and incomparability. On the other hand, the unification evaluation should be carried out among the alternatives based upon the utility, and the superior or inferior of the certain alternative is the result of comparison among the alternatives to be selected. This is the relative efficiency of the unified evaluations.

In this article, as far as the multiple attribute decision making (MADM) and analysis problems whose utility is known are concerned, a method called relative efficiency evaluation method is developed, and its application results are discussed.

2. RELATIVE EFFICIENCY

The multiple attribute decision making and analysis problems in the form of the above Table can be formally expressed as follows:

$$\text{(MADM)} \quad \text{Order-Satisfying } \{ O_1(A), O_2(A), ..., O_m(A)\}$$
$$\text{Subject to} \quad A \in \{ A_1, A_2, ..., A_n \}$$

That is, select and/or rank the alternatives $A_1, A_2, ..., A_n$ according to the attributes $O_1, O_2, ..., O_m$, to determine the decision maker's satisfying alternative, where $O_j(A_i) = U_{ij}$. For (MADM). The following linear programming to optimize the weight coefficients can be formalized:

$$(\text{MADM}_i) \quad \text{Max} \quad \sum_j W_j U_{ij} = W^T U_i = F_i$$
$$\text{s.t.} \quad \sum_j W_j U_{kj} = W^T U_k \le 1, \text{ for } k=1,2,...,n; \quad W_j \ge 0 \text{ for } j=1,2,...,m.$$

where $U_k = (U_{k1}, U_{k2}, ..., U_{km})$ for k=1,2,...,n.

DEFINITION (Relative Efficiency and Weak Relative Efficiency) The alternative A_i is called weak relative efficient, if the optimal solution W^* of the linear programming problem (MADM_i) satisfies $F_i = W^{*T}U_i = 1$. Furthermore, if there exists at least one optimal solution W^0 such that $W^0 > 0$ and $F_i = W^{0T} U_i = 1$, then the alternative A_i is said relative efficient. We call F_i the relative efficiency index of the alternative A_i.

By means of the following Lemma, we can easily prove Theorem 1.

LEMMA Suppose matrix B is reversible, then

$$\text{(I)} \quad \max f(x)=V_I \quad \text{subject to} \quad x \in X$$

and

$$\text{(II)} \quad \max f(x)=V_{II} \quad \text{subject to} \quad Bx \in X$$

have the same optimal value, that is, $V_I = V_{II}$.

THEOREM 1. (Dimensionally Constant Property) The relative efficiency index F_i of the alternative A_i is dimensionally constant, that is, F_i has nothing to do with the dimension selection of the objectives (O_i, O_2, ..., O_m).

The relationship between the relative efficiency and the Pareto efficiency is described in the following Theorem.

THEOREM 2. (1) If A_i is relative efficient, then it must be Pareto efficient; (2) If A_i is weak relative efficient, then it must be weak Pareto efficient; (3) If A_i is weak efficient and $W^{*T} U_j < 1$ for $\forall\, j \neq i$ and $j \in \{1,2,...,n\}$, where W^* is the optimal solution of ($MADM_i$), then A_i must be Pareto efficient.

It is worth noticing that the inverse of Theorem 2 is not true. In fact, the following inverse example shows that the Pareto efficient alternative may not be the relative efficient alternative, and the weak Pareto efficient alternative may not be the weak relative efficient alternative.

INVERSE EXAMPLE: Consider the following three-alternative and two-attribute (max-oriented) decision making problem:

Table 2. Example's Decision Making Information

	O_1	O_2
A_1	2	2
A_2	3	1
A_3	1	4

Here, $O(A_1)=(2,2)$, $O(A_2)=(3,1)$, $O(A_3)=(1,4)$, $A=\{A_1, A_2, A_3\}$, $O=\{O_1, O_2\}$. It is easily proved that

A_1, A_2 and A_3 are all Pareto efficient alternatives, of course, which are all weak Pareto efficient alternatives. But, consider the following efficiency problem of A_1 :

$$\max \quad 2W_1 + 2W_2 = F_1$$

$$\text{s.t.} \quad 2W_1 + 2W_2 \le 1,\ 3W_1 + W_2 \le 1,\ W_1 + 4W_2 \le 1,\ W_1 \ge 0,\ W_2 \ge 0.$$

Solve this programming to get $W^* = (3/11, 2/11)$, $F_1 = 10/11 < 1$, which means A_1 is not relative efficient, and not weak relative efficient.

3. ECONOMIC MEANING AND GEOMETRIC INTERPRETATION OF RELATIVE EFFICIENCY

Let $Y=\{ y \in R^m : y=O(A),\ A \in \{A_1, A_2, ..., A_n\}\}$. Y will be called the set of production effects.

$T=\{ y \in R^m$: there exist $y^1, y^2, ..., y^k \in Y$ and $\lambda_1, \lambda_2, ..., \lambda_k \ge 0$, $\lambda_1 + \lambda_2 + ... + \lambda_k \le 1$ such that $y = \lambda_1 y^1 + \lambda_2 y^2 + ... + \lambda_k y^k$, and k is some positive integer $\}$

It is easily shown that T is a convex polyhedron in R^m , which consists of the elements in Y and the origin. We will call T the possible set of the production effects Y. T's Pareto efficient face will be called the efficient face of the production effects Y. T's edge, the border of the set of weak efficient alternatives of (MADM), will be called the efficient edge of the production effects Y.

THEOREM 3. A_i is the (weak) relative efficient alternative of (MADM) if and only if $y^0 = O(A_i)$ is the (weak) Pareto efficient alternative of the following multiple attribute programming decision problem:

$$\text{Max} \ (y_1, y_2, ..., y_m) = y \qquad \text{subject to} \quad y \in T$$

THEOREM 4. (Existence Theorem) (1) There exists at least one Pareto efficient alternative $y^* \in T$ and $y^* \neq 0$ for (MADM); (2) there exists at least one alternatives of (MADM) which is relative efficient; (3) there exists as least one weak relative efficient alternative for (MADM).

4. THE BOUNDED RELATIVE EFFICIENT EVALUATION MODEL

The above section gives the concept of the relative efficiency in the multiple attribute unified evaluation and decision making and analysis. In the practical application, the method based on the concept at least has the following aspects of shortcomings: (1) The weight vector for evaluating the alternative A_j is the optimal solution of the ($MADM_j$), which is most beneficial solution for the alternative A_i . Here, the model does not consider the difference between the practical importance degrees among the attributes, which leads to some evaluating attributes which are carefully chosen sometimes have zero weights. This, on the one hand, reflects A_i's property of stressing the superior and avoiding the inferior, on the other hand, forms a sharp contrast against the objective of the unified evaluation. (2) The same evaluation attribute sometimes has very different weights with respect to different alternative evaluation, which, in general sense, is unacceptable. (3) When there are many attributes in the evaluation program, the solving of ($MADM_i$) is time- consuming, and sometimes the number of the relative efficient alternatives may be increased greatly, which conversely decreases the meaning of the evaluation.

One possible way to overcome the above shortcomings is to designate the proper range the weights variation, that is, to narrow the feasible range of the weight vectors to be selected, and build up a bounded relative efficiency evaluation model as follows:

$$(\text{B-MADM}_i) \qquad \max \quad \sum_j W_j\ U_{ij} = F_i$$

$$\text{s. t} \quad \sum_j W_j\ U_{kj} \leq 1 \text{ for } k=1,2,...,n; \ \ W_j \in [LW_j\ , SW_j\ ] \text{ for } j=1,2,...,m$$

where LW_j and SW_j are the positive lower bound and upper bound for the attribute O_j , respectively.

The concrete weight vectors to evaluate the alternatives can be different, but their alteration range and bounds are the same. $LW=(LW_1 ,LW_2 ,..., LW_m)$ and $SW=(SW_1 , SW_2 ,...,SW_m)$ can be determined according to the practical needs. The several possible determining methods could be as follows: (1) Solve the models ($MADM_i$) for i=1,2,...,n to get a group of weight vectors, which forms a weight matrix. LW and SW can be supposed from this weight matrix. (2) Solve the models ($MADM_j$) for j=1,2,...,n, and designate the acceptable change rate for each weight (e.g., 1:2), then the value range of the weight can be determined according to the rate. Actually, this needs to evaluate the relative importance among the objectives or attributes, and AHP method can be used jointly. (3) Designate a known feasible public weight, then allow every alternative evaluating weight to vary according to this group of public weights range in proportion to the certain percentage (for example, 30%). The public weight can be taken as the geometric average value or weighted value of the unbounded models ($MADM_i$) (i=1,2,...,n), or determined by means of AHP method using the importance ranking techniques. It is worthwhile to mention that the special case for the bounded models is the linear weighted ranking method. So, we can say that the bounded models have broader application prospects. In addition, the organic combination of the unbounded models with the AHP method will make the ranking results more practical.

5. UNIFIED EVALUATION AND DECISION ANALYSIS METHOD

Based on the relative efficiency concept and the model discussion in the above sections, a relative efficiency method labeled RED for the unified evaluation and decision making and analysis can be developed. Consider the multiple attribute evaluation and decision problem expressed by the Table. The implementation steps of the REM method are as follows.

$0 \Rightarrow 1$, $1 \Rightarrow k$.

STEP ONE. Solve ($MADM_k$) . If $F_k = 1$, suppose the optimal solution to ($MADM_k$) be $W^k = (W^k_1, W^k_2, ..., W^k_m)$, then standardize W^k to be $\underline{W}^k = (\underline{W}^k_1, \underline{W}^k_2, ..., \underline{W}^k_m)$ such that $\underline{W}^k_1 + \underline{W}^k_2 + ... + \underline{W}^k_m = 1$. Go to step three. Otherwise, go to step two.

STEP TWO. Judge whether k=n. If yes, go to step four; otherwise, $k+1 \Rightarrow k$, and go to step one.

STEP THREE. Interact with the decision maker to judge whether the decision maker is satisfactory with the weight W^k or $\underline{W}^k$ or not. If yes, then the alternative A_k is the decision maker's compromised satisfying alternative with respect to the objective weight W^k or $\underline{W}^k$, and stop. Otherwise, let $B^L = \underline{W}^k$, $L+1 \Rightarrow L$, and go to step two.

STEP FOUR. Coordinate process for the L groups of weights $B^1, B^2, ..., B^L$. In this stage, the decision maker has given all the alternatives a throughout examination for the relative efficiency, but under the corresponding weights, there is none the decision maker is satisfied. to Make the coordination for the weights, let $W = (B^1 + B^2 + ... + B^L)/L$, then a coordination alternative A* can be reached from the primal problem. A* obtained in this way satisfies $W^T O(A^*) = \max\{ W^T U_1, W^T U_2, ..., W^T U_m \}$. At this time, the decision maker will be asked either to select the alternative A* as the coordinating alternative or return to step five.

STEP FIVE. Feedback process. At this stage, the decision maker should be able to pick out the least satisfying coordinating weight element , say W_r from $W = (W_1, W_2, ..., W_m)$, and supply the decision analyst with the threshold or range to take the value, that is, to specify $LW_r, SW_r \in [0,1]$ such that $LW_r \leq SW_r$. Then add the constraint $W_r \in [LW_r, SW_r]$ into the constraint conditions of ($MADM_i$) for i=1,2,...,n, and begin a new iteration and interaction.

Above process continues until the decision maker finds his or her satisfying alternative. It can be shown that this satisfying alternative must be a relative efficient or weak relative efficient alternative. Notice that the coordination could have a variety of forms, such as the weight preference information from the decision maker.

6 . RELATIVE EFFICIENCY CLUSTER ANALYSIS METHOD

The concept of the relative efficiency can be used to carry out not only the alternative ranking analysis, but also the cluster analysis. A feasible relative efficiency cluster analysis method is to assign several critical values to the alternatives relative efficiency indexes, that is, separate the interval [0,1] into several segments, and let each segment correspond to a class of the alternatives.

We might as well assume that the alternatives are going to be divided into the K classes, and the corresponding critical values are $C_0 = 0$, $C_K = 1$, $C_i < C_{i+1}$ for i=0,1,...,K-1. First, the relative efficiency models bounded ($B\text{-}MADM_j$) or unbounded ($MADM_j$) is operated to get the corresponding relative efficiency index F_j . If $F_j \in [C_i, C_{i+1}]$ for some $i \in \{1,2,...,K-1\}$, then we can classify the alternative A_j into the i^{th} class. The above idea to develop the relative efficiency cluster analysis method can be completed just after operating the models bounded or unbounded only one time. But the difficulty is to determine the critical values C_i (i=1,2,...,K-1), and the class number K. The class number K often can be determined according to the practical meaning and evaluation standard for the certain specific problem. But the C_i often needs to be determined according to the experience

To overcome the above shortcomings, we can carry out the cluster analysis in the following way.

STEP ONE. Operate the models ($MADM_j$) to find the relative efficiency index F_j for each alternative A_j , then cluster the alternatives which have the indexes value equal to 1 as the first class.

STEP TWO. Eliminate the alternatives that are in the first class to form the new alternative evaluation group, for this alternatives group, say $\{A^1, A^2, \ldots, A^k\}$ operate the corresponding relative efficiency evaluation models to get the relative efficiency index for each A^j , j=1,2, ... , k, then cluster the alternatives which have the index value 1 as the second class.

STEP THREE. This cluster process continues until all the alternatives being evaluated have the same index value 1. In this case, cluster this group of the alternatives as the last class, and the method ends.

7. CONCLUSION

The concepts of the relative efficiency, the relative efficiency decision making, ranking and cluster analysis methods have been successfully applied to the decision analysis for the R&D projects selection and evaluations and the science and technology achievement evaluations, and have achieved better effects, from which the following several enlightenments are reached:

1st. The relative efficiency concept and the relative efficiency decision making method based on this concept can be directly used to deal with the multiple attribute decision making problems which have different dimension or noncommon measurability.

2nd. The relative efficiency concept can be very conveniently used to exert the alternatives or projects ranking and cluster analysis. So it is powerful in the applications of the evaluation problems, such as industrial engineering evaluation, systems engineering evaluation, and the corresponding decision support systems as a useful tool in the model databases.

3rd. The relative efficiency decision making analysis method only needs the preference information that the decision maker can be easily supplied with, so it has better decision support properties of both simple operation and easy acceptation by the decision maker.

4th. The organic combination between the relative efficiency method and AHP method can reach a more practical decision and evaluation results.

5th. A special case of the bounded relative efficiency evaluation model is the linear weighted evaluation model. So, the former has a broader application prospect.

6th. The relative efficiency concept can be easily extended to the case of multiple objective programming and decision making problems with continuous decision variables, and the corresponding decision method also can be generalized.

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