Research on Pricing Strategy of Online Reverse Auction Based on Complete Information

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Abstract
Aiming at the problem of reverse auction which involves one buyer and multiple sellers in procurement market, this paper studies about online reverse auction via internet during which different sellers arrive at different time and bid, and the buyer makes decision whether to purchase after receiving each bid. And then, the random pricing strategy of online reverse auction is researched. After the compare with single pricing strategy, it shows that the random pricing strategy using the market information to make a procurement price can avoid the waste of cost and incomplete procurement, and a case test is provided in the end.

Key words: Online reverse auction; Random pricing strategy; Competitive analysis

INTRODUCTION
How to use the new procurement technology and operational mode to transform and manage the supply chain and procurement processes, then reduce the procurement costs and improve efficiency is becoming more and more concerned and paid attention. The FreeMarkets Company which was established in 1995 by Glen Meakem is the earliest one who used online reverse auction, which is innovative for traditional procurement mode. Reverse auction is one kind of procurement which makes a decision after the end of bidding. With the intensification of the time effect on the procurement cost, reverse auction participants are not willing to wait for the results for a long time. Waiting means that the time cost increases, as well as loss new purchases and sales opportunities, so online reverse auction was proposed. Online reverse auction is that sellers arrive at different time and bid, the decision whether to buy the bidders’ goods needs to be made immediately after the buyer receives each bid. Reverse auction in application process gradually gets into transparent equalization. Buyers make public supplier’s information and bidding to change the incomplete information into complete information. The supplier’s competition is more intense under the complete information, which will bring lower prices, higher quality suppliers, and reduce the procurement costs. In practice, the online reverse auction becomes prevalent in Europe and the U.S. since 2000, which has a reverse bid process via internet. Compared with traditional negotiation, this kind of auction can save costs up to 11%-12% for buyers. In this paper, we will mainly study the random pricing strategy of online reverse auction, with comparative analysis to the single pricing strategy.

1. LITERATURE REVIEW
With the development of E-business, it is common to buy and sell goods via internet. The researches on online reverse auctions began from auction research in the first deriving from McAfee, R. P. and J. McMillan’s (1986) agent competitive strategy analysis and government procurement mathematical model in the commissioned agency theory. Subsequently, Dasgupta. S and D. F. Spulber (1989) analyze the reverse auction strategy problem of the procurement management. CHEN (1999)
summarizes the quantities of the reverse auction and strategic equilibrium of price auction on the basis of the aforementioned studies, and comparatively analyzes the efficiency of the different auction mechanisms. The research of online reverse auction model and algorithm mostly focus on the online algorithms and competitive analysis methods growing up in computer science. Ron Lavi and Noam Nisan (2000) study the online auctions that after bidders arriving at different time the auction mechanism must make decision immediately after receipt of each bid, they also take advantage of the competition analysis in the worst case to identify the auctioneer optimal supply curve, based on which the online auction mechanism is very competitive. Alan Smart and Alan Harrison (2003) study the role in buyer-supplier relationships of online reverse auctions. Avrim Blum, Tuomas Sandholm and Martin Zinkevich (2006) put online learning applications in digital goods online auction, get a new digital goods auction model and a constant competition ratio on the optimal auction revenue under fixed price, and apply the technology applications in the design of online auction mechanism.

The goods purchased through reverse auction are generally standardized goods, of which the true valuation of the supplier is generally converge, so the majority of the bids usually fluctuate within a relatively small range. Buyers provide complete information to suppliers, and then a phenomenon appear that all bids of suppliers are concentrated in a range because of the fierce competition to get the object with smaller advantage. Because buyers can not understand this small range, their valuations of the commodity usually belong to a large range, in which a single pricing is likely to be too high to cause waste of cost, or be set so low that reverse auction comes to nothing, so that the procurement tasks can not be completed. With the help of researches on online algorithms and competitive analysis methods by many other scholars (XU, XU, & LU, 2005; DING & XU, 2007; XIN, XU, & YI, 2007; XU & XU; 2008), this paper studies the random pricing strategy of online reverse auction in which all bids in competition are more concentrated, and comparative analysis with a single pricing proves that when a random pricing strategy of online reverse auction takes advantage of the market price information, the disadvantages of single pricing caused such as the waste of cost or unable to complete the task of procurement can be overcome.

2. PROBLEM DESCRIPTION
Assuming buyers want to buy a certain type of standardized non-market exclusive products, and select a purchase items from a number of suppliers, and suppliers can meet the number of purchasing items of buyers. According to the complete information provided by the buyer, in each stage of the bidding process of online reverse auction, suppliers give different bids in different time, and change the bids in accordance with the changes of other suppliers. Suppliers bid on the one hand are based on their own reservation price, on the other hand are based on the bids of other suppliers at any time to adjust, and maximize their expected utility with lower prices. When the expected utility falls to the minimum margin, the price will no longer be reduced. According to the pre-determined procurement strategy, the buyer can analysis the bids of suppliers, determine the most suitable purchase price, and close a deal with a supplier. Overall, in this process, the buyer always expects that the price is more and more lower to achieve the maximum expected utility by reducing procurement costs.

3. MODELING AND ANALYSIS
A buyer usually determines his own pricing strategy independently, but as a seller, the supplier can refer to the bids of other multiple suppliers to determine his own bidding strategy, whose goal of participating in the auction is to maximize expected utility (DING & XU, 2007). In general, it should be set up in advance when the bid of online reverse auction begins. In this article, it is assumed that the buyer can end the auction at any time based on the price situation during the online reverse auction process, also he can end the auction if his bid price is high enough for the supplier to win the bidding, or end the auction because one buyer of urgent demand for the goods is willing to purchase at the bid price of the time.

Given an online reverse auction $A$, in which there are $n$ suppliers. Assume that the $i$-th bidder’s real valuation of goods is $v_i (i = 1, 2, ..., n)$, his quoted price is $b_i$, and the buyer pay the $i$-th bidder a price of $p_i$. If $p_i \geq b_i$, transaction maybe occurred, then the $i$-th bidder’s utility is $U_i = p_i - v_i$. If $p_i < b_i$, transaction will not occur, the bidder’s utility is 0.

Now let’s consider about an online reverse auction problem with the objective of minimum cost. For any bid sequence $L$, $C(L)$ is taken to be the cost that the online reverse auction $A$ makes an instant decision of bidding sequence $L$, and $C_\alpha(L)$ indicates the decision-making cost if the bidding sequence $L$ is known in advance. If there is a constant $r$ independent of the bidding sequence $L$, which meets $C_\alpha(L) \leq r C(L)$, then the competitive ratio is $\alpha$, which shows the better between the optimal procurement costs in the two cases. This analysis process of the online algorithms performance is called competitive analysis.

Therefore, before the analysis of random pricing strategy of online reverse auction, it needs firstly to study the single pricing condition, and thus to analyze the competitive performance of random pricing strategy.

3.1 Optimal Single Pricing
If the buyer has determined a purchase price before online
reverse auction, the first supplier whose bid price is less than or equal to this purchase price will close a deal with the buyer. This online reverse auction strategy is called online single pricing strategy (abbreviated as the online SP strategy).

If the buyer has an estimated range \([B_1, B_2]\) of the commodity price, then assumed that the single price is set to \(B_1\), if the price is set too low, there is probably not one supplier can win the bidding all the way, the buyer has to purchase at the last-minute bid price because of the procurement time limit, and if \(B_1\) is set too high, the online reverse auction may soon close a deal, but can not achieve the goal of saving costs.

From the competitive analysis point of view:

If the bid price \(B_1\) doesn’t appear in procurement time, the cost ratio of online and offline is \(B_1/B_2\).

If the bid price below the single fixed price doesn’t appear all the way during the procurement time, and the final bid price comes to the ceiling, then the cost ratio of online and offline is \(B_1/B_2\).

So given the single fixed price \(B_1\), the competition ratio of this online reverse auction is \(r_{SP} = \sup\{B_1/B_2, B_1/B_3\}\), and the optimal \(B_1\) should be the solution of the equation

\[ B_1/B_2 = B_1/B_3 \]

Because the price makes the \(r_{SP}\) minimum value, then \(B_1 = \sqrt{B_2/B_3} = B_1 = \sqrt{B_2/B_3} \).  

### 3.2 Random Pricing Strategy and Competitive Analysis

#### 3.2.1 Random Pricing Strategy

With complete information, if the bid prices of suppliers are more concentrated, the buyer needs to analyze all the bid prices of small difference. Then, we will apply the random pricing strategy of online reverse auction (abbreviated online reverse auction \(RP\) strategy) to analyze the supplier’s bids in competition.

Let the estimated range \([B_1, B_2]\) of the buyer about the commodity price be separated into \(m\) subintervals, \(\Delta 1, \Delta 2, \ldots, \Delta m\), where \(\Delta i = (c_{i-1}, c_i), \ i = 1, 2, \ldots, m\). Assuming the selected number of bid prices up to \(n\) (that is, up to the \(n\)-th bid auction will end), now let the right end point \(c_i\) of the first subinterval \(c_{i-1}, c_i\) be the minimum reserve price, and assuming the price making sequence is \(p_1, p_2, \ldots, p_n\) that is to make the price \(p_i\) for the commodity of the \(i\)-th bidder, then the random pricing strategy of online reverse auction is as follows:

- The probability of price \(p_1\) made for the commodity of the first supplier meets \(P_r = 1\), if the first bid price \(b_1 \leq c_1\), the purchase price is \(c_1\). Otherwise, the transaction will be unsuccessful, and then look at the second bid.
- Assuming \(b_2\) falls into the subinterval \(\Delta k (k \neq 1)\), then the price \(p_2\) made for the commodity of the second supplier satisfies \(P_r = \frac{n-1}{n}\), \(P_r = \frac{n-1}{n}\).
- For the price \(c_{i-1} = c_1 = 1/n\), and if the second bid price \(b_2 \leq c_i\), then the transaction will be successful, so the probability of price \(c_i\) in the successful deal is \(n-1/n\), as the probability of price \(c_i\) in the deal is \(1/n\). If \(c_i < b_2 \leq c_i\), then the probability of a successful deal is \(1/n\), and the bargain price is \(c_i\). If \(b_2 > c_i\), the transaction can not be reached, then see the third bid.

- Usually, the probability of the price \(p_i\) made for the commodity of the \(i\)-th supplier satisfies:

\[
P_r(p_i = c_i) = \frac{N_j}{n} \quad j=2,3,\ldots,m
\]

where \(N_j\) represents the number of bid prices falling into the \(j\)-th subinterval of the front \(i-1\) suppliers.

#### 3.2.2 Competitive Analysis

If bid prices of all suppliers concentrate in a subinterval with a length of \(\lambda\) within the valuation interval \([B_1, B_2]\) of the buyer, then split \([B_1, B_2]\) to several small intervals with each length of \(\lambda/k\), and suppose that there are \(n\) bid prices concentrating in a small area \([a, a+\lambda]\), then all bid prices will fall into at most \(k+1\) segmented subintervals.

According to the random pricing strategy of online reverse auction, the worst case is that the transaction is concluded at the price of \(a+\lambda/2\) with the probability of \(n-1/n\), and no deal is closed with the probability of \(1/n\), then the buyer has to purchase at the price ceiling \(B_2\). So the expected cost

\[
E(C_{RP}) = \frac{a + \lambda + (n-1)(a + \lambda/ \lambda)}{n}
\]

While the offline optimal cost is \(\alpha\), then the ratio of expected cost \(E(C_{RP})\) and offline optimal cost \(\alpha\) is:

\[
E(C_{RP}) = \frac{\alpha}{a} = \frac{n-1}{n} \frac{n}{n}
\]

When the interval is split up into much smaller subintervals, namely, if \(k \rightarrow \infty\), then:

\[
\lim_{k \rightarrow \infty} \frac{E(C_{RP})}{\alpha} = \lim_{k \rightarrow \infty} (1 + \frac{\lambda}{B_1} + \frac{n-1}{n} \frac{\lambda}{kB_1}) = 1 + \frac{\lambda}{B_1}, \quad \text{and d}
\]

\[
r_{RP} = \sup_{\alpha} \frac{E(C_{RP})}{\alpha}, \quad \text{that is}, \quad \text{when the valuation range is split up infinitely, } r_{RP} \text{ approximates to } 1 + \frac{\lambda}{B_1}.
\]

If \(k \rightarrow \infty\), then:

- When \(\lambda=0\), \(r_{RP} \rightarrow B_2/B_1\).
- When \(\lambda=\infty\), \(r_{RP} \rightarrow B_2/B_1\).
When $1+ \frac{\lambda}{B_1} < \sqrt{\frac{B_2}{B_1}}$, i.e. when $\lambda \in [0, \sqrt{B_1B_2} - B_1]$, \(r_{SP} < r_{RP}\), now the competitive performance of random pricing strategy is better than that of optimal single pricing strategy. Therefore, the smaller the \(\lambda\) is, the smaller \(r_{RP}\) is too. That is, the more concentrated the bid prices are, the closer the expected cost of random pricing strategy is to the offline optimal cost. So, even if the buyer does not know the fluctuant cost range of suppliers, he can also adjust gradually the probability of bid price making by random pricing strategy, so that the expected cost of the online close offline optimal cost.

4. CASE TEST

4.1 Case Description
An assembly production enterprise wants to procure a number of parts from multiple suppliers. Because assembled commodities parts has the characteristics such as purchasing large quantities and high standardization, the enterprise decides to use multi-stage online reverse auction by using bulk procurement of multi-channel. During online reverse auction, the buyer will firstly audit and evaluate the suppliers who applied online to determine their participation eligibility, then get price information of each supplier and eliminate weaker competitive suppliers to determine those outright ones. Finally, the buyer can distribute the order form to complete the procurement for the minimum purchase cost.

The detailed procurement process of this enterprise is described as follows:

(1) The number attributes of supplier
Because of bulk purchasing, the buyer needs to divide an order into multi orders to respectively procure. According to the number attributes of the procurement goods and each batch procurement number range, in this case, 3 suppliers are needed, and at most one supplier is selected in each procurement stage, so at least there are 3 stages of the bidding. For the simplicity of test, assuming that one supplier is chosen in each stage and the procurement needs 3 stages to be finished.

(2) Assuming that that buyers’ understanding of this product price is limited to a certain price range \([48, 72]\), the initial price is set at \(b_0=60\), because the supplier’s bid would be more concentrated in the less bid interval, the buyer will put every 10 bids into a stage. To easily establish the model, we assume that the bidding sequence number is the serial number of suppliers, which is shown in Table 1, 2, 3.

| Table 1  |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Series number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Supplier       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Competitive tender | 63 | 64 | 65 | 63 | 61 | 62 | 61 | 60 | 62 | 60 |

| Table 2  |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Series number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Supplier       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Competitive tender | 56 | 59 | 58 | 58 | 57 | 55 | 56 | 54 | 55 | 54 |

| Table 3  |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Series number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Supplier       | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Competitive tender | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

4.2 Pricing
As we can see the bidding sequence from Table 1, 2, 3, in this transaction period if we consider the optimal offline purchasing transaction price that it is clearly 59. The optimal single price is \(\sqrt{B_1 \cdot B_2} = \sqrt{48 \times 72} = 58.8\), and the ratios of the optimal single pricing strategy to the optimal offline purchases is \(r_{SP} = \sqrt{B_2 / B_1} = 1.22\). Obviously, \(59 \leq 58.8 \times 1.22\), that is to say that applying single pricing strategy to procure is better than offline purchases. But according to single pricing strategy to procure, apparently it can not quickly make deal. Because the previous bidding prices are not lower than 59, the buyer may lose the opportunity after waiting for a long time. Even the procurement can not be completed during the limited time of online reverse auction.

(1) Using the \(RP\) strategy of online reverse auction to price the first phase bids, the analysis is as follows:
Divide \([48, 72]\) into 8 small pieces \([48, 51]\), \([51, 54]\), \([54, 57]\), \([57, 60]\), \([60, 63]\), \([63, 66]\), \([66, 69]\), \([69, 72]\), the pricing and probability sequence is as follows:
1) \(P(p_1=51)=1, and b_1=63\), So the transaction can not be reached.
2) \(P_1(P_1=51)=9/10, P_1(p_1=63)=1/10, and b_2=64\), So
the transaction can not be reached.

3) \( P_r(p_5=51)=8/10 \), \( P_r(p_6=63)=1/10 \), \( P_r(p_7=66)=1/10 \), and \( b_5=65 \), so the probability of the deal is 1/10, the transaction value is 66.

4) \( P_r(p_5=51)=7/10 \), \( P_r(p_6=63)=1/10 \), \( P_r(p_7=66)=2/10 \), and \( b_5=63 \), so the probability of the deal is 1/10, the transaction value is 63.

5) \( P_r(p_5=51)=6/10 \), \( P_r(p_6=63)=2/10 \), \( P_r(p_6=60)=2/10 \), and \( b_5=61 \), so the probability of the deal is 2/10, the transaction value is 63.

6) \( P_r(p_5=51)=5/10 \), \( P_r(p_6=63)=3/10 \), \( P_r(p_6=60)=2/10 \), and \( b_5=62 \), so the probability of the deal is 3/10, the transaction value is 63.

7) \( P_r(p_5=51)=4/10 \), \( P_r(p_6=63)=4/10 \), \( P_r(p_7=66)=2/10 \), and \( b_5=61 \), so the probability of the deal is 4/10, the transaction value is 63.

8) \( P_r(p_5=51)=3/10 \), \( P_r(p_6=57)=4/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=54 \), so the probability of the deal is 4/10, the transaction value is 57.

9) \( P_r(p_5=51)=2/10 \), \( P_r(p_6=57)=5/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=55 \), so the probability of the deal is 5/10, the transaction value is 57.

10) \( P_r(p_5=51)=1/10 \), \( P_r(p_6=57)=6/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=54 \), so the probability of the deal is 6/10, the probability of the deal is 7/10, the probability of the transaction value which equals 57 is 6/10, the probability of the transaction value which equals 60 is 3/10.

The ratio of the expected cost in this procurement stage to the optimal offline cost is competitive ratio:

\[
r_{RP} = \sup \frac{E(C_{mp})}{a} = \frac{a + \lambda}{na} + \frac{n - 1}{n} \frac{ka + k\lambda + \lambda}{ka} = 1.06
\]

2) Using the \( RP \) strategy of online reverse auction to price the third phase bids, the analysis is as follows:

Divide \([48, 72]\) into 8 small pieces \([48, 51]\), \([51, 54]\), \([54, 57]\), \([57, 60]\), \([60, 63]\), \([63, 66]\), \([66, 69]\), \([69, 72]\), the pricing and probability sequence is as follows:

1) \( P_r(p_5=51)=1 \), and \( b_5=56 \), so the transaction can not be reached.

2) \( P_r(p_5=51)=9/10 \), \( P_r(p_6=57)=1/10 \), and \( b_5=59 \), so the transaction can not be reached.

3) \( P_r(p_5=51)=8/10 \), \( P_r(p_6=57)=1/10 \), \( P_r(p_6=60)=1/10 \), and \( b_5=58 \), so the probability of the deal is 1/10, the transaction value is 60.

4) \( P_r(p_5=51)=7/10 \), \( P_r(p_6=57)=1/10 \), \( P_r(p_6=60)=2/10 \), and \( b_5=58 \), so the probability of the deal is 2/10, the transaction value is 60.

5) \( P_r(p_5=51)=6/10 \), \( P_r(p_6=57)=2/10 \), \( P_r(p_5=60)=2/10 \), and \( b_5=56 \), so the probability of the deal is 1/10, the transaction value is 57.

6) \( P_r(p_5=51)=5/10 \), \( P_r(p_6=57)=3/10 \), \( P_r(p_6=60)=2/10 \), and \( b_5=55 \), so the probability of the deal is 2/15, the transaction value is 57.

7) \( P_r(p_5=51)=4/10 \), \( P_r(p_6=57)=3/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=56 \), so the probability of the deal is 3/10, the transaction value is 57.

8) \( P_r(p_5=51)=3/10 \), \( P_r(p_6=57)=4/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=54 \), so the probability of the deal is 4/10, the transaction value is 57.

9) \( P_r(p_5=51)=2/10 \), \( P_r(p_6=57)=5/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=55 \), so the probability of the deal is 5/10, the transaction value is 57.

10) \( P_r(p_5=51)=1/10 \), \( P_r(p_6=57)=6/10 \), \( P_r(p_6=60)=3/10 \), and \( b_5=54 \), so the probability of the deal is 6/10, the probability of the transaction value which equals 57 is 6/10, the probability of the transaction value which equals 60 is 3/10.

The ratio of the expected cost in this procurement stage to the optimal offline cost is competitive ratio:

\[
r_{RP} = \sup \frac{E(C_{mp})}{a} = \frac{a + \lambda}{na} + \frac{n - 1}{n} \frac{ka + k\lambda + \lambda}{ka} = 1.06
\]
in the first stage is 63, and the second phase of the price is set at 57, and the price of the third stage is 54. At this time, due to the supplier's bids are intensive, it may appear that several suppliers bid the same price, so the buyer can make the decision based on the evaluation of other aspects of the suppliers.

**CONCLUSION**

Aiming at the procurement problem that using online reverse auction to select a supplier, this paper studies the optimal single pricing strategy and random pricing strategy, using which the price made for the supplier is independent of has bid price. The latter can take advantage of market price information, correct the defects caused by static single pricing, besides, it has many other advantages such as full market information, low cost of transaction management, and excellent operability, etc. However, this strategy is applicable to the online reverse auction for standard commodities with bid prices more concentrated. It still needs further studies on the problem of more dispersive bid prices and big competitive ratio.

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