How to Coordinate Integrated Supply Chain When Demand and Cost Disruptions Occur Simultaneously

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INTRODUCTION

Disruptions such as natural disasters, terrorism attacks, major public health events, financial crisis, machine failures and strikes etc. may have certain impact on the operation of supply chain. An originally-coordinated supply chain cannot be coordinated because of some disruptions and the survival of the supply chain members can be influenced by other disruptions. For example, Ford’s procurement system reacts to “9 • 11” attacks slowly, its supply of engines and drive parts suspends for a long time, which makes five companies affiliated with Ford in north America close temporarily. Therefore, the losses in this accident are heavy. The demand for tents and moving shelters increases dramatically in the disaster areas after the Wenchuan Earthquake on May 12th, 2008. And, a lot of enterprises in manufacturing tents have to work overtime in order to meet the demand, which affects the operation of the related supply chain system. Recently, with the development of e-commerce, shopping online is widely spread all over China. According to the statistics, the total sale volume in Alibaba is about 57.1 billion RMB on 11th November, 2014, which has great impact on the online and offline supply chain system. The online retailers need to order a large number of products in advance in order to meet the coming huge demand and the logistics system is greatly influenced by the related activities such as purchasing, storing, transporting and distributing.

As can be seen above, supply chain members’ decision, total supply chain profit and even the survival of enterprises in supply chains can be affected by disruptions.
1. LITERATURE REVIEW

Disruption management is firstly put forward by Clausen (2001). It is used to solve the operations of Continental Airlines when it faces emergency (Yu, 2003) and the airlines company benefits a lot from utilizing the concept. A two-stage supply chain when disruption occurs is studied by Qi (2004), Xu (2006) and Huang (2006), and quantity discount contract is used to coordinate the supply chain. The major differences in those studies lie in that the market demand functions used are a different form and the disruption is different. Qi (2004) studies how to coordinate the supply chain when the retailer faces a linear demand function and demand disruption occurs. Huang (2006) studies the similar problem when the retailer faces an exponential demand function. Xu (2006) studies a kind of supply chain coordination problem when cost disruption when the production cost function is a linear function and a convex function, respectively. Kleindorfer (2005) puts forward a conceptual model to analyze the supply chain disruptions and an example is also used to show how to use the conceptual model. Xiao (2005) studies a supply chain consisting of one manufacturer and two retailers when demand and cost disruption occurs and extends the study of a more complex problem which exists price competition between the two retailers. Tomlin (2006) studies a supply chain which exists dual-sourcing supply when the supply chain faces disruption risks. Recently, Lei (2012) uses a linear contract to coordinate a one-supplier-one-retailer supply chain under asymmetric information when demand and cost disruptions happen simultaneously.

2. BASIC MODEL

This paper studies a supply chain composed of one supplier and one retailer, in which the supplier is the leader of a Stackelberg game. The transaction between the supplier and the retailer is done under symmetric information. It means that the supplier knows the cost structure and profit function, and vice versa. The supplier sells a kind of short-life-cycle product to the retailer according to a production plan which is formulated according to market forecast. The retailer decides whether or not to buy the product according to the contract the supplier offers.

Suppose that \( p \) is the retail price and the demand function that the retailer faces is a exponential function, i.e., \( d = D e^{kr} \). \( D \) is the market scale and \( c \) is the supplier’s unit production cost. The unit retail price is \( p \) and the price sensitivity coefficient is \( k \) \((k>0)\). \( Q \) is the real demand under the retail price \( p \). Then, the demand function is \( Q = D e^{kr} \) and the retail price is \( p = \frac{1}{k} \ln \frac{D}{Q} \). The total profit of the supply chain is

\[
\begin{align*}
f(Q) &= Q \left( \frac{1}{k} \ln \frac{D}{Q} - c \right),
\end{align*}
\]

From the first-order condition, we obtain that the optimal retail price is

\[
- \frac{1}{k} \ln \frac{D}{Q} + c = 0,
\]

the optimal production quantity is

\[
Q^* = D e^{-\frac{k}{1+c}},
\]

and the corresponding total supply chain profit function is

\[
f(Q^*) = \frac{1}{k} \ln \frac{D}{Q^*} - c - \lambda_i (Q^* - Q^*) - \lambda_2 (Q^* - Q^*)^2.
\]

3. INTEGRATED SUPPLY CHAIN COORDINATION WHEN DEMAND AND PRODUCTION COST DISRUPTIONS OCCUR SIMULTANEOUSLY

Integrated supply chain is a system which a supplier and a retailer act as a single decision-maker and their decisions are made under the same decision-maker. Some disruptions occur after the supplier’s production plan is formulated. It results in not only the change of the price sensitivity coefficient but also the change of the supplier’s production cost. The disruptions are captured by the terms of \( \Delta k \) and \( \Delta c \) if and only if \( k+\Delta k>0 \) and \( c+\Delta c>0 \), which ensure they have real meaning. Therefore, the discussions followed are based on the conditions mentioned above.

After the disruptions occur, the demand function is \( d = D e^{(k+\Delta k)r} \). The market scale is \( Q = D e^{(k+\Delta k)r} \) and the retail price is \( p = \frac{1}{k+\Delta k} \ln \frac{D}{Q} \). Thus, the corresponding total supply chain profit function is written as

\[
f(Q) = Q \left( \frac{1}{k+\Delta k} \ln \frac{D}{Q} - c - \lambda_i (Q - Q^*) - \lambda_2 (Q - Q^*)^2 \right).
\]

The parameters \( \lambda_i > 0 \) and \( \lambda_2 > 0 \) in the equation are the marginal costs related to the change of the market scale and \( (x) = \max \{0, x\} \). \( \lambda_i \) is extra increased unit cost due to increase production plan and \( \lambda_2 \) is the extra unit disposal cost due to sell the remained products in the secondary market at the price lower than the marginal production cost when the supply is greater than the demand. In order to further discuss the effect of the disruption mentioned above on the original production plan, we put forward the lemma below.

**Lemma.** If the disruptions of the supplier’s production cost and the price sensitivity coefficient occur simultaneously, we assume that \( Q^* \) is the optimal production quantity which maximizes the supply chain profit function shown in Equation (5). Then \( Q^* \geq Q \) if \( k+\Delta k<0 \) and \( \Delta c<0 \), and \( Q^* \leq Q \) if \( \Delta k>0 \) and \( \Delta c>0 \).

The lemma illustrates the following results. When disruptions make the demand increase and the production...
cost decrease in the supply chain, the supplier needs to meet the enlarged market scale by increasing production quantities. When disruptions make the demand decrease and the production cost increase in the supply chain, the supplier needs to meet the shrunk market scale by decreasing production quantities.

According to the lemma, when \( \Delta k < 0 \) and \( \Delta c < 0 \), \( Q' \geq \bar{Q} \).

Then optimizing the total supply chain profit function \( f(Q) \) is equal to optimize the strictly concave function

\[
f_i(Q) = Q(\frac{1}{k + \Delta k} \ln \frac{D}{Q} - c - \Delta c) - \lambda_i(Q - \bar{Q}).
\]

Subject to \( Q' \geq \bar{Q} \).

We consider two cases below according to the first-order necessary condition.

Case 1: \( -k < \Delta k \leq -k + \frac{c k}{c + \Delta c + \lambda_1} \).

When this condition is true, \( Q' \geq \bar{Q} \). The optimal production quantity is \( Q_{\text{case}1} = D e^{-(c + \Delta c + k) / (k + \Delta k)} \). The corresponding optimal retail price \( p_{\text{case}1} \) and the optimal supply chain profit \( f'_{\text{case}1} \) are shown below:

\[
p_{\text{case}1} = c + \Delta c + \lambda_1 + \frac{1}{k + \Delta k} = p + \lambda_1 + \Delta c - \frac{\Delta k}{k(k + \Delta k)}
\]

\[
f'_{\text{case}1} = Q_{\text{case}1}(p_{\text{case}1} - c - \Delta c) - \lambda Q_{\text{case}1} - \bar{Q}
\]

\[
= \frac{1}{k + \Delta k} D e^{-(c + \Delta c + k) / (k + \Delta k)} + \lambda_1 D e^{(1+\Delta k)}.
\]

As a consequence, the optimal retail price in this case is influenced by the change of the supplier’s production cost and the price sensitivity coefficient, and the optimal production quantity and the optimal supply chain profit are also influenced by the change of the price sensitivity coefficient and the supplier’s production cost.

Case 2: \( -k + \frac{c k}{c + \Delta c + \lambda_1} < \Delta k < 0 \).

When this condition is true, \( Q' \) does not satisfy \( Q' \geq \bar{Q} \).

According to the characteristics of concave function, \( f_i(Q) \) decreases monotonously when \( Q \in [-\infty, \bar{Q}] \). Thus, \( f_i(Q) \) is maximized by \( \bar{Q} \) and \( Q_{\text{case}2} = D e^{-(1+\Delta k)} \). The corresponding optimal retail price \( p_{\text{case}2} \) and the optimal supply chain profit \( f'_{\text{case}2} \) are shown below:

\[
p_{\text{case}2} = c + \frac{1 - \Delta k}{k + \Delta k} = p - \Delta k(1 + c k) ,
\]

\[
f'_{\text{case}2} = Q_{\text{case}2}(p_{\text{case}2} - c - \Delta c) = \frac{1 - \Delta k}{k + \Delta k} D e^{(1+\Delta k)}.
\]

As a consequence, the optimal supplier’s production quantity in this case is not influenced by the change of the price sensitivity coefficient and the supplier’s production cost, and they are the original production plan. It means that the supplier does not need to change production plan in this case but the retail price in this case needs to adjust according to the change of the price sensitivity coefficient.

We now consider the case: Where \( \Delta k > 0 \) and \( \Delta c > 0 \), \( Q' \leq \bar{Q} \). Optimizing the total supply chain profit function \( f(Q) \) is equal to optimize the strictly concave function

\[
f_i(Q) = Q(\frac{1}{k + \Delta k} \ln \frac{D}{Q} - c - \Delta c) - \lambda_i(Q - \bar{Q})
\]

subject to \( Q' \leq \bar{Q} \).

When the disruptions occur, we also consider two cases (Case 3 and Case 4). Similar to the analysis used above, the corresponding retail price and production quantity are obtained.

Case 3: \( \Delta k \geq -k + \frac{c k}{c + \Delta c - \lambda_2} \).

When this condition is true, \( Q' \leq \bar{Q} \). The optimal production quantity is \( Q_{\text{case}3} = D e^{-(1+\Delta c - \Delta k)(1+\Delta k)} \). The corresponding optimal retail price \( p_{\text{case}3} \) and the optimal supply chain profit \( f'_{\text{case}3} \) are shown as below:

\[
p_{\text{case}3} = c + \Delta c - \lambda_2 + \frac{1}{k + \Delta k} = p - \lambda_2 + \Delta c - \frac{\Delta k}{k(k + \Delta k)}
\]

\[
f'_{\text{case}3} = Q_{\text{case}3}(p_{\text{case}3} - c - \Delta c) - \lambda_2 Q_{\text{case}3} - \bar{Q}
\]

\[
= \frac{1}{k + \Delta k} D e^{-(c + \Delta c - \lambda_2)(1+\Delta k)} - \lambda_2 D e^{(1+\Delta k)}.
\]

Case 4: \( 0 < \Delta k < -k + \frac{c k}{c + \Delta c - \lambda_2} \).

When this condition is true, \( Q' \) does not satisfy \( Q' \leq \bar{Q} \).

According to the characteristics of concave function, \( f_i(Q) \) increases monotonously when \( Q \in [-\infty, \bar{Q}] \). Thus, \( f_i(Q) \) is maximized by \( \bar{Q} \) and \( Q_{\text{case}4} = D e^{-(1+\Delta k)} \). The corresponding optimal retail price \( p_{\text{case}4} \) and the optimal supply chain profit \( f'_{\text{case}4} \) are shown as below:

\[
p_{\text{case}4} = c + \frac{1 - \Delta k}{k + \Delta k} = p - \Delta k(1 + c k) ,
\]

\[
f'_{\text{case}4} = Q_{\text{case}4}(p_{\text{case}4} - c - \Delta c) = \frac{1 - \Delta k}{k + \Delta k} D e^{-(1+\Delta k)}.
\]

Summarizing the results above, we have the following theorem which shows the decision-maker’s optimal decisions in the integrated supply chain when the disruptions of the price sensitivity coefficient and the supplier’s production cost occur simultaneously.

**Theorem.** When disruptions make the price sensitivity coefficient and the supplier’s production cost change simultaneously (the deviations of the two parameters are \( \Delta k \) and \( \Delta c \)) and the demand function is \( d = D e^{d t + \Delta k p} \), the decision-maker in the integrated supply chain needs to adjust the optimal retail price and the optimal production quantities in order to optimize the total supply chain profit and realize the supply chain coordination. According to different disruptions, the optimal retail price \( p^* \) and the optimal production quantity \( Q^* \) are shown as follows:
sensitivity coefficient and the production cost change at the same time, there exist four different disruptions in the supply chain system. The supply chain performances are shown in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Case</th>
<th>d</th>
<th>c</th>
<th>p'</th>
<th>q'</th>
<th>f'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.04</td>
<td>-0.5</td>
<td>9.35</td>
<td>17.61</td>
<td>84.14</td>
</tr>
<tr>
<td>2</td>
<td>-0.04</td>
<td>0.5</td>
<td>9.62</td>
<td>16.42</td>
<td>67.56</td>
</tr>
<tr>
<td>3</td>
<td>0.04</td>
<td>-0.5</td>
<td>7.35</td>
<td>16.42</td>
<td>46.84</td>
</tr>
<tr>
<td>4</td>
<td>0.04</td>
<td>0.5</td>
<td>7.44</td>
<td>15.93</td>
<td>30.44</td>
</tr>
</tbody>
</table>

As it can be seen from case 2 and case 3 in Table 1, if the disruptions make the price sensitivity coefficient decrease and the production cost increase or the disruptions make the price sensitivity coefficient increase and the production cost decrease, there exists no impact on the original production plan under certain conditions. It means that there exists a balanced relationship between the price sensitivity coefficient and the production cost. In order to balance the impact of the price sensitivity coefficient on the market demand, the original optimal production plan can remain unchanged but the corresponding retail prices need to adjust. When the disruptions make the price sensitivity coefficient and the production cost increase/decrease simultaneously and the changes of the two parameters exceed a certain range (see Case 1 and Case 4), the optimal production quantity, the optimal retail price need to adjust. As it can be seen from Case 1 in Table 1, when the disruptions make the production cost decrease and the market scale increase, the optimal production quantities in the disruptions are greater than those in the basic case, and the supplier increases the production quantity. As is shown in Case 4, when the disruptions make the production cost increase and the market scale decrease, the optimal production quantities in the disruptions are less than those in the basic case, and the supplier decreases the production quantity. The supplier’s production plan and coordination strategy need to adjust in both cases, which can maximize the total supply chain profit.

### 4. NUMERICAL EXAMPLES

We will analyze the effect of the disruptions on the supply chain profit by using numerical examples in this section.

Let \( D=200 \), \( c=5 \) and \( k=0.3 \). In the scenario, it is shown that the supplier’s optimal production quantity is \( Q=16.42 \), the optimal retail price is \( p^*=9.35 \) and the optimal supply chain profit is \( f=54.72 \).

Suppose that \( \lambda_1=\lambda_2=1 \), \( d=\{-0.04,0.04\} \) and \( c=\{-0.5,0.5\} \). When the disruptions make the price

\[
p^* = \begin{cases} 
-p + \lambda_1 + c - \frac{\Delta k}{k + \Delta k} & \text{if } -k < \Delta k \leq -k + \frac{ck}{c + \Delta c + \lambda_1}; \\
-p - \frac{\Delta k(1 + ck)}{k + \Delta k} & \text{if } -k + \frac{ck}{c + \Delta c + \lambda_1} < \Delta k < -k + \frac{ck}{c + \Delta c - \lambda_2}; \\
-p - \lambda_2 + c - \frac{\Delta k}{k + \Delta k} & \text{if } \Delta k \geq -k + \frac{ck}{c + \Delta c - \lambda_2}. 
\end{cases}
\]

\[
Q' = \begin{cases} 
De^{-[1+(c+\Delta c+\lambda_1)(k+\Delta k)]} & \text{if } -k < \Delta k \leq -k + \frac{ck}{c + \Delta c + \lambda_1}; \\
De^{-[1+ck]} & \text{if } -k + \frac{ck}{c + \Delta c + \lambda_1} < \Delta k < -k + \frac{ck}{c + \Delta c - \lambda_2}; \\
De^{-[1+(c+\Delta c-\lambda_2)(k+\Delta k)]} & \text{if } \Delta k \geq -k + \frac{ck}{c + \Delta c - \lambda_2}. 
\end{cases}
\]

The theorem illustrates the following results. When disruptions make the price sensitivity coefficient and the supplier’s production cost change simultaneously, there exists certain robustness in the original production plan. When \( d \) is in certain range, the original production plan does not need to change but the retail price needs to adjust in order to compensate for the extra cost derived from the disruption. The retail price is only influenced by the price sensitivity coefficient and is not correlated to the supplier’s production cost. If \( d \) exceeds certain range, the original production plan needs to adjust and the retail price also needs to adjust according to the change of the market scale.

### CONCLUSION

This paper studies how to coordinate a one-supplier-one-retailer intragated supply chain when some disruptions make the price sensitivity coefficient and the production cost change simultaneously. The supplier needs to increase the production quantities in order to meet the enlarged market demand when the disruptions make the market demand increase and the production cost decrease. There exists certain robustness in the original
production plan when making decisions. That is to say, when the disruptions of the price sensitivity coefficient and the production cost satisfy a given condition, the original production plan does not need to be adjusted and the supplier only need to adjust the retail price in order to compensate for the deviation cost derived from the disruptions. The production plan and the retail price need to adjust only if the disruptions of the price sensitivity coefficient and the production cost exceed a certain critical point. Finally, some numerical examples are given to analyze the supply chain performances when we adopt the adjusted strategy.

REFERENCES


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