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# Discussion and Application Study on GDP Combination Forecasting Method

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## Abstract

A combination model of GM (1, 1) model, nonlinear and chaotic dynamics model, and nonlinear quadratic autoregressive model for forecasting GDP has been built. As the GDP data for the test in 2012 of Henan Province, results showed that the three prediction models having higher prediction accuracy can be used for long-term GDP forecasts. Then GDP in 2013-2022 of Henan Province was forecasted by these three forecasting models and the GDP forecast results in 2013-2022 of Henan Province with geometric average of the forecasting outcome of these three forecasting models was given.

**Key words:** GDP; GM (1, 1) model; Logistic mapping; Nonlinear autoregressive model

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## INTRODUCTION

GDP is an important index in economic life and the barometer of social economic development. The timely, effective and appropriate regulation of macroeconomic operation is the necessary means to guarantee the steady, healthy and rapid development of national economy. The research on the rules of the development or change of GDP or accurate GDP prediction is one of the most important bases for macroeconomic regulation policy. Many scholars have done researches on GDP Forecasting Method (Long & Wang, 2008; Wei, He, & Jiang, 2007; Cao, He, & Li, 2008; Xu, Yan, & Yan, 2008; Xiao & Wu, 2008; Liu & Xuan, 2004; Su, 2003). In general, the forecasting method can be classified into single model forecasting and combination model forecasting. In 1969, Bates put forward Combination Forecasting Method for the first time which combined different prediction methods and useful information they provided to improve the prediction accuracy (Bates & Granger, 1969). He has been paid wide attention from scholars both at home and abroad for his research which had incomparable advantage on prediction accuracy improvement compared with others (Bunn, 1989; Lam, Mui, & Yuen, 2001; Tang, Zhou, & Shi, 2002). Typical GDP combination model, such as gray linear regression model, particle swarm optimization combined with partial least squares regression method is based on the ARMA model, ARCH model, and etc of BP neural network.

Time series data of GDP such as discrete data are influenced by many factors. Interpreting the value of the GDP and its influence factors for the grey number or whiting values of interval numbers are more reasonable and scientific. Historical data show that GDP changes have trends and laws, and generally, prediction methods tend to have difficult fitting parameters, extrapolation forecast spreads fast, the error rapidly increase, this is similar to the chaotic dynamics in the butterfly effect. Due to economic development in recent years influencing the current state of the economy heavily, on the other hand, economic situation in those years far from now has a little effect on the current economic operation or basically does not have any effect, as a result, too much historical GDP data is of no help to increase GDP prediction accuracy, sometimes it even has negative impact on the accuracy of the prediction result. Based on above characteristics of GDP sequence, using the grey system theory and the method of chaotic dynamics theory and model GDP changing law is described more scientifically. So this article will put forward the combination forecast method based on GM (1, 1), chaos dynamics model and quadratic regression model, and presents the combined forecast GDP of Henan province from 2013 to 2022, its forecasting result is scientific and reasonable, it has a certain reference value for macroeconomic policy formulation of Henan province.

## 1. COMBINATORIAL FORECASTING THEORY AND METHOD FOR GDP

### 1.1 GM (1,1) Model of GDP Prediction and Its Calculation Method

The grey system theory is that the volatility and randomness of the accumulation generation sequence $\{x^{(1)}(k)\}$ of discrete GDP sequence $\{x^{(0)}(k)\}$ will greatly abate, whose changes obey the grey exponential law, GM(1,1) grey differential equation $\frac{dx^{(1)}(k)}{dk} + \alpha x^{(1)}(k) = \beta$ is available. For the time response equation of the model $\hat{x}^{(1)}(k+1)=[x^{(0)}(1)-\frac{\beta}{\alpha}]\cdot e^{-\alpha\cdot k}+\frac{\beta}{\alpha}$, we can get the time response equation of the GDP through process of accumulative generation for time response equation. GDP grey forecasting methods and steps are the following 8 points[3, 12]:

(1) Set the GDP original data sequence $\{x^{(0)}(k)\}$ ($k$=1,2,⋯,$n$, $n$ is the number of data).

(2) Calculate the accumulation generation sequence $\{x^{(1)}(k)\}$ ($k$=1,2,⋯,$n$) of original data sequence $\{x^{(0)}(k)\}$, here $x^{(1)}(k)=\sum_{m=1}^{k} x^{(0)}(m)$.

(3) Structure matrix $B$ and $Y$, among them

$$B=\begin{pmatrix} -0.5(x^{(1)}(1)+x^{(1)}(2)) & 1 \\ -0.5(x^{(1)}(2)+x^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -0.5(x^{(1)}(n-1)+x^{(1)}(n)) & 1 \end{pmatrix}, \quad Y=\begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}$$

(4) According to the least squares estimate the parameters$(\alpha, \beta)T=(B^TB)^{-1}B^TY$, here, $-\alpha$ is development coefficient, $\beta$ is gray action. When the parameter $-\alpha \le 0.3$, the GM (1, 1) model can be used for medium and long-term prediction; When the parameter $0.3<-\alpha \le 0.5$, the GM (1, 1) model can be used for short-term forecasting; When the parameter $0.5<-\alpha \le 0.8$, the GM (1, 1) model for short-term prediction should be careful; When $0.8<-\alpha \le 1$, the GM (1, 1) model should make residual error correction; When $-\alpha >1$, the GM (1, 1) model is unfavorable for forecasting, we should switch to other methods.

(5) Write time response expressions of GM (1, 1) model.

$\hat{x}^{(1)}(k+1)=[x^{(0)}(1)-\frac{\beta}{\alpha}]\cdot e^{-\alpha\cdot k}+\frac{\beta}{\alpha}$, among them $\hat{x}^{(1)}(1)=x^{(0)}(1)$, $k$=1,2,⋯2, (1)

(6) Work out time response equation of GDP data sequence.

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)=(1-e^{\alpha})(x^{(0)}(1)-\frac{\beta}{\alpha})e^{-\alpha k}$$

$k$=1,2,⋯, $n$-1, (2)

(7) Inspect the fitting precision of GDP time response equation.

a. Calculate variance $s_0^2$ and standard deviation $s_0$ of the original data sequence $\{x^{(0)}(k), k=2,3,\cdots,n\}$, among them $\bar{x}^{(0)}=\frac{1}{n-1}\sum_{k=2}^{n} x^{(0)}(k)$.

b. Calculate the residual sequence $\{\varepsilon^{(0)}(k)\}=\{x^{(0)}(k)-\hat{x}^{(0)}(k)\}$ and the relative error sequence $\{e(k)\}=\left\{\left|\frac{\varepsilon^{(0)}(k)}{x^{(0)}(k)}\right|\right\}$, $k$=2,3,⋯,$n$.

c. Calculate variance $s_1^2$ and standard deviation $s_1$ of the residual sequence $\{\varepsilon^{(0)}(k)\}, k=2,3,\cdots,n\}$

Here $s_1^2=\frac{1}{n-2}\sum_{k=2}^{n}\{\varepsilon^{(0)}(k)-\bar{\varepsilon}^{(0)}\}^2$ (among them, $\bar{\varepsilon}^{(0)}=\frac{1}{n-1}\sum_{k=2}^{n}\varepsilon^{(0)}(k)$ is the average value of residual sequence）。

d. Calculate posteriori difference ratio $C$ and small error probability $P$.

The posteriori difference ratio $C=\frac{s_1}{s_0}$, the small error probability $P=P\{|\varepsilon^{(0)}(k)-\bar{\varepsilon}^{(0)}|<0.6745.s_0\}$. Setting the number of satisfying $|\varepsilon^{(0)}(k)-\bar{\varepsilon}^{(0)}|<0.6745.s_0(k=2,3,\cdots,n)$ as $m$, and $P=\frac{m}{n-1}$. GM (1,1) model prediction accuracy hierarchy standard is shown in Table 1.

**Table 1**
**GM (1, 1) Model Prediction Accuracy Grade Division Standard**

| Prediction accuracy level | Good | Barely qualified | Unqualified |
|---|---|---|---|
| $P$ value | >0.95 | >0.70 | ≤0.70 |
| $C$ value | <0.35 | <0.56 | ≥0.65 |

If established GM (1,1) prediction model does not meet the accuracy requirement, the prediction model will be error corrected.

Setting $k>k_0$, if symbol of $\varepsilon^{(0)}(k)$ is consistent and $n-k_0$, $\{\varepsilon^{(0)}(k) \mid k \ge k_0\}$ is called Modeling Residual Tail Section. for the absolute value of Modeling Residual Tail Section GM (1, 1) model is established according to the above steps, calculate the estimation of residual, then residual error correction model of the GM (1, 1) prediction model can be expressed as

$$\hat{\hat{x}}^{(0)}(k+1)=\begin{cases}\hat{x}^{(0)}(k+1) & ,\ k<k_0\\ \hat{x}^{(0)}(k+1)\pm\hat{\varepsilon}^{(0)}(k+1) & ,\ k\ge k_0\end{cases},\tag{3}$$

It is required that symbols before $\hat{\varepsilon}^{(0)}(k+1)$ and symbols of the original residual sequence here are consistent, and $\hat{\hat{x}}^{(0)}(k+1)$ is on behalf of the modification value.

### 1.2 Quadratic Regression Model and Chaotic Dynamics Model for GDP Forecast

Preprocess series are the known GDP time $\{x^{(0)}(k)\}$ ($k$=1,2,⋯$n$), that can be $\{R_k\}=\left\{\dfrac{x^{(0)}(k)}{x^{(0)}(k-1)}\right\}$ ($k$=2,3,⋯$n$). Use the quadratic curve regression model for the sequence $\{R_k\}$.

$$R_{k+1}=aR^2_k-bR_k+c \tag{4}$$

to fit (Here, parameters $a$, $b$, $c$ be estimated)

Nonlinear chaotic dynamics model is built on the basis of the type (4). For eliminating the constant term. formula which is plugged $R_k=dY_k+m$ into is reduced

$$Y_{k+1}=adY_k^2-(b-2am)Y_k+\frac{am^2-(b+1)m+c}{d} \tag{5}$$

Thus, under the condition of $\Delta=\sqrt{(b+1)^2-4ac}\ge 0$, by taking $m=\dfrac{b+1+\sqrt{\Delta}}{2a}$ the constant term in the Formula (5) can be eliminated. To transform $Y_k=\dfrac{2am-b}{ad}X_k$, and note $2am$-b=$\mu$, Formula (5) can be turned into

$$X_{k+1}=uX_k(1-X_k). \tag{6}$$

The Formula (6) is the famous Logistic mapping. By chaotic dynamics theory (Jaditz, & Sayers, 1993; Li & Yorke, 1975), as the parameter $\mu$ increases from 0 to 4, Logistic mapping process through the period-doubling bifurcation route to chaos. When 0< $\mu$ <1, the Formula (6) is the only stable fixed point $X_F$=0; when 1< $\mu$ <3, Formula (6) is the only stable fixed point $X_F=\dfrac{\mu-1}{\mu}$; When after 3, Logistic mapping period-doubling bifurcation occurs, in the type (6) Two stable fixed point; ...... ; when $3<\mu<\sqrt{6}$ ,in Formula (6) there are two stable fixed points $X_F=\dfrac{\mu+1\pm\sqrt{(\mu+1)(\mu-3)}}{2\mu}$ ……, When $\mu>\mu_\infty$ = 3.5699456, the Logistic map increases with increasing $\mu$, appearing in turn $2^n$($n$=1,3,5,⋯) period-doubling bifurcation; When $\mu>\mu_\infty$, the Logistic mapping solution within a certain range is random, namely the chaos. When $\mu$ = 4, the chaotic degree is the strongest (Su, 2003; Liu & Peng, 2004). Logistic mapping period-doubling bifurcation process with the parameter increasing is shown in Figure 1.

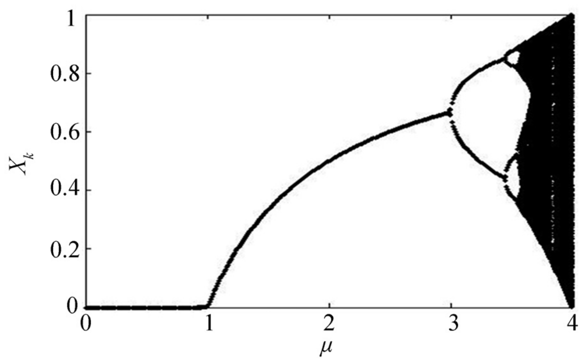


**Figure 1**
**The Logistic Map Period-Doubling Bifurcation Diagram**

## 2. THE EMPIRICAL ANALYSIS OF GDP FORECAST

### 2.1 Test GDP Forecast Model

To test the accuracy and reliability of above models with GDP from 2006 to 2011 in Henan province as the model test data and 2012 GDP as testing data, to verify the prediction accuracy of forecasting model, in this way, the forecast of GDP from 2013 to 2025 of Henan province was conducted. GDP data of Henan province during 2006-2012 are shown in Table 2 ( XXXX, 2013).

By the GM (1, 1) prediction model, calculation method and step, parameters($\alpha$, $\beta$)$^T$=(-0.1427,12252.5631) $^T$ are obtained. Time response equation for GDP is $x^{(0)}$ ($k$+1)=12252.5631$e^{0.1427}$, ($k$=1,2,⋯). Small error probability of the response equations $P=1$,A posteriori difference ratio 0.0799,parameter $\alpha$ > -0.3, This shows that the response equation has good prediction accuracy, which can be used for medium and long term forecast GDP. The average relative error for GDP simulating In Henan province during 2006-2011 was 1.58%, relative error between GDP predictive value of 30750.1274 and the actual value of 29599.31 in 2012 is only 3.89%.

The regression equation is calculated by the GDP data during 2006-2011in Henan province.

$$R_{k+1}=4.1574R^2_k-1.7368R_k+3.6392 \tag{7}$$

then calculate the coefficient of Logistic mapping $\mu$=1.3175. As 1< $\mu$ < 3, by the chaotic dynamics $\{X_k\}$is converged to $\dfrac{\mu-1}{\mu}$ = 0.241, further convergence can be obtained $d\cdot\dfrac{2am-b}{ad}\cdot\dfrac{\mu-1}{\mu}+m=\dfrac{2am-b}{a}\cdot\dfrac{\mu-1}{\mu}+m=1.0991$. Thus GDP in 2012 can be predicted to 26931.03×1.0991=31087.5157 billion yuan. The relative error between the predictive value and the actual value is only 5.03%. It is directly calculated $R_7$ = 1.1528 by the nonlinear regression Formula (7), GDP in 2012 can be predicted 31046.7906 billion yuan, it has to do with the actual value of the relative error is only 4.89%. The test showed that the three kinds of prediction methods have good prediction effect and can be used for medium and long term forecast GDP. The relative error between this and the actual value is only 4.89%.Above test shows that three kinds of prediction methods have good prediction effect and they can be used for medium and long term forecast GDP.

**Table2**
**GDP Data of Henan Province During 2006-2012 (Units: billion yuan)**

| Year | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|
| Serial number | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| GDP | 12363.79 | 15012.46 | 18018.53 | 19480.46 | 23092.36 | 26931.03 | 29599.31 |

### 2.2 GDP Forcast of Henan Province During 2013-2023

Using the GDP data of Henan in 2013-2022 to estimate the GM (1, 1) model parameters, and using the equation to simulate the GDP data from 2006 to 2012, the GDP average relative error is 1.58%. So the model has good fitting precision. In addition, by the least squares estimation and the nonlinear regression model equation the GDP from 2013 to 2022 in Henan province can be predicted as well as the GDP chaotic prediction. Despite three prediction methods have better prediction precisions, but due to different starting point of modeling and prediction model, there must exist differences between the results of GDP forecast. As can be seen in Table 3, 2013-2022 annual GDP gray forecast value are less than that of chaotic prediction, and the chaotic prediction value is less than the quadratic curve regression model prediction value. Because of linear transformation between the Logistic map (6) and a quadratic curve regression model (4), the prediction results are about the same. Because the above three kinds of forecasting methods have distinguishing features and good prediction effect, to make the average value of the above three kinds of prediction results' combination value of Henan province in 2013-2022 GDP forecast will be more reliable, and grey model predicted value of GDP, chaotic model prediction, secondary nonlinear autoregressive model prediction and combination forecasting results are shown in Table 3.

**Table 3**
**The Forecasting GDP of Henan From 2013 to 2022 ( Units: billion yuan)**

| Year | GDP grey model prediction | GDP chaotic model prediction | GDP autoregressive nonlinear model predictive value | Combination forecasting value |
|---|---|---|---|---|
| 2013 | 34287.7983 | 33643.5910 | 33947.3972 | 33958.5764 |
| 2014 | 39223.2083 | 38240.4595 | 38552.4286 | 38669.8647 |
| 2015 | 44869.0247 | 43465.4179 | 43825.2189 | 44049.2172 |
| 2016 | 51327.5040 | 49404.2849 | 49812.4580 | 50174.6377 |
| 2017 | 58715.6213 | 56154.6047 | 56618.6684 | 57152.1925 |
| 2018 | 67167.1895 | 63827.2499 | 64354.7022 | 65100.0314 |
| 2019 | 76835.2823 | 72548.2415 | 73147.7647 | 74153.1401 |
| 2020 | 87895.0073 | 82460.8196 | 83142.2579 | 84465.2145 |
| 2021 | 100546.6770 | 93727.7960 | 94502.3421 | 96211.3333 |
| 2022 | 115019.4370 | 106534.2278 | 107414.6035 | 109590.9209 |

## CONCLUSION

Use the Combination Forecast Method of GM (1,1), nonlinear chaotic dynamics model and the second nonlinear autoregressive model to predict the GDP of Henan from 2013 to 2022. The model test shows that all of the three above prediction methods have higher prediction accuracy. The Gray forecasting GDP is smaller than the chaos prediction values and the chaos prediction GDP is smaller than the quadratic regression model prediction values. The combined forecasting GDP of Henan from 2013-2022 will be more reliable by using the geometric average of the three prediction values which combine all their advantages. It is important to note that when we do GDP extrapolation forecast, the forecast results of recent years are relatively accurate. After long extrapolation time, the predicted results can only reflect the changing trend of GDP. In addition, we need to adjust the parameters of the prediction model and correct the predicted results over a period of time (two or three years or four or five years).

 

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