Outsourcing Decision Model of Sewage Treatment Based on Emissions Trading

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Abstract
This paper considers the professional treatment manufacturers to participate in emissions trading under the condition of the emissions trading which establish the supply chain between the manufacturers and the providers of professional sewage treatment. By using the strategy analysis method, under the certain conditions this study analyze the difference of the outsourcing decisions on the treatment price fluctuating. This study also conclude that the expected return of the supply chain on the strategy of all manufacturers choose to outsourcing the sewage treatment is less than the expected return on the integration supply chain strategy. Also, this research documents the existence of the supply chain. The conclusions of this paper contribute to the establishment of emissions trading under the conditions of wastewater treatment supply chain and future contributing to the sewage treatment industry with the application.

Key words: Emission trading; Outsourcing decision; Sewage treatment

INTRODUCTION
Franchising is a major mode of using and operating mechanisms in the current reform of public utilities such as sewage treatment in China (YANG, 2010). These patterns changed in the past investment in sewage treatment by the government exclusive, exclusive building and self-management system, and gradually formed a unified planning and management in the government, the diversified investment, multi-mode operation and the broad participation of society new system. Li Fang, Huang Tongcheng, Gu Mengdi and Wang Mengguang (2006) discussed the use of two-stage game model of emissions trading under the control of the object identified, efforts to control the arrangement of the penalties for illegal sewage behavior and effective control of the target design and other issues (LI, HUANG, GU, 1978). Harford (1978) put forward a simple theoretical model in pollution emission standards and tax firm behavior under the conditions (Harford. J. D., 1978). Keeler (1991) studied the pollution discharge enterprise through the concrete fine function model to carry out the pollution discharge standard to evade taxes incompletely the behavior, when the control cannot obtain coordinates completely, to the pollution discharge power transaction and the standard carries on the concrete comparison (Keeler A G., 1991). Malik (1990) and Li Shoude with Huang Tongcheng (2003) discussed the general enforcement of violations under the emissions trading market and its control mechanisms (Malik, Aroun S., 1990; DU, DONG, LIANG, ZHANG, 2009). Sun Wei (2010) introduces the pollution discharge reduction quantity and the transaction scale of charges parameter, the application real options method, has established the non-transaction expense separately and has under the transaction expense condition to monopolize the manufacturer anti-pollution technology investment...
decision-making model (SUN, SHANG, LIANG, 2010). Yang Song (2010) based on the economical optimality, fairness and the production continuous principle, has constructed an initial pollution discharge power free assignment multi-objective decision making model, has carried on the discussion to this model related nature. (LI, HUANG, 2003) Li Fang (2006) analyzed in the sewage treatment to purify the cost to the enterprise the influence, established the enterprise to produce the optimized model, obtained had under the emissions quota most superior production strategy. (WANG, LIU, 2008) Most of papers in the references mainly studied the sewage treatment question under the pollution discharge power transaction condition between the pollution discharge enterprise or the pollution discharge enterprise and the government, The paper (WANG, LIU, 2008) tell us the general manufacturer non-core spare part outside package of strategy question. In this paper, giving a professional treatment to participate in emissions trading model for sewage treatment, sewage treatment research findings to promote the industrialization of a reference value, based on the literatures above.

1. MODEL NOTATION AND ASSUMPTIONS

Consider a single manufacturer and single sewage treatment enterprises consisting of a single period of the supply chain. Manufacturer produces a single product, and a certain amount of sewage discharged. manufacturers could either process polluted water by themselves or outsource to specialized wastewater treatment plant through pollution rights trading system. The market demand be faced by the manufacturer is stochastic, but probability distribution is known. \( \lambda \) is the manufacturer’s production capacity (in production that), as the manufacturer’s core production capacity for decision-making variables. is the product manufacturer’s market demand, \( D \leq \lambda \); \( y \) is random variable of \( D \), \( \lambda(y) = Y \) is the amount of sewage discharge of the finished product \( y \) is the production function. Without loss of generality, we consider the discharge per unit of production volume \( \lambda(y) > 0 \), that is \( Y = \lambda(y) = \lambda y \); Apparently the manufacturer’s total sewage discharge \( \lambda(D) \), and \( \lambda(D)\leq \lambda (\lambda) = X_1 \). Set \( X_1 \) the manufacturer to handle the sewage capacity, so there is \( X_1 = \lambda(y) \), \( 0 \leq \lambda \leq D \leq \lambda (\lambda) \); as the manufacturer’s sewage treatment capacity, is non-core production capacity, but also decision variables. \( f(\lambda) \), \( F(\lambda) \) respectively is the produce market demand probability density and the distribution function. \( p \) is the manufacturer’s unit sales price, unit of emissions trading, \( p_i \) is emissions trading market price of the product manufacturer or sewage treatment business unit of purified water discharge of sewage obtained the right to save. After \( a \) is the pollution discharge power which the product manufacturer or the sewage treatment business unit sewage withdrawal purifies obtains saves, also is called the sewage treatment purification level, \( 0 \leq \alpha < 1 \). \( c_i \) is the manufacturer unit product production cost, \( c_i(a) \) is the product manufacturer under purification level unit sewage withdrawal processing cost, continuously differentiable, and \( c_i(0) = 0 \), \( c_i(1) = \infty \), \( c_i'(a) > 0 \), \( c_i''(a) > 0 \), \( c_i(a) \) is sewage treatment business under the purification level \( a \) unit sewage withdrawal purification processing cost, continuously differentiable also satisfies \( c_i(0) = 0 \), \( c_i(1) = \infty \), \( c_i'(a) > 0 \), \( c_i''(a) > 0 \). \( \Pi_m(X) \) is the manufacturer’s expected profit. \( \Pi_m(X') \) is the maximum expected return of the manufacturer, \( \Pi(X) \) is the expected return of sewage treatment enterprises, \( \Pi(X) \) is the maximum expected return of the sewage treatment enterprise. \( \Pi(X) = \Pi_m(X) + \Pi_m(X) \) is the overall supply chain benefits in distributed systems. \( \Pi(X) \) is the expected return when the sewage treatment business and professional enterprises as a whole . We have some of the assumptions are as follows:

Assumption 1: Emissions trading market is improve and perfect, the manufacturers and the professional treatment providers can enter the market for emissions trading at any time, is the unit price of emissions trading in emissions trading market .

Assumption 2: Supply chain parties (the manufacturers and the wastewater treatment providers) are neutral attitude toward risk, that decision is based on both their expected revenue maximization.

Assumption 3: The Manufacturer’s purpose is to complete the sewage treatment emission reduction targets for government issued, not to sell emission rights.

Assumption 4: Professional sewage treatment capacity of sewage treatment business unlimited.

Assumption 5: Do not consider the product manufacturer lead time.

Assumption 6: \( c_i(a) < c_i(\alpha) \).

2. MODELLING AND ANALYSIS

Manufacturer’s decision-making process is: first, the core product in the beginning of the production capacity and sewage treatment capacity to make decisions; the second step, according to market demand determine the actual situation of the number of wastewater treatment outsourcing. Because of \( Y = \lambda(y) = \lambda y \), and \( f(y) \) is the probability density of demand random variable \( Y \), is

\[ g(Y) = f(Y) \frac{dy}{dY} = \frac{1}{\lambda} \frac{dY}{dY} \]

Let us make the following tag:

\[ \lambda (\lambda) = X_1, \lambda (\lambda) = X_1, X = (\lambda, \lambda), (X_1, X_2) = \lambda X. \] So, the expected profits of the manufacturers and the sewage treatment business should be:

\[ \Pi_m(X) = \int [\alpha f(y) dy - \int x_t f(y) dy + p \lambda (y) \int (x_t - x_s) f(y) dy] \]

\[ \lambda x_t \leq x_s, x_t \geq 0, x_s \geq 0 \]  

\[ \Pi_m(X) = [p - \alpha] \int [\alpha y - \int x_t f(y) dy + \int (x_t - x_s) f(y) dy] \]

\[ \lambda x_t \leq x_t, x_s \geq 0, x_s \geq 0 \]  

(1)

(2)
Lemma. If $0 < \lambda < \frac{p}{\mu_p}$, than the expression (1) is a concave function in $R_c = \{(X_{1}, X_{2})|X_{1}, X_{2} \geq 0, X_{1} \geq X_{2}\}$.

Proof: The calculation yields:

$$\frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} = (p_{\lambda} - \rho)F(x_{i}) + (p - p_{\lambda} - c_{i})$$

$$\frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} = p_{\lambda} \int q_{i}(x)dx - p_{\lambda}F(x_{i}) + p - c_{i} \otimes \lambda$$

$$\frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} = 0 \quad \frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} = (p_{\lambda} - \rho)F(x_{i}) < 0$$

$$\frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} = - \rho F(x_{i}) < 0$$

So the Hesse matrix $\nabla \prod_{i=1}^{2} (X_{i}) = \begin{bmatrix} \nabla \prod_{i=1}^{2} (X_{i}) \nabla \prod_{i=1}^{2} (X_{i}) \nabla \prod_{i=1}^{2} (X_{i}) \nabla \prod_{i=1}^{2} (X_{i}) \end{bmatrix}$

Because $\frac{\partial \prod_{i=1}^{2} (X_{i})}{\partial x_{i}} < 0$, $\nabla \prod_{i=1}^{2} (X_{i}) > 0$, so $\nabla \prod_{i=1}^{2} (X_{i})$ will be the positive definite matrix. Therefore, $\prod_{i=1}^{2} (X_{i})$ is a strictly concave function based on the $R_c$. The Lemma has been proved.

We use the Kuhn - Tucker conditions for solving the most advantages of the equation (1). That is, the manufacturer’s optimal decision on treatment. The formula (1) standard form deformation as follows:

$$\min[\prod_{i=1}^{2} (X_{i})]$$

$$g_{1}(x) = x_{1} - x_{2} \geq 0$$

$$g_{2}(x) = x_{2} \geq 0$$

We can get the manufacturer’s integration by solving equations (3):

Strategy I. When $\mu_{2}^* > 0, \mu_{2}^* > 0 > \chi^*$, we can get

$$x_{1}^* = F^{-1}(\frac{p - p_{\lambda} - c_{i}}{p_{\lambda}})$$

and $\mu_{2}^* = c_{i}(\alpha)\lambda - p_{\lambda}$ from (4). And because $\chi^* \geq 0, \mu_{2}^* > 0$, so that $0 < p_{s_{2}} \leq \min \{c_{i}(\alpha)\lambda - \frac{p - c_{i}}{\lambda}\}.$

Strategy II. When $\mu_{2}^* > 0, \mu_{2}^* = 0$, we can get

$$x_{1}^* = F^{-1}(\frac{p - p_{\lambda} - c_{i}}{p_{\lambda}})$$

and $\mu_{2}^* = \frac{c_{i}(\alpha)\lambda}{p_{\lambda}}$ from (5), and because $\chi^* \geq 0$, we get

$$\frac{p - p_{\lambda} - c_{i}}{p_{\lambda}} - \frac{p - c_{i}(\alpha)\lambda}{p_{\lambda}} \geq 0$$

$$\frac{p - p_{\lambda} - c_{i}}{p_{\lambda}} - \frac{p - c_{i}(\alpha)\lambda}{p_{\lambda}} \geq 0$$

We can get the following results from the latter two in equalities: $p_{s_{2}} e[c_{i}(\alpha)\lambda - \frac{p - c_{i}}{\lambda}]$. The first inequality can be organized into the following inequality:

$$(\lambda^2 - \lambda)p_{s_{2}}^2 + [c_{i}(\alpha)\lambda^2 + p + c_{i}(\alpha)\lambda - p_{\lambda}]p_{s_{2}} - p_{c_{i}(\alpha)\lambda} \leq 0$$

Let $\Delta = [c_{i}(\alpha)\lambda^2 + p + c_{i}(\alpha)\lambda - p_{\lambda}]^2 + 4(\lambda^2 - \lambda)p_{c_{i}(\alpha)\lambda}$.

$\omega(p_{s_{2}}) = (\lambda^2 - \lambda)p_{s_{2}}^2 + [c_{i}(\alpha)\lambda^2 + p + c_{i}(\alpha)\lambda - p_{\lambda}]p_{s_{2}} - p_{c_{i}(\alpha)\lambda}$. Obviously $\Delta > 0$, then $\omega(p_{s_{2}}) > 0$ has two unequal real roots. May wish to assume there roots are $p_{s_{2}}, p_{s_{2}}^*, p_{s_{2}}^*$, because $p_{s_{2}}, p_{s_{2}}^*, p_{s_{2}}^* < p_{\lambda}$. So there must be $p_{s_{2}}^* < 0, p_{s_{2}} > 0$, and we can get the following results:

$$\lambda > 1, p_{s_{2}} \in [c_{i}(\alpha)\lambda - \frac{p - c_{i}}{\lambda}] \cap (0, p_{\lambda})$$

$$\lambda < 1, p_{s_{2}} \in [c_{i}(\alpha)\lambda - \frac{p - c_{i}}{\lambda}] \cap [p_{\lambda}, p_{\lambda}]$$

$$\lambda = 1, p_{s_{2}} \in [c_{i}(\alpha)\lambda - \frac{p - c_{i}}{\lambda}]$$

strategy III. When $\mu_{2}^* > 0, \mu_{2}^* = 0$, we can get the following result from (3):

$$x_{1}^* = F^{-1}(\frac{p - p_{\lambda} - c_{i}(\alpha)\lambda}{p_{\lambda}})$$

and $\mu_{2}^* = - p_{\lambda}F(\chi_{1}^*) + p_{\lambda} - c_{i}(\alpha)\lambda$. because $\mu_{2}^* = - p_{\lambda}F(\chi_{1}^*) + p_{\lambda} - c_{i}(\alpha)\lambda > 0$.

To solve the above Inequalities we have the following results:

When $\lambda > 1, p_{s_{2}} \in (0, p_{\lambda} - c_{i}(\alpha)\lambda)] \cap [p_{\lambda} - 1, \lambda^{-1}]$; When $\lambda < 1$ and

$\Delta > 0$, $p_{s_{2}} \in (0, \frac{c_{i}(\alpha)\lambda + c_{i}(\alpha)\lambda}{1 - \lambda}] \cap [p_{\lambda} - 1, \lambda^{-1}]$, if $\Delta < 0, p_{s_{2}}$ will be no solutions; When $\lambda = 1, p_{s_{2}} \in (0, \frac{c_{i}(\alpha)\lambda + c_{i}(\alpha)\lambda}{1 - \lambda}) - p_{\lambda}$.

strategy IV When $\mu_{2}^* > 0, \mu_{2}^* > 0$, the following results can be deduced from equation (3):

$$\chi_{1}^* = x_{1}^* > \lambda > \chi_{2}^*$$

$$\lambda > 1, then p_{s_{2}} \in (\max \{\frac{p - c_{i}(\alpha)\lambda}{1 - \lambda}, \frac{p - c_{i}(\alpha)\lambda}{p_{\lambda}}\}) \cap \chi_{2}^*$$

$\lambda = 1, then p_{s_{2}} > p_{\lambda}$; and If $\lambda < 1, then p_{s_{2}}$ will be no solutions.

Proposition 1. If the manufacturers choose outsourcing all the task of sewage treatment, the maximum expected profit of supply chain in a decentralized system would be less than the maximum expected profit of supply chain in integration model.

Proof: In the decentralized system, there must be $p_{s_{2}} > c_{i}(\alpha)$. When all of the sewage treatment companies have chosen outsourcing, we know that from the integration strategy

$$\lambda_{2}^* = 0 \text{ and } x_{1}^* = F^{-1}(\frac{c_{i}(\alpha)\lambda}{p_{\lambda}}).$$

Thus the operation maximum expected gain of the supply chain would be that as follow:

$$\Pi_{s}(X) = \Pi_{a}(X) = \Pi_{s}(X) = [p - c_{i}(\alpha)\lambda][x_{1}^* - \frac{1}{\lambda}F(y\lambda) - c_{i}(\alpha)\lambda]$$

(6)

In the case of integration, $\chi_{2}^* > 0$, and no trading of emission rights, that is $p_{s_{2}} = c_{i}(\alpha)$. At this point, the supply chain expected profit is
\[
\Pi_n(X) = [p - c_s(\alpha)\lambda] \int x_i - \frac{1}{\alpha} F(y)dy - c_r x_i \quad (7)
\]

It is clearly that the formula (9) satisfies the lemma conditions. To consider
\[
\frac{d\Pi_n(X)}{dx} = [p - c_s(\alpha)\lambda] \int 1 - F(y)dy - c_r = 0,
\]
then we will have \( F(x^*_{\lambda}) = \frac{p - c_s(\alpha)\lambda - c_r}{p - c_s(\alpha)\lambda} \),
and thus, the maximum expected return of the supply chain in the integration case should be
\[
\Pi_n(\bar{x}^*_{\lambda}) = \left[ p - c_s(\alpha)\lambda \right] \int_{x^*_{\lambda}}^{\bar{x}^*_{\lambda}} 1 - F(y)dy - c_r x^*_{\lambda} \quad (8)
\]
The inequality \( F(x^*_{\lambda}) > F(x^*_{\lambda}) \) or \( x^*_{\lambda} > x^*_{\lambda} \) can be proved easily.

Let
\[
W(x) = \left[ p - c_s(\alpha)\lambda \right] \int x - \frac{1}{\alpha} F(y)dy - c_r x
\]
\[
\frac{dW(x)}{dx} = \left[ p - c_s(\alpha)\lambda \right] \int 1 - F(y)dy - c_r = \frac{c_r\lambda p - c_r(\alpha)}{p - c_s(\alpha)\lambda} > 0, \quad \text{So } W(x^*_{\lambda}) > W(x^*_{\lambda})
\]
To consider the formulas (6), (8), and (9), we have
\[
\Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) - \Pi_n(x^*_{\lambda}) + \Pi_n(x^*_{\lambda}) = \Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) > 0
\]
This completes the proof.

Proposition 2. In this paper’s conditions, if the manufacturer pay a fixed fee to the professional treatment providers in a period advance:
(i) The manufacturer pay a fixed fee to the professional treatment providers to resale their emission rights with cost-price to the manufacturer, that is \( p_\lambda = c_s(\alpha) \), and the number does not exceed the maximum set by the manufacturer at the beginning emissions.

Proof: Just prove the existence of a constant \( \lambda \) and the supply chain after being coordinated equals to the profit of the supply chain in integration. \( \Pi_n(X) \) is the expected profit function after coordination of the manufacturer (sewage companies) and \( \Pi_n'(X) \) respect the expected profit function after coordination of the professional sewage treatment business.
\[
\Pi_n'(X) = p\int x_i - \frac{1}{\alpha} F(y)dy - c_r(x_i - \frac{1}{\alpha} \int p_\lambda Y dy) - c_r x_i - M = (p - c_s(\alpha)\lambda)\int x_i - \frac{1}{\alpha} F(y)dy - c_r x_i - M
\]
\[
\Pi_n'(X) = 0 + M
\]

The formula (12) apparently to meet the lemma conditions available. From the first order conditions, the yield optimal product of \( \Pi_n'(X) \) should be
\[
\hat{x}^*_n = \frac{\frac{p - c_s(\alpha)\lambda - c_r}{p - c_s(\alpha)\lambda}}{0} = \hat{x}^*_n
\]
To consider formulas (12) and (13), we know that the optimal supply chain expected profit after coordination, that is
\[
\Pi_n(\bar{x}^*_{\lambda}) = \Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) - \Pi_n(x^*_{\lambda}) + \Pi_n(x^*_{\lambda}) = \Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) > 0
\]
As the maximum expected income gains between the two sides after coordination are the increase than those in pre-coordinated, and \( \Pi_n(\bar{x}^*_{\lambda}) > M > \Pi_n(\bar{x}^*_{\lambda}) \), therefore
\[
\Pi_n(\bar{x}^*_{\lambda}) = \Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) - \Pi_n(x^*_{\lambda}) + \Pi_n(x^*_{\lambda}) > 0
\]
\[
\Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) = \Pi_n(\bar{x}^*_{\lambda}) - \Pi_n(x^*_{\lambda}) > 0
\]
This completes the proof.

**CONCLUSION**

(1) According to the above analysis, it is not difficult to see that professional treatment providers in order to enter the sewage treatment market should strive to achieve \( c_s(\alpha)\lambda - p_\lambda > 0 \), allowing the manufacturer to select the integration strategy I, that is outsource all of its wastewater treatment business. This need to reduce treatment costs, and to reduce the emissions trading price to the following bottom line the manufacturer can select the integration strategy I.

(2) The government should waste water treatment industry’s supply chain to provide the necessary legal conditions. The first is to increase the energy saving law enforcement, sewage discharge limits, the implementation of total control; The second is to establish emissions trading mechanisms, so that pollution industries to buy emission rights according to their own emission reduction benefits, while professional sewage treatment enterprises can make full and timely sale of emission rights. The third is to use administrative means to control the price through emissions trading sewage treatment benefit distribution mechanism, through the purification of the levels of sewage treatment standards for sewage treatment, thus promoting the orderly operation of sewage treatment industry.

(3) The businesses of sewage treatment are likely to cheat the government arbitrarily reduced wastewater treatment standards in order to gain their maximum economic benefits. The is a indicator controlled by the government, therefore, the government can take effective measures to control the and prevent professional sewage treatment enterprises the behavior to get the benefits of enterprise through damage public interest.

(4) In this study, no government department, the manufacturer (sewage enterprises), professional treatment...
providers around the emissions trading price of the conduct of the game. In addition, government departments how to use emissions trading prices, the level of sewage purification and sewage charges through the sewage treatment industry on the implementation of effective policy guidance and regulation, is the next step of the important issues.

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