CSCanada
International Business and Management
Vol. 5, No. 2, 2012, pp. 111-116
DOI:10.3968/j.ibm.1923842820120502.ZR0220

ISSN 1923-841X [Print]
ISSN 1923-8428 [Online]
www.cscanada.net
www.cscanada.org

Statistical Analysis of the Second Type of Sales Diffusion Model

YU Xiaoyang[a],*; XU Xiaoling[a]; GU Beiqing[a]

[a] Business Information Management School, Shanghai Institute of Foreign Trade, Shanghai, 201620, China.

*Corresponding author.

Supported by organizations as follows: "2011 annual Shanghai college students' innovative activity programs" by Shanghai municipal education commission; Natural Science Foundation of China (11141002); Shanghai half analytical research key laboratories of galaxies and cosmology (SKLA1101); and Special Research Foundation of Shanghai university for excellent young teachers (swm10010).

Received 3 September 2012; accepted 7 November 2012

Abstract

Based on the second type of sales diffusion curve proposed by the professor Zheng Zukang, the existence of the moment estimation is demonstrated through researching the digital characteristics of parameters in the model. Besides, the moment estimations of parameters are calculated. The precision of estimation is investigated by Monte-Carlo simulation, and some examples are used to validate this model.

Key words: Impulse purchase behavior; Sales diffusion; Moment estimation; Monte-Carlo simulation

YU Xiaoyang,XU Xiaoling,GU Beiqing(2012). Statistical Analysis of the Second Type of Sales Diffusion Model. *International Business and Management, 5*(2), 111-116. Available from: http://www.cscanada.net/index.php/ibm/article/view/j.ibm.1923842820120502.ZR0220
DOI: http://dx.doi.org/10.3968/j.ibm.1923842820120502.ZR0220

INTRODUCTION

With the development of economy and living standard, impulsive purchasing is becoming a main stream of modern consuming idea, due to continuous change in shopping taste and concept. Iyer, an employee of Dupont Company (1989), thought that almost all consumers have experienced at least one unplanned purchasing. Additionally, a study shows that impulsive purchasing has more influence on consuming behavior (Sfiligoj, 1996). Analyzing reasons and main factors of impulse purchase can help companies take corresponding marketing strategies, so as to stimulate impulsive purchasing and expand sales volume.

Many scholars, these years, have been conducting deep researches in consumers' impulsive purchasing as well as new products' sales diffusion. XUE Ming gave definition of impulsive purchasing in his article. It is a specific and unplanned purchasing behavior which is instant, emotional, regardless of consequences. This behavior has been classified in four categories: pure, indicative, inspired and planned buying, and XUE listed corresponding marketing strategies. It has got cognitive aspects such as lack of planning and deliberation (Kim Ramus & Niels Asger Nielsen, 2005). LU Xiaomin and XUE Yunjian reclassified impulse purchase into catalyzed, compensatory, penetrative, and blind impulse purchasing. Main factors affecting the behavior are also analyzed in depth with features as follows: first, detailed research target instead of ambiguous consumer image; second, more comprehensive incentives combining examples, theories and data, some most typical incentives are summed as follows: attitude towards money, use of credit cards, life experience, bundled price promotion, self-construction, allopatry and anticipated regret. Thus, we can give an outlook of Chinese impulse purchase researches, and continue studying "re-impulse purchase" which will shift focus from traditional shopping to E-shopping. Impulse buying behaviors are presumed to be universal in nature (Mai *et al.*, 2003; Rook, 1987). However, although the impulse behavior as such may not deviate much between countries, it is suggested that local market conditions as well as social and cultural factors affect consumers' propensity to make such purchases (Mai *et al.*, 2003; Rook, 1987; Shamdasani & Rook, 1989). There are several areas of consumers' consumption and patronage behavior that are in need of additional exploration (Pan & Zinkhan, 2006; Brown & Dant, 2008).

While ZHENG paid emphasis on mathematical analysis of Sales diffusion curve in order to explain how impulse affects the change of sales diffusion curve. The "impulse" can be described by survival analysis in the Statistics. The method in this article is to constitute three types differential equation by integrating sales diffusion curve, average speed of sales diffusion and time of market. Thus, we can anticipate some information such as peak sales, time of over-half sales, according to the results of differential equations.

On the basis of the second sales diffusion curve proposed by professor ZHENG, this article aims to calculate moment estimation through researching models' numerical characteristics and proving the existence of moment estimation. The article also uses Monte-Carlo to gain simulate estimation accuracy, and verify models with examples.

1. MODELS AND NUMERICAL CHARACTERISTICS

In this article, we have a specific discussion on the second model, combining equation (3) with average ratio $\frac{F(t)}{t}$ or instantaneous ratio $f(t)$. As a consequence, we get probability distribution function, show that

$$F(t) = 1 - \frac{1}{1+\alpha t^{\beta}} = \frac{\alpha t^{\beta}}{1+\alpha t^{\beta}}, \quad t > 0, \alpha > 0, \beta > 0$$ where

t denotes time

α, β are parameter

Then we get its probability density function

$$f(t) = \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2}, \quad t > 0, \alpha > 0, \beta > 0 \tag{4}$$

Now, we can utilize the above contents to examine the image characteristics of $f(t)$

For given β, we find these six limits

$$\lim_{t\to 0} f(t) = \lim_{t\to 0} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = +\infty, \quad \lim_{t\to +\infty} f(t) = \lim_{t\to +\infty} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = 0$$

where $0 < \beta < 1$

$$\lim_{t\to 0} f(t) = \lim_{t\to 0} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = \alpha, \quad \lim_{t\to +\infty} f(t) = \lim_{t\to +\infty} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = 0$$

where $\beta = 1$

$$\lim_{t\to 0} f(t) = \lim_{t\to 0} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = 0, \quad \lim_{t\to +\infty} f(t) = \lim_{t\to +\infty} \frac{\alpha\beta t^{\beta-1}}{\left(1+\alpha t^{\beta}\right)^2} = 0$$

where $1 < \beta < +\infty$

Then we take the derivative of probability density function, show that

$$f'(t) = \frac{\alpha\beta t^{\beta-2}\left[(\beta-1) - \alpha(\beta+1)t^{\beta}\right]}{\left(1+\alpha t^{\beta}\right)^3} \tag{5}$$

For given β ($0 < \beta < 1$), we know $f'(t) < 0$.Hence, $f(t)$ is a strictly monotonic decreasing function. See Figure 1

For given β ($\beta = 1$), we also know $f'(t) < 0$.Hence, $f(t)$ is a strictly monotonic decreasing function .See Figure 1

But for β ($1 < \beta < +\infty$), there is something different. Let

$$(\beta-1) - \alpha(\beta+1)t^{\beta} = 0$$

Then seeking the solution of the equation, we get t_0

$$t_0 = \left[\frac{\beta-1}{\alpha(1+\beta)}\right]^{\frac{1}{\beta}} \tag{6}$$

From that we know the probability density function of this model, $f(t)$, is strictly monotonic increasing where t (denotes time) is from 0 to t_0.Correspondingly, $f(t)$ is strictly monotonic decreasing where t is greater than t_0. So $f(t)$ take the peak $t = t_0$ and the value is

$$f(t_0) = \frac{\alpha^{\frac{1}{\beta}}}{4\beta}(\beta-1)^{1-\frac{1}{\beta}}(1+\beta)^{1+\frac{1}{\beta}} \tag{7}$$

Characteristics of $f(t)$ see Figure1

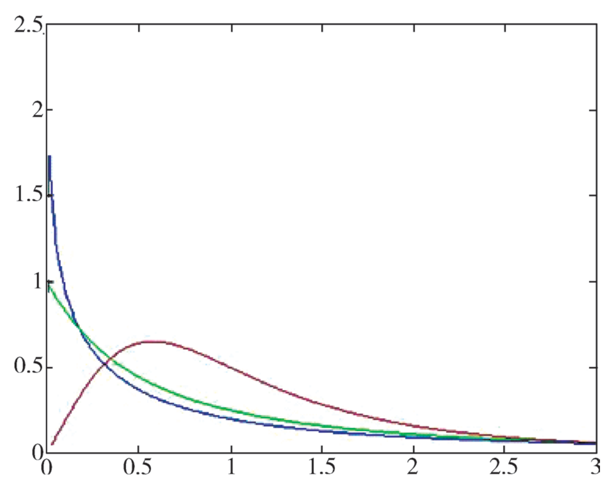

Figure 1
Probability Density Function $f(t)$ where $\alpha = 1$

The failure rate function of the above probability distribution is

$$\lambda(t)=\frac{f(t)}{1-F(t)}=\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}\ ,\ t>0,\alpha>0,\beta>0 \tag{8}$$

where t denotes time

α,β are parameters

Now, we start to examine the image characters of $\lambda(t)$ just as probability density function.

For given β , we find these six limits

$$\lim_{t\to 0}\lambda(t)=\lim_{t\to 0}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=+\infty\ ,\ \lim_{t\to+\infty}\lambda(t)=\lim_{t\to+\infty}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=0$$

where $0<\beta<1$

$$\lim_{t\to 0}\lambda(t)=\lim_{t\to 0}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=\alpha,\ \lim_{t\to+\infty}\lambda(t)=\lim_{t\to+\infty}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=0$$

where $\beta=1$

$$\lim_{t\to 0}\lambda(t)=\lim_{t\to 0}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=0\ ,\ \lim_{t\to+\infty}\lambda(t)=\lim_{t\to+\infty}\frac{\alpha\beta t^{\beta-1}}{1+\alpha t^{\beta}}=0$$

where $1<\beta<+\infty$

Then we take the derivative of failure rate function

$$\lambda'(t)=\frac{\alpha\beta\left(\beta-1-\alpha t^{\beta}\right)t^{\beta-2}}{\left(1+\alpha t^{\beta}\right)^{2}} \tag{9}$$

For given β ($0<\beta<1$), we know $\lambda'(t)<0$. Hence, $\lambda(t)$ is a strictly monotonic decreasing function. See Figure 2

For given β ($\beta=1$), we also know $\lambda'(t)<0$. Hence, $\lambda(t)$ is a strictly monotonic decreasing function too. See Figure 2

But for β ($1<\beta<+\infty$), the result is different.

Let

$$\beta-1-\alpha t^{\beta}=0$$

Then seek the solution of the equation, we get t_0'

$$t_0'=\left(\frac{\beta-1}{\alpha}\right)^{\frac{1}{\beta}} \tag{10}$$

From that we know failure rate function of the model, $\lambda(t)$, is strictly monotonic increasing where t (denotes time) is from 0 to t_0'. Correspondingly, $\lambda(t)$ is strictly monotonic decreasing where t is greater than t_0'. So $f(t)$ take the peak $t=t_0'$ and the value is

$$\lambda\left(t_0'\right)=(\beta-1)^{1-\frac{1}{\beta}}\alpha^{\frac{1}{\beta}} \tag{11}$$

Characteristics of $\lambda(t)$ are shown as Figure 2

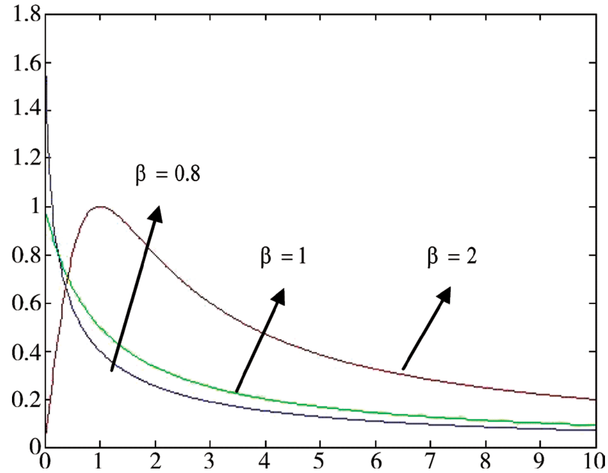

Figure 2
Failure Rate Function $\lambda(t)$ **Where** $\alpha=1$

The P-Quartile Chart of the Above Distribution

For given $0<p<1$, we know that $F(t)=p$ and just equals to

$$\frac{\alpha t^{\beta}}{1+\alpha t^{\beta}}=p$$

Solve the equation, then we get t_p

$$t_p=\left[\frac{p}{\alpha(1-p)}\right]^{\frac{1}{\beta}} \tag{12}$$

Numerical Characteristics of the Distribution

The k-moment of parameter t

$$ET^k=\int_0^{+\infty}t^k\frac{\alpha\beta t^{\beta-1}}{(1+\alpha t^{\beta})^2}dt$$

We simplify that expression as

$$ET^k=\int_0^{+\infty}t^k\frac{\alpha\beta t^{\beta-1}}{(1+\alpha t^{\beta})^2}dt=\int_0^{+\infty}\alpha^{-\frac{k}{\beta}}x^{\frac{k}{\beta}}\frac{dx}{(1+x)^2}$$

If k-moment of parameter t exists, we can get the equation

$$ET^k=\alpha^{-\frac{k}{\beta}}\mathrm{B}\left(\frac{k}{\beta}+1,2-\left(\frac{k}{\beta}+1\right)\right)$$

$$=\alpha^{-\frac{k}{\beta}}\mathrm{B}\left(1+\frac{k}{\beta},1-\frac{k}{\beta}\right)$$

where $\beta>k$

Especially, the mathematical expectation of parameter t is

$$ET=\alpha^{-\frac{1}{\beta}}B\left(1+\frac{1}{\beta},1-\frac{1}{\beta}\right)=\frac{\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)}{\alpha^{\frac{1}{\beta}}\Gamma(2)}$$

$$=\frac{\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)}{\alpha^{\frac{1}{\beta}}}$$

where $\beta>1$

Second-moment of parameter t is

$$ET^2=\alpha^{-\frac{2}{\beta}}B\left(1+\frac{2}{\beta},1-\frac{2}{\beta}\right)=\frac{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}{\alpha^{\frac{2}{\beta}}\Gamma(2)}$$

$$=\frac{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}{\alpha^{\frac{2}{\beta}}}$$

where $\beta>2$

2. MOMENT ESTIMATORS OF THE PARAMETERS

Suppose $T_1,T_2,\cdots,T_n$ is a simple random sample of population T, whose size is n. We can establish the following equation if we are notified $\beta>2$, that is

$$\frac{\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)}{\alpha^{\frac{1}{\beta}}}=\overline{T}\ ,\quad \frac{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}{\alpha^{\frac{2}{\beta}}}=\overline{T^2}$$

where
$$\overline{T}=\frac{1}{n}\sum_{i=1}^{n}T_i$$
$$\overline{T^2}=\frac{1}{n}\sum_{i=1}^{n}T_i^2$$

Let us establish a new equation

$$\frac{\left[\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)\right]^2}{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}=\frac{\left(\overline{T}\right)^2}{\overline{T^2}}$$

Lemma: the equation (13) for β has unique positive real root, where $\beta>2$

$$\frac{\left[\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)\right]^2}{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}=\frac{\left(\overline{T}\right)^2}{\overline{T^2}} \tag{13}$$

To prove the lemma, let us establish an auxiliary function $G(\beta)$

$$G(\beta)=\frac{\left[\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)\right]^2}{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)}$$

where $\beta>2$

Just because of $\beta>2$, we can infer that both $\frac{1}{\beta}$ and $\frac{2}{\beta}$ are not whole numbers. So we can construct two equations according to the properties of Gama function. Show that

$$\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)=\frac{1}{\beta}\Gamma\left(\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)=\frac{1}{\beta}\frac{\pi}{\sin\left(\frac{\pi}{\beta}\right)} \tag{14}$$

$$\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)=\frac{2}{\beta}\Gamma\left(\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)=\frac{2}{\beta}\frac{\pi}{\sin\left(\frac{2\pi}{\beta}\right)} \tag{15}$$

After plugging in (14), (15), we simplify the auxiliary function. That is

$$G(\beta)=\frac{1}{\beta^2}\frac{\pi^2}{\left[\sin\left(\frac{\pi}{\beta}\right)\right]^2}\cdot\frac{\beta}{2}\frac{\sin\left(\frac{2\pi}{\beta}\right)}{\pi}=\frac{\pi}{2}\frac{1}{\beta}\frac{\sin\left(\frac{2\pi}{\beta}\right)}{\left[\sin\left(\frac{\pi}{\beta}\right)\right]^2}$$

$$=\frac{\pi}{2}\frac{1}{\beta}\frac{2\sin\left(\frac{\pi}{\beta}\right)\cos\left(\frac{\pi}{\beta}\right)}{\left[\sin\left(\frac{\pi}{\beta}\right)\right]^2}=\frac{\pi}{\beta}\frac{\cos\left(\frac{\pi}{\beta}\right)}{\sin\left(\frac{\pi}{\beta}\right)}$$

$$=\frac{\pi}{\beta}\operatorname{ctg}\left(\frac{\pi}{\beta}\right)$$

Next, let us establish another auxiliary function $g(x)$

$$g(x)=x\operatorname{ctg}x$$

where $0<x<\frac{\pi}{2}$

$g'(x)$ is the derivation of function $g(x)$, show that

$$g'(x)=\operatorname{ctg}x-x(\csc x)^2=\frac{\cos x}{\sin x}-\frac{x}{(\sin x)^2}$$

$$=\frac{\cos x\sin x-x}{(\sin x)^2}=\frac{1}{(\sin x)^2}\left[\frac{\sin(2x)}{2}-x\right]$$

Let new function $g_1(x)$

$$g_1(x) = \frac{\sin(2x)}{2} - x$$

where $0 < x < \frac{\pi}{2}$

Similarly, we take the derivative of $g_1(x)$, that is

$$g_1'(x) = \cos(2x)$$

Because trigonometric function is smaller than real number 1, we can easily get the result

$$g_1'(x) = \cos(2x) - 1 < 0$$

Find the limit of $g_1(x)$ as x approaches zero and $\frac{\pi}{2}$

$$\lim_{x\to 0} g_1(x) = 0, \lim_{x\to \pi/2} g_1(x) = -\frac{\pi}{2}$$

So we get the result $g_1(x) < 0$, furthermore $g'(x) < 0$, thus $G'(\beta) > 0$

We also find the limit of $g(x)$ as x approaches zero and $\frac{\pi}{2}$, that is

$$\lim_{x\to 0} g(x) = 1, \lim_{x\to \pi/2} g(x) = 0$$

Moreover, we get two limits

$$\lim_{\beta\to 2} G(\beta) = 0, \lim_{\beta\to +\infty} G(\beta) = 1$$

Considering the inequality $0 < \frac{(\overline{T})^2}{\overline{T^2}} < 1$, so the equation

$$G(\beta) = \frac{(\overline{T})^2}{\overline{T^2}}$$

have unique solution where $\beta > 2$

Hence, the unique root of equation (16), $\hat{\beta}$, is just the moment estimation of β.

$$\frac{\left[\Gamma\left(1+\frac{1}{\beta}\right)\Gamma\left(1-\frac{1}{\beta}\right)\right]^2}{\Gamma\left(1+\frac{2}{\beta}\right)\Gamma\left(1-\frac{2}{\beta}\right)} = \frac{(\overline{T})^2}{\overline{T^2}} \tag{16}$$

As a consequence, parametric estimate of α is

$$\hat{\alpha} = \left[\Gamma\left(1+\frac{1}{\hat{\beta}}\right)\Gamma\left(1-\frac{1}{\beta}\right)\Big/\overline{T}\right]^{\hat{\beta}} \tag{17}$$

In order to examine the accuracy of the moment estimation, we make a simulation to display it concretely. Let the sample size n = 10(5)30, real values are α = 1, β = 3. Then we take 1000 times Monte Carlo simulation and we get moment estimation for mean and mean-square error of parameter α, β, see Table 1. From that we find the estimation precision is satisfied and mean-square error decreases as sample size increases.

Table 1
Simulation Results of Moment Estimation

n	$\hat{\alpha}$		$\hat{\beta}$	
	mean	**mean-square error**	**mean**	**mean-square error**
10	1.2347	1.9482	4.1304	2.2980
15	1.0571	0.6162	3.9172	1.5964
20	1.0140	0.3306	3.7395	1.1113
25	0.9664	0.2043	3.6487	0.8625
30	0.9535	0.1794	3.5647	0.6840

For example: Let the sample size n =10(5)30, real values are $\alpha = 1$, $\beta = 3$. Take Monte Carlo simulation and we get a set of simple random sample from population T, show that

0.6533, 0.8704, 0.5044, 0.6698, 1.1526, 0.7570, 0.7539, 1.4736, 2.3477, 1.1282,

0.5108, 0.7828, 0.3145, 0.6842, 0.7627, 0.7788, 0.8156, 1.9681, 0.6543, 0.9698

Thus we get the moment estimation of parameter α, β

$\hat{\alpha} = 2.0541$, $\hat{\beta} = 4.0202$.

CONCLUSION

Based on the properties of probability distribution function and failure rate function, while in method of moment estimation, researches in this article estimate α, β, variables of model, through analyzing numerical characteristics of variables in model. This article also testifies moment estimation accuracy of α, β, by means of using simulate simple random samples from population T, and these samples are produced by Monte-Carlo. From the result, the moment estimation accuracy is convincible. Thus, we can put this model into practice to help solve realistic problems.

REFERENCES

Brown, J. R., & Dant, R. (2008). Scientific Method and Retailing Research: A Retrospective. *Journal of Retailing*, *84*(1), 1-13.

Kim Ramus, & Niels Asger Nielsen. (2005). Online Grocery Retailing: What do Consumers Think? *Internet Research*, *15*(3), 335-352.

LU, X. M., & XUE, Y. J. (2011). The Influence Factors of Impulse Purchasing Behavior. *Enterprise research*, (13), 50-52.

Mai, N. T. T., Jung, K., Lantz, G., & Loeb, S. G. (2003). An Exploratory Investigation into Impulse Behavior in a Transitional Economy: A Study of Urban Consumers in Vietnam. *Journal of International Marketing*, *11*(2), 13-35.

Pan, Y., & Zinkhan, G. W. (2006). Determinants of Retail Patronage: A Meta-Analytical Perspective. *Journal of Retailing*, *82*(3), 229-243.

Shamdasani, P. N., & Rook, D. W. (1989). An Exploratory Study of Impulse Buying in an Oriental Culture: The Case of Singapore. *Singapore Marketing Review*, *4*, 7-20.

XUE, M. (2010). Consumers Impulse Shopping Behavior and Marketing Countermeasure Analysis. *Business Herald*, (11), 113-114.

ZHENG, Z. K. (2010). The Mathematical Analysis of Sales Diffusion Processes. *Mathematical Theory and Applications*, *30*(3).

