Retailer’s Optimal Ordering Policies with Two Stage Credit Policies and Imperfect Quality

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Abstract

Two levels of trade credits refers that the supplier provides to his/her retailer a permissible delay period ($M$) in paying for purchasing items and the retailer also in turn provides a permissible delay period ($N, M > N$) to his/her customer to stimulate his product demand. When lot received by retailer, it may be contain some imperfect quality of goods by the causes of non-ideal production process or other causes. So retailers perform a screening process to find the imperfect items and returned to the supplier immediately. Therefore, an attempt is made in this paper to develop the retailer’s optimal ordering policies in supply chain coordination with upstream and downstream trade credits and imperfect quality. The propose paper considers two cases $N \leq M$ and $M \leq N$ that is more near to real world cases. Some numerical examples are used to be show validity of this paper.

Key words: Inventory; Imperfect items; Up-stream and down-stream trade credits and supply chain

INTRODUCTION

Several research articles have been published in various international journals by taking the assumption of permissible delay in payment in supply chain coordination. It is tacitly assumed that the retailer would pay for the items as soon as the items were received. In general, the supplier offers a grace period term as trade credit to his/her retailer to increase of sale and joined the new customer. In this credit policy, the supplier does not charge any interest if the payment is made before credit period. However, if the payment is paid beyond the predefined period, a high interest is charged. Some of the prominent paper related to trade credit as follows.


In above models, the supplier adopts a business strategy of permissible delay period in paying the purchasing cost to attract more customers. The retailer takes the benefit of trade credit and sells the product and earns the interest by putting generated revenue in an interest bearing account. They implicitly assumed that the
buyer would pay the supplier as soon as he/she receives the items. As mentioned earlier, supplier only offers a trade credit to the retailer but retailer does not provide any trade credit to his/her customer which means we deal with one level of trade credit. However, in many real-world cases, this condition will not hold. Recently, Huang (2003) modified this assumption by assuming that the retailer will also adopt the trade credit policy to stimulate his/her customer demand to develop retailer replenishment model and this method is called two level of trade credit policy.

In Huang’s (2003) model the retailer starts to accumulate the revenue from O to T. But in real world it is not always true. Then Goyal and Teng (2007) developed a model in which the retailer starts to accumulate revenue from N to T+N, that is more match real life cases and selling price is higher than the cost price.

High quality always translates into high volume and high quality of products produce to customers is key of maintaining in marketing world. But, in sometimes production assembly may be influence by so many causes that result the imperfect items produced. A lot of number interesting papers published in various journals by taking assumption of imperfect items is presented in lots and imperfect items is found by a screening process. Some of the related papers of imperfect items are as follows.


This paper extends Goyal and Teng (2007) model in the light of imperfect items and lot contains some imperfect items which is found by a screening process and returned to supplier immediately when full screening process ends. In this paper, we developed retailer’s optimal ordering policies for imperfect items in supply chain coordination. The basis objective of this paper is cost minimization for retailer. Numerical examples are illustrated to managerial insight to proposed problem.

1. ASSUMPTIONS AND NOTATIONS

For convenience, most Assumption and Notations similar to Goyal and Teng (2007) will be used in the present paper.

1.1 Assumptions

1) The demand rate and defective rate are constant over time.

2) Lead time zero and Shortages are not allowed.

3) Replenishment rate is infinite.

4) Imperfect items are treated as a single batch and returned to the supplier immediately when the 100% screening process ends. The supplier will discount all defective items for sale to customers.

5) The delay period is larger than the screening time.

6) To avoid shortages for the retailer, the expected quantity of perfect items should be greater than or equal to the demand during the screening time.

7) Trade credit between supplier and retailer as follows:

- If the retailer pays by M, then supplier does not charge the retailer any interest.

- If the retailer pays after M but before N, he keeps his/her profit and sells revenue is utilized to earn interest with annual rate Ic. Then the supplier charges the retailer an interest rate of Ic on the balance amount.

- If the retailer pays after second permissible delay period N, then supplier charges the retailer an interest rate of Ic on the balance amount, with Ic > Ic.

1.2 Notation

1) D The demand rate per year

2) h Unit stock- holding cost per year excluding interest charge

3) P The selling price per year

4) c The unit purchasing cost, with c < P

5) A The ordering cost per order

6) T The replenishment cycle

7) M The retailer’s trade period offered by supplier in years

8) N The customer’s trade period offered by retailer in years M > N

9) Ie The interest earned per S in stock per year

10) Ic The interest charged per S in stocks per year by supplier in years.

11) Q The order quantity

12) T* The optimal cycle time of TC(T)

13) χ The screening rate

14) y The random variable representing the percentage of defective items

15) Y The expected value of y i.e. Y=E(y)

16) ν The unit warranty cost for defective items including the penalty cost for the supplier

17) K The required time for screening the defective items, K=Q

18) TC The total cost of an inventory system/unit time

2. MATHEMATICAL FORMULATION

The total relevant cost consists following elements:

Expected annual ordering cost per unit time is $A T$ (1)

Expected screening cost per unit time is $dQ/dT = dD(1-Y)$
Expected holding per unit time is
\[ \frac{h\left(\frac{Q(1-Y)}{2} + \frac{YD}{2}\right)}{T} \]  (2)

Expected interest earned and additional sales for defective items per unit time is
\[ \frac{(v-y)Q}{T} + \frac{v(1-Y)Q(M-k)}{T} \]
\[ = \frac{(v-y)DY}{(1-Y)} + \frac{v(1-Y)D(M-k)}{(1-Y)} \]  (4)

Case 1 \( N \leq M \)

Sub-case 1-1 \( M \leq T + N \)

In this paper, the sales revenue is utilized to earn interest during the period of \((M, T + N)\) when the account is settled, the items which are still in inventory have to be financed with annual rate. Therefore, the annual interest payable is as follows in figure 1.

The interest charged per year is
\[ \frac{cilD}{2T} [T + N - M]^2 \]  (5)

The annual total relevant cost for the retailer as
\[ TC_1(T) = \frac{A}{T} + \frac{dD}{T} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T + \frac{cilD}{2T} [T + N - M]^2 \]  (6)

Sub-case1-2 \( M > T + N \)

In this sub-case, the retailer has enough money to pay full the supplier. So, there is no interest charge. While retailer receives the total amount at time \( T + N \) and can pay full at time \( M \). Therefore, interest earned in shaded region is as follows

The annual interest earned is
\[ = \frac{cilD}{2T} \left[2(N-M) + T\right] \]  (7)

Thus, the annual total relevant cost is
\[ TC_2(T) = \frac{A}{T} + \frac{dD}{T} + h\left(\frac{D}{2} + \frac{YD^2}{x(1-Y)^2}\right)T + \frac{cilD}{2T} \left[2(N-M) + T\right] \]  (8)

3. DETERMINATION OF THE OPTIMAL CYCLE TIME \( T^* \)

In this section, we determine the optimal value of \( T \) which minimizes \( TRC(T) \). The necessary and sufficient condition for the optimality is \( TRC'(T) = 0 \) and \( TRC''(T) < 0 \). Taking the first order and second order derivatives of \( TRC_1(T)TRC_2(T) \) and \( TRC_3(T) \) with respect to \( T \), we obtain.
Theorem 2

When \( N \leq M \), it is clear from that \( TRC_A(T) \) is a strictly convex function in \( T \); consequently, we obtain the corresponding unique cycle time \( T_1^* \) as

\[
T_1^* = \sqrt{\frac{2A + D(M-N)^2}{(h+cle)D^2Y}} \geq 0
\]  
(18)

\( T_1^* \) will satisfy the condition \( M \leq T+N \) provided if and only if

\[
\Delta_1 = 2A - (M-N)^2 \left\{ A + 2D + \frac{h+cle}{D} \right\} \leq 0
\]  
(19)

Therefore, the optimal order quantity \( Q_1^* = DT_1^* \).

Consequently, we know that \( TRC_A(T) \) is also strictly concave function in \( T \). Likewise, we can easily obtain the unique optimal replenishment cycle time \( T_2^* \) as

\[
T_2^* = \sqrt{\frac{2A}{D(h+cle)} + \frac{h D^2 Y}{x(1-Y)^2}} \geq 0
\]  
(21)

\( T_2^* \) will satisfy the condition \( M \leq T+N \) provided, if and only if

\[
\Delta_1 = 2A - (M-N)^2 \left\{ A + 2D + \frac{h+cle}{D} \right\} \geq 0
\]  
(22)

Hence, the optimal order quantity \( Q_1^* \) is \( Q_1^* = DT_1^* \).

From the above argument, we obtain the following results.

Theorem 1

(A) if \( \Delta_1 \geq 0 \), then \( T_1^* = T_1^* \) and \( Q_1^* = Q_1^* \).

(B) if \( \Delta_0 \leq 0 \), then \( T_1^* = T_1^* \) and \( Q_1^* = Q_1^* \).

(C) if \( \Delta_0 = 0 \), then \( T_1^* = T_1^* \) and \( Q_1^* = Q_1^* \).

In classical economic order quantity model assumed that the retailer’s and the customer would be pay for the items as soon as the items received by him. Therefore, \( M=N=0 \), the classical optimal EOQ is

\[
Q_1^* = \sqrt{\frac{2AD}{(h+cle)}}
\]  
(23)

As a result, we can easily obtain the following theoretical result.

Theorem 2

When \( N \leq M \)

(A) if \( pe \leq cIc \), then both \( Q_1^* \) and \( Q_2^* \) are larger than \( Q_4^* \).

(B) if \( pe > cIc \), then both \( Q_1^* \) and \( Q_2^* \) are larger than \( Q_4^* \).

(C) if \( pe = cIc \), then \( Q_1^* = Q_2^* = Q_4^* \).

Next, let us discussed the case in which \( N \geq M \), when \( N \geq M \), we know that from (17) that \( TRC_A(T) \) is a strictly convex function in \( T \); consequently we obtain the unique optimal cycle time \( T_2^* \) as

\[
T_2^* = \sqrt{\frac{2A + D(M-N)^2}{(h+cle)D^2Y}} \geq 0
\]  
(24)

Therefore, the optimal order quantity \( Q_2^* \) is \( Q_2^* = DT_2^* \).

As a result, if \( N \geq M \), then the retailer’s optimal order quantity is exact the same as the classical EOQ.

4.  NUMERICAL EXAMPLE

(1) \( D = 4200 \), \( h = 4 \), \( A = 150 \), \( x = 175 \), \( d = 0.4 \), \( Ie = 0.09 \), \( N = 60/365 \), \( M = 90/365 \), \( Ic = 0.12 \), \( Y = 0.02 \), \( c = 20 \), \( P = 40 \), \( v = 30 \)

\[
\Delta_1 < 0, \quad T_2 = 0.07068, \quad Q_2 = 296.86, \quad k = 1.70, \quad TC_1 = 4651.65
\]

(2) \( Ie = 0.15 \) and other parameters is same as in example 1

\[
\Delta_1 > 0, \quad T_1 = 0.10594, \quad Q_1 = 444.95, \quad k = 2.54, \quad TC_1 = 7116.39
\]
CONCLUSION
This paper extends Goyal and Teng (2007) model by assuming imperfect items is presented in lots and imperfect items found by a screening process. In this paper, the retailer accumulated revenue from \(N\) to \(T+N\) that is more matches to real world problem and selling price is significantly higher than the unit cost. There are two cases \(M > N\) and \(N > M\) discussed with \(I_c > I_e\) and variety of examples are investigated for validity of model.

REFERENCES


