Study on Buyback Contract in Supply Chain With a Loss-Averse Supplier and Multiple Loss-Averse Retailers Under Stockout Loss Situation

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Abstract
According to the prospect theory and the loss-aversion function, this paper developers the buyback contract model in a two-stage supply chain with a loss-averse supplier and multiple loss-averse retailers. Under the stockout loss setting, we analyze the effect of the loss aversion on the behavior from the retailers and the supplier, and then the buyback contract has been shown to be able to coordinate the supply chain. Furthermore, the number of retailers and loss aversion coefficient meet a certain range, there will be a unique optimal buyback price to achieve supply chain coordination.

Key words: Stockout loss; Loss aversion; Buyback contract

INTRODUCTION
As the market competition intensifies gradually, with the progress of science and technology, more and more product life cycle shortens constantly, the perishable goods have become an important research subject in supply chain field. For its unstable demand and the short sales cycle, the risk preference of supply chain participants plays an important role in the supply chain decision. However, most researchers usually assume that the participants are risk neutral, in other words, in accordance with the expected utility theory, its decision-making goal is expected to maximize profits. Faced with the risk in the stochastic market, the participants’ decisions would be different, in turn, it would affect the supply chain performance. Therefore, the risk preference becomes an important factor for supply chain.

In recent years, scholars begin to study the risk preference of the supply chain participants. Choi, Li, and Yan (2008) measure the risk aversion with the MV model which is developed by Markowitz (1952), under the supply chain with a retailer and a manufacturer. In the background of the supply chain composed by a single supplier and a single retailer, Eeckhoudt, Gollier, and Schlesinger (1995) and Agrawal and Seshadri (2000) predict the expected returns and risk for the risk aversion newsboy with the expected utility function. Jacobson and Roszbach (2003) study the relationship among the bank loan, the credit and the risk by the VR method. Chen, Xu and Zhang (2009) and Yang, Xu and Yu (2009) analyze the risk aversion with CVaR method. Although these studies made up for the inadequacy of expected utility, they could not explain some behavior of policymakers, prospect theory put forward by Kahneman and Tversky (1979) is good enough to describe and explain the decision-making behavior the supply chain members, which overthrew the “rational man” assumption in traditional economics, and it argues that: (a) most people is risk aversion in the face of the gain; (b) most people seek risk when facing loss; (c) people are more sensitive to loss than gain. (d) decision-makers are loss averse. The size of the loss produces pain to the decision-maker is more than happy for the same size of revenue. That is to say, decision-maker is bounded rationality in the decision, and different decision-makers will make different response to the uncertainty. Hence, prospect theory makes a more comprehensive explanation why participants evade the risk but sometimes will not.
Researchers generally study prospect theory from the newsboy model. Daniel and Amos (1979) discuss the participants’ decision when risk emerges. Schweitzer and Cachon (2000) find that under a known demand distribution, the loss-averse newsboy’ order is always lower than that in the expected utility. Wang and Webster (2009) conclude that loss-aversion plays a restricted role in the goods quantity by studying loss-averse newsboy. Wang (2010) studies the game among the multiple loss aversion newsboys, according to the proportion distribution requirements principle, proves that the influence of the loss-aversion is so strong that the total inventory in the competition will be less than the inventory in integrated supply chain.

Its purpose is how to coordinate the supply chain participants and achieve the optimal supply chain system, after analyzing the loss-aversion influence, some scholars achieved the coordination by the supply chain contracts, which include wholesale price contract, buyback contract, revenue sharing contract and quantity flexibility contract, etc. The buyback contract is of great practical value in the supply chain contract coordination mechanism, It can not only share the risks of the market uncertainty, also motivate retailers to increase the order quantity. Pasternack (1985) is the first researcher to put forward the concept of the buyback contract, points out that the partial buyback strategy (the buyback price is the wholesale price) can make the supply chain system to realize the coordination. Howard (1995) and Mantrala and Raman (1999) show that the buyback contract mechanism can encourage retailers to increase product quantity, to improve the interests of the whole supply chain, but different buyback price has different effect on the manufacturer. Cachon (2003) realizes that under the certain condition, the buyback contract distributes the channel profit arbitrarily between suppliers and retailers. As the retailer’s sales efforts have an impact on the demand, Taylor (2001) combines a proper objective feedback strategy with buyback contract to achieve win-win result. He (2005) also gets this conclusion with the newsboy Xu et al. (2008) design the restrictive buy-back contract which also coordinates the supply chain, and has a practical significance to verify the sale effort level. Lau et al. (2001) develop the principal - agent and buyback contract model to solve the supply chain coordination in the asymmetric cost information. Refer to Lau et al., Cao (2012) takes the emergency situation into consideration to study the buyback contract, the optimal buyback strategy also emerges.

From the above, the relevant contract documents for the loss-aversion are not in consideration of one-to-many supply chain under stockout loss situation. This paper will consider the coordination between multiple loss-averse retailers and a loss-averse supplier in the supply chain. With the prospect theory, we will analyze the retailer’s ordering behavior and decision-making behavior and the influence of buyback contract to supply chain, under stockout loss situation. This paper is organized as follows: We will formulate the model and propose assumption in section 2; the section 3 will analyze the integrated supply chain; the retailers and supplier’s decision will be presented in section 4; we will investigate the buyback contract coordination mechanism in section 5; the section 6 shows the summary.

1. MODEL DEVELOPMENT

To facilitate the model and theoretical analysis, we assume that multiple loss-averse retailers selling the same type of the newsboy product, the product information is completely open in the open market. Faced with the stochastic market demand, each loss-averse retailer will forecast the market need before the sales period, and order products to the supplier. For meeting requirements of the total order quantity, supplier provides products to each retailer respectively. As sales period is over, supplier buys the rest of products back.

The retail price is exogenously given and denoted by \( p \). The supplier produces the product at a unit cost \( c \), and sells the product to the retailer at a unit wholesale price \( w \). Let \( q_{ir} \) be the order quantity of the retailer in the integrated supply chain, \( q \) be the total quantity of retailers in the decentralized supply chain, the retailer \( i \) orders \( q_{i} \), so we get \( q_{ir}=q_{i}-q \). In addition, we use \( b \) MERGEFORMAas the buyback price. \( s \) is the unit stockout cost ; In this two-echelon supply chain, demand is random and all unsatisfied demands are lost at the end of the season. Denote by \( x \) the market demand with PDF \( f(x) \), and CDF \( F(x) \) is defined to be differentiable, invertible, and strictly. that is \( F(0)=0 \), and \( F(x)=1-F(x) \). Let \( G(x) \) be the \( i \) retailer’s stochastic demand distribution function, \( g(x) \): the first one retailer’s density function.

We also assume that:

For the retailer \( i \), the market demand is proportional to its order quantity, that is, \( \frac{x_{i}}{q_{i}}=\frac{q_{i}}{q} \), so we get that function:

\[
\frac{x_{i}}{q_{i}}=\frac{q_{i}}{q} \quad , \quad \frac{x_{i}}{q_{i}}=\frac{q_{i}}{q} \quad \text{(1)}
\]

(b) Gain (or loss) is perceived if the final wealth is higher (or lower) than the initial wealth, at the end of the selling season. We define the loss-aversion utility function of the retailer to be piecewise linear as follows

\[
U(\Pi_{w})=\begin{cases} \Pi_{w}-\Pi_{w}^{\lambda}, & \Pi_{w} \geq 0 \\ \lambda_{ir}(\Pi_{w}-\Pi_{w}^{\lambda}), & \Pi_{w} < 0 \end{cases}
\]

(2)

Where \( \lambda_{w} \) is defined as the retailer’s loss-aversion level and \( \Pi_{w}^{\lambda} \) is the final wealth of the retailer after the selling season. If \( \lambda=1 \), then the retailer is risk neutral. If \( \lambda>1 \), then a slope change occurs at the reference level, and the higher value of \( \lambda \) implies the higher level of loss aversion, in order to simplify the model, let denote each
retailer’s loss aversion coefficient is the same, that is, \( \lambda_w = \lambda \). Without loss of generality, we normalize \( \Pi_0 = 0 \), if \( \Pi_0 \geq 0 \), then retailer perceive the gain, at that time, \( \Pi_L = 0 \), retailers perceive the loss. As well as the supplier, and the supplier’s loss aversion is \( \lambda_s \).

(c) To avoid unrealistic and trivial cases, we assume that the following relationship is maintained: \( p > w > c \).

## 2. INTEGRATED SUPPLY CHAIN

In integrated supply chain, retailers and supplier belong to an entirety system, the loss reference has no effect on the total supply chain, and we need not consider the loss aversion. The supplier’s output is the total retailers’ Sales. So, expected gross profit of the integrated supply chain is as follows:

\[
E(\Pi_{sc}) = (p-v)q_{sc} - \int_0^q F(x)dx - (c-v)q_{sc}.
\]

After taking derivative of the expected utility (3) with respect to \( q \) yields, we get:

\[
\frac{\partial E(\Pi_{sc})}{\partial q_{sc}} = (p-v)[1-F(q_{sc})] - (c-v)q_{sc}.
\]

We find that expect supply chain profit function is concave function in \( q_{sc} \). Hence, the optimal production quantity \( q_{sc}^* = F^{-1}(\frac{p-c}{p-v}) \).

Put the Formula (1) into the retailer’s expected utility function, and \( x_i(q) = \frac{w-b}{p-b}(q_i + q_{sc}) \), \( x_i(q) = \frac{p-w+s}{s}(q_i + q_{sc}) \), \( q_i = q_i^* \), so we will simplify the \( E[U(\Pi_i)] \).

\[
E(U(\Pi_i)) = -\lambda \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx - \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx + \frac{\lambda s q_i^{\frac{1}{\gamma q_i}}}{\gamma q_i} \int_0^{x_i(q)} F(x)dx.
\]

Taking the first derivative and second derivative, then we get:

\[
\frac{\partial E[U(\Pi_i)]}{\partial q_i} = -\lambda \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx\left( \frac{1}{\gamma q_i} \right) F(x)dx - \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx\left( \frac{1}{\gamma q_i} \right) F(x)dx + \frac{\lambda s q_i^{\frac{1}{\gamma q_i}}}{\gamma q_i} \int_0^{x_i(q)} F(x)dx.
\]

\[
\frac{\partial^2 E[U(\Pi_i)]}{\partial q_i^2} = \frac{2q_i}{\gamma q_i} \left[ -\lambda \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx + 2\left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx \right] + \frac{\lambda s q_i^{\frac{1}{\gamma q_i}}}{\gamma q_i} \int_0^{x_i(q)} F(x)dx - \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx\left( \frac{1}{\gamma q_i} \right) F(x)dx\left( \frac{1}{\gamma q_i} \right) F(x)dx + \frac{\lambda s q_i^{\frac{1}{\gamma q_i}}}{\gamma q_i} \int_0^{x_i(q)} F(x)dx.
\]

For \( p > b, \lambda \geq 1 \), \( \frac{\partial^2 E[U(\Pi_i)]}{\partial q_i^2} < 0 \), that is \( E(U(\Pi_i)) \) is concave function in \( q_i \), hence, the \( n \)-1 retailers’ order is \( q_{i-1} \).

## 3. DECENTRALIZED SUPPLY CHAIN

In the decentralized decision-making mode, suppliers and retailers are independent and limited rational individual. In the trade process, they will attempt to maximize their own interests as the goal. In this case, suppliers and retailers is a master-slave relationship, there is the Nash equilibrium problem among retailers, it is that the retailers and supplier both pursue their expected utility maximization.

### 3.1 The Retailers’ Decision

According to the wholesale price and the buyback price offered by supplier, coupled with stockout loss, retailers make their own sale price, order and profit.

\[
\Pi_w = \left\{ \begin{array}{ll}
px - wq_i + b(q_i - x_i), & x_i \leq q_i, \\
(p-w)q_i - s(x_i - q_i), & x_i > q_i.
\end{array} \right.
\]

If \( \Pi_w = 0 \), the retailer’s surplus balance is

\[
x_i(q_i) = \frac{w-b}{p-b}q_i, \quad x_i(q_i) = \frac{p-w+s}{s}q_i,
\]

then we can know that \( x_i < x_i^* \), then \( \Pi_w < 0 \); \( x_i^* < x_i \), then \( \Pi_w > 0 \); \( q_i = x_i < x_i^* \), then \( \Pi_w < 0 \). Mapping the retailer’s expected profit function into the Its utility function (2), we can get the retailer’s expected utility profit is:

\[
E(U(q_i)) = \lambda \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx - \left( \frac{p-b}{q} \right) q_i^{\frac{1}{\gamma q_i}} \int_0^{x_i(q)} F(x)dx + \frac{\lambda s q_i^{\frac{1}{\gamma q_i}}}{\gamma q_i} \int_0^{x_i(q)} F(x)dx.
\]

For \( p > b, \lambda \geq 1 \), there is a unique optimal order \( q_{i_{opt}}^* \) strategy for the retailer to obtain the optimal value to make Formula (6) to be zero.
\[
\begin{align*}
\lambda \left( p-b \right) q + \lambda \int_0^{q \lambda} \left( p-b \right) q + \int_0^{q \lambda} F(x)dx \left( p-b \right) q + \int_0^{q \lambda} F(x)dx - (p-b) F(q) q_n \\
+ s q_n \int_0^{q \lambda} F(x)dx + \int_0^{q \lambda} (p-b) F(q) q_n = 0.
\end{align*}
\]

**Theorem 1:** For \( n \) retailers, the Nash equilibrium exists and \( q^*_{1:n} = q^*_{2:n} = \ldots = q^*_{n} \), the optimal the retailers’ total order \( q^* \), meets:

\[
-\lambda \left( p-b \right) \frac{(n-1)q^*}{nq^*} \int_0^{q^*} F(x)dx + \int_0^{q^*} F(x)dx - (p-b) \frac{n-1}{n} F(q^*) - \frac{2}{n} \frac{\lambda q_n^2}{q^*} = 0.
\]

**Proof:** This article assumes retailers to be players who have to participate in a game, in a particular case, if none of the players can act alone to increase their earnings, then the formation of the decision-making combination of the retailers produces Nash equilibrium. According to Nash equilibrium existence theorem (Lee & Zhou, 2013): “In a discrete strategy type game, if each player’s pure strategy space \( S_i \) is a non-empty, closed on Euclidean space, the bounded payment function is continuous and to be quasi-concave, then the pure strategy’s Nash equilibrium exists in this game.” and when retailers order goods from the supplier, their game strategies and the corresponding payment function space satisfy the pure strategy Nash equilibrium condition.

For any retailer’s optimal order quantity can be represented by the Formula (8) and retailers make the decision simultaneously. According to the symmetry, the Nash equilibrium exists, then there must be \( q^*_{1:n} = q^*_{2:n} = \ldots = q^*_{n} \), that is: \( q^*_{1:n} = \frac{1}{n} q^* \), \( q^*_{2:n} = (n-1)q^* \), and formula (9) is tenable.

**Theorem 2:** \( \frac{d q^*}{d s} \) is an increasing function in \( s \), is decreasing function in \( \lambda \) and \( w \).

\[
\frac{d q^*}{d s} = \frac{\lambda q_n^2}{q^*} \left( \frac{1}{\lambda} \right) \int_0^{q^*} F(x)dx + \frac{(p-b) q^*}{q^*} \left( p-b \right) q + \frac{\lambda q_n^2}{q^*} F(x(q^*)) + \frac{2}{n} \frac{\lambda q_n^2}{q^*} \frac{\partial q^*}{d s} > 0,
\]

then \( \frac{d q^*}{d s} > 0 \), that is: When stockout loss is gradually increasing, the retailer will stimulate increasing order to reduce losses for stockout. Similarly, \( \frac{\partial q^*}{\partial \lambda} < 0 \), loss aversion and wholesale prices increase, retailers will make the movement to reduce the order quantity.

**Theorem 3:** For each retailer, there is a unique optimal unit stockout loss, so that the retailers can obtain the maximum expected utility in \( s = s^* \), that is \( E[U(\Pi_s(s^*))]=E[U(\Pi_s(q^*))] \).

\[
\begin{align*}
\frac{\partial^2 E[U(\Pi_s)]}{\partial s^2} &= -\lambda \left( p-b \right) \frac{q}{q^*} \int_0^{q^*} F(x)dx + \int_0^{q^*} F(x)dx - (p-b) \frac{n-1}{n} F(q^*) - \frac{2}{n} \frac{\lambda q_n^2}{q^*} \frac{\partial q^*}{d s} > 0,
\end{align*}
\]

\[
\frac{\partial^2 E[U(\Pi_s)]}{\partial q^*} > 0,
\]

for \( \frac{\partial^2 E[U(\Pi_s)]}{\partial q^*} > 0 \), numerical corresponding relation shows, if \( s = s^* \), then \( q^* = q^* \).

**3.2 The Supplier’s Decision**

As a dominant, supplier decides on the wholesale price and the buyback price, but the supplier’s profit will be affected by the retailer’s order quantity.

\[
\Pi_s = \left\{ (w-b-c)q + bx \mid x \geq q^* \right\}
\]

If \( w \geq b + c \), then \( \Pi_s > 0 \), if \( w < b + c \), then \( \Pi_s \leq 0 \), the supplier’s breakeven point is \( x^* \), in order to fully reflect the characteristics of loss aversion suppliers, we assume \( w < b + c \), thus \( x = \frac{w-b-c}{b} q^* \), the supplier’s expected utility function is as follows:

\[
\begin{align*}
E(U(\Pi_s)) &= \lambda s \left( \frac{w-b-c}{b} q^* + bx \right) f(x)dx + \int_0^{q^*} f(x)dx + \int_0^{q^*} (w-c) f(x)dx.
\end{align*}
\]

After taking the first partial derivative of (12), with respect to \( w \), we get:

\[
\frac{\partial E[U(\Pi_s)]}{\partial s} = \left( \lambda \frac{w-b-c}{b} q^* + F(q^*) + (w-c) \right) \frac{\partial q^*}{\partial w} = 0.
\]

\[
\text{Theorem 4:} \text{ The larger the retailers 'loss-aversion degree becomes, the larger the buyback price given by supplier turns; the smaller the buyback price given}
\]
by supplier is, the smaller the buyback price become.

Proof: for \( p>w>b \) and \( \lambda \geq 1 \), we get as following with the implicit function rule:

\[
\frac{\partial b}{\partial \lambda} = \frac{(\lambda+1) \int_{a}^{b} F(x) \, dx + (1-\lambda)(p-b)(w-p)b + (w-b)q^{\lambda} - (p-b) \int_{a}^{b} F(x) \, dx}{\partial b} > 0
\]

in the same way, \( \frac{\partial b}{\partial \lambda} < 0 \).

Theorem 4 describes that when retailers are loss averse, the supplier’s buyback price should increases along with the increase of loss aversion coefficient, and irrelevant with shortage cost, and the degree of loss aversion of supplier will affect their buyback price decisions. So, supplier as leader, on the one hand should consider the loss of retailer’s preferences, on the other hand prefer to try to minimize his loss aversion and formulate buyback price in rational attitude which will be beneficial to the whole supply chain coordination.

\[
\lambda = \left[ F(q_{n}^{*}) - \frac{w}{c} \right] b \left( n+1 \right)
\]

\[
\lambda = \frac{(p-b)(n-1) \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx - (w-b) F(x) \, dx - \lambda \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx}{\partial b} > 0
\]

\[
\lambda = 1 + \frac{(\lambda-1)q_{n}^{*} \left[ (w-b) F(x, (q_{n}^{*})) - (p-w-s) F(x, (q_{n}^{*})) + q_{n}^{*} (p-b+s) F(q_{n}^{*}) \right]}{(p-b) \left( \lambda - \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx - \lambda \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx \right)}
\]

For, \( E[U(\Pi_{n})] \) is known to be strictly increasing function in \( b \). if \( b=w \), then \( T(q_{n}^{*})=0 \); if \( \lambda > \lambda c \) and \( n<n_{c} \), then \( b=s \), \( T(q_{n}^{*})<0 \). So, \( b \in (p-s,w) \) exists and meets

\[
b(w) = \frac{q_{s}^{*} F(x_{s}(q_{s}^{*})) + s q_{n}^{*} \left[ (w-b) F(x_{s}(q_{n}^{*})) + q_{n}^{*} (p-b+s) F(q_{n}^{*}) \right]}{(\lambda-1)(n-1) \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx + s \int \frac{\partial [F(x) - s(n-1) \int F(x) \, dx]}{\partial n} + \int F(x) \, dx}
\]

4. SUPPLY CHAIN COORDINATION MECHANISM BASED ON BUYBACK CONTRACT

As retailers and supplier are each a separate entity, they will maximize their own interests in the supply chain transaction, which will cause the supply chain to perform worse, compared to integrated supply chain, and therefore, the supplier should adjust buyback price to inspire and guide retailers to order quantity which is equal to the order quantity of integrated supply chain.

Prove: For Formula (9) and Formula (13), we can gain the critical loss aversion coefficient of the retailers and supplier \( \lambda_{r}, \lambda_{s} \) and the critical number of the retailers \( n_{c} \).

\[ q_{s}^{*} = q_{n}^{*}. \] By reverse extrapolation to solve \( b^{*} \), put \( q_{s}^{*} = q_{n}^{*} \) into the Equation (9), we can get the optimal buyback price:

\[
b(w) = \frac{q_{s}^{*} F(x_{s}(q_{s}^{*})) + s q_{n}^{*} F(q_{n}^{*}) - s(n-1) \int \frac{\partial [F(x) + (\lambda-1)(p-w-s) q_{s}^{*} F(q_{s}^{*}) - \lambda q_{n}^{*}]}{\partial n} + \int F(x) \, dx + (\lambda-1)(p-w-s) q_{s}^{*} F(q_{s}^{*}) - \lambda q_{n}^{*}}{(\lambda-1)(n-1) \int \frac{\partial [F(x) + (\lambda-1)(p-w-s) q_{s}^{*} F(q_{s}^{*}) - \lambda q_{n}^{*}]}{\partial n} + \int F(x) \, dx)}
\]

CONCLUSION

In this paper, a two-stage supply chain system as a background, under the stockout loss situation, we discuss the effect of loss aversion on the behavior of decision makers and the role of buyback contract coordination, so we get results as follows:

(a) There is the optimal stockout loss to make utility maximized and to get the optimal order quantity.

(b) In the uncertain and competitive market, Nash equilibrium can be achieved among the multiple loss averse retailers.

(c) Further study shows that order of retailers will increase in pace with the increase of buyback price, while
supplier is more loss averse, the buyback price is lower, which also shows the suppliers should try to reduce the effect of loss aversion preferences in the transaction process, but also concerns the retailer’s loss aversion preferences, so as to achieve more satisfactory results, making the whole supply chain system to achieve coordination.

REFERENCES


