The Study on Long-Term Dynamics in China’s Coal Prices Based on Jump-Diffusion Models

JIN Hao[a]*, ZHANG Si[a]

[a]School of Science, Xi’an University of Science and Technology, Xi’an, China.
*Corresponding author.

Supported by NSFC (71103143, 71473194) and NSFS (2013KJXX-40).

Received 27 September 2015; accepted 18 November 2015
Published online 31 December 2015

Abstract
This paper analyzes the evolution process of China’s coal prices formation mechanism and the trend of dynamic weekly data of Datong mix coal in Qinhuangdao market from January 2004 to December 2013. Resorting to the jump diffusion models and cumulants estimation, the empirical study on the phenomenon of coal prices fluctuations is executed. These results show that coal prices dynamics are characterized by high volatility, high intensity jumps, and upward drifts, and are concomitant with underlying fundamentals of coal markets and China’s economy. Furthermore, markets expected coal prices to still remain volatile and jumpy with higher probability and stay in jump for the next couple of years.

Key words: Coal prices; Volatility; Jump-diffusion models; Cumulants estimation

INTRODUCTION
Coal is the affordable foundation energy for the sustaining development of economy, and remains more than 65% in the structure of primary energy production and consumption in China. The coal prices in China were in a high stage in recent years and reached the historical record in the first half of 2008. Qinhuangdao mixed coal weekly closing prices reached 1000 Yuan/ton in July 14, 2008, and the annual average prices 770 Yuan/ton averagely increases 53.5% comparing with the 502 Yuan/ton in 2007. Coal prices dynamics is relevant for hedging, forecasting and making policy, thus it is important for coal enterprises to find the characteristics of prices fluctuations and analyze the long term dynamic in coal prices.


Study on the nonlinear features of coal prices in these articles is not enough, especially less attention on the fluctuation of coal prices. Hence, we will analyze the long term dynamic of coal prices to make the best use of the current market situation and provided data support in making decisions for the coal enterprises.

1. HISTORICAL COAL PRICES ANALYSIS
With a view to concentrating on recent coal prices dynamics, we choose weekly spot prices of Datong mixed
coal delivered at Qinhuangdao Port from 2004 to 2013, containing 488 observations. Data are collected from China Coal Market Online (http://www.cctd.com.cn).

Figure 1
Observed Weekly Spot Prices of Datong Mixed Coal

Figure 1 shows spot prices for the full sample period. It clearly shows that fluctuation of coal prices is very large from 2004 to 2013. The coal prices are moving upward, and have become predictable. After each peak, coal prices seemed to retreat temporarily then re-trend toward higher peaks. Let \( P_t \) be the coal prices in Yuan/ton, an augmented Dickey-Fuller test (Table 1) indicated that \( P_t \) possessed a unit root; it was pulled by a strong upward trend, showing no sign for mean reversion. However, the first difference \( P_t \), defined as \( \Delta P_t = P_t - P_{t-1} \), was stationary.

Table 1
Time-series Properties of Coal Prices

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller (ADF) unit-root test on coal prices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null hypothesis: ( P_t ) has a unit root</td>
</tr>
<tr>
<td>ADF test statistic=-0.59; probability value=0.85</td>
</tr>
<tr>
<td>Test critical values: 1% (-3.44); 5% (-2.86); 10% (-2.57)</td>
</tr>
<tr>
<td>Null hypothesis: ( \Delta P_t = P_t - P_{t-1} ) has a unit root</td>
</tr>
<tr>
<td>ADF test statistic=-24.35; probability value=0.00</td>
</tr>
<tr>
<td>Test critical values: 1% (-3.44); 5% (-2.86); 10% (-2.57)</td>
</tr>
</tbody>
</table>

Additional insight into coal prices dynamics is gained by analyzing log return defined as \( \Delta \log P_t = \log P_t - \log P_{t-1} \). The graph for these changes (Figure 2) shows that jumps in coal prices were frequent and had a relatively high probability. The distribution was right-skewed, implying that upward jumps of larger size were more frequent than downward jumps of small size; as the mean was positive and high, smaller jumps were outweighed by larger jumps. The distribution had also fat tails, meaning that large jumps tended to occur more frequently than in the normal case. These empirical findings on coal prices were typical of financial time series as noted in Clark (1973), Fama (1965) and Mandelbrot (1963). These facts strongly suggested modeling the coal prices process as a jump-diffusion.

Volatility measures uncertainty and sensitivity of prices to news and shocks, and is a key parameter in option pricing. Volatility was also computed using a GARCH model for data on weekly coal prices covering full sample in Figure 3. The fitting of the GARCH model showed high prices volatility, periods of volatility clustering, followed by some reversion to a mean volatility estimated at 18 percent. GARCH volatility was rising during period of large prices shocks, simulating speculation and leading to volatility clustering; it was, however, receding during periods of prices retreat. It implied that coal markets were constantly experiencing large uncertainties and were affected by frequent shock.

Figure 2
Coal Prices Return Distribution. Note: mean=0.15, standard deviation=2.06, skewness=1.53, kurtosis=39.72, Jarque-Bera normality statistics=275, p-value=0.0.

Figure 3
Estimated GARCH (1,1) Volatility

2. COAL PRICES AS A JUMP-DIFFUSION PROCESS

2.1 Jump-diffusion Process

Based on the empirical findings discussed in the previous section, namely the presence of skewness and kurtosis in the empirical distribution of coal prices returns, an adequate model for coal prices would be a jump-diffusion model. In fact, it is well-known that options have market implied volatilities that exhibit a significant skew across strikes. In this connection, Bakshi, Cao, & Chen (1997) argued that pure diffusion based models could not adequately explain the smile effect in option prices and
emphasized the importance of adding a jump component in modeling asset prices dynamics. In the same vein, Bates (1966) also suggested that diffusion-based stochastic volatility models could not explain skewness in implied volatilities, except under implausible values for the model’s parameters. Models with jumps generically lead to significant skews for maturities. More generally, adding jumps to returns in a diffusion-based stochastic volatility model, the resulting model can generate sufficient variability and asymmetry to match implied volatility skews for maturities.

According, the continuous-time stochastic process driving coal prices can be stated as J-D process given by a stochastic differential equation (SDE): 

$$\frac{dP_t}{P_t} = \alpha dt + \sigma dW_t + (\exp(J_t) - 1)dN_t,$$

where $P$ denotes the weekly coal prices, $\alpha$ is the instantaneous return, and $\sigma$ is the instantaneous variance. The continuous component is given by a standard Brownian motion $\frac{dW_t}{\sqrt{\Delta t}} \sim N(0,\Delta t)$. The discontinuities of the prices process are described by a Poisson counter $N_t$, characterized by its intensity $\lambda$ and jump size $J_t$. The Brownian motion and the Poisson process are independent. The intensity of the Poisson process describes the mean number of arrivals of abnormal information per unit of time expressed as $P(N_t = n) = e^{-\lambda t} \frac{\lambda^t}{n!}$. Defining the characteristic function of $\frac{dP_t}{P_t}$ as $\phi(x,t) = \mathbb{E}[e^{ix\frac{dP_t}{P_t}}]$, we have:

$$\phi(x,t) = e^{\frac{\sigma^2 x^2}{2} + \lambda \frac{t}{\Delta t} (\frac{1}{2} - x^2 + \frac{x^2}{\Delta t} J_t)}.$$ 

Putting $\Delta t = 1$, i.e., the time interval is $(t,t+1)$, the density function becomes

$$f(x) = \sum_{n=0}^{\infty} \frac{(\lambda)^n}{n!} \frac{1}{\sqrt{2\pi(\sigma^2 + n\delta^2)}} \exp\left(-\frac{(x - \mu - n\beta)}{2(\sigma^2 + n\delta^2)}\right),$$

where $x = \frac{\Delta P}{P}$, $\mu = \frac{\Delta \ln P}{\Delta t}$, $\sigma$, $\lambda$ and $\beta$ are the mean, standard deviation, intensity, and jump size, respectively. The log-return, $\frac{dP_t}{P_t}$, is independent of $\frac{dW_t}{\sqrt{\Delta t}}$ in independent random variables and can be expressed as:

$$f(x) = \sum_{n=0}^{\infty} \frac{(\lambda)^n}{n!} \frac{1}{\sqrt{2\pi(\sigma^2 + n\delta^2)}} \exp\left(-\frac{(x - \mu - n\beta)}{2(\sigma^2 + n\delta^2)}\right).$$

### 2.2 The Method of Cumulants

Press (1967) used the method of cumulants as described in Kendall and Stuart (1977) to estimate the J-D models. Define the characteristic function of $x$, $u$ as:

$$\phi(u) = E[\exp(iux)] = \int \exp(iux) f(x) dx,$$

where $f(x)$ is the probability density function of $x$, $u$ is the transform variable, and $\mathbb{E}[x^k]$. The cumulants of $x$, denoted by $\omega_n$, $n = 0,1,2, \ldots$ are the coefficients in the power series expansion of the logarithm of the characteristic function of $x$, expressed as:

$$\ln\phi(u) = \sum_{n=0}^{\infty} \omega_n \frac{(iu)^n}{n!} = 1 + \omega_1 \frac{(iu)^1}{1!} + \omega_2 \frac{(iu)^2}{2!} + \cdots$$

Obviously, the cumulants enable to recover J-D parameters from sample moments (Cumulants one). In order to avoid using higher order cumulants, Press (1967) imposed the restriction $\mu = 0$ and derived the following relations (Cumulants two):

$$\hat{\beta}^2 - 2\frac{\omega_2}{\omega_1} \beta^2 + 2\frac{\omega_3}{\omega_1} \beta - \frac{\omega_2^2}{2\omega_1^2} = 0, \quad \lambda = \frac{\omega_2}{\beta^2},$$

$$\hat{\delta}^2 = \frac{\omega_3 - \hat{\beta}^2 \omega_2}{3\omega_1}, \quad \hat{\sigma}^2 = \frac{\omega_2}{\hat{\beta}} - \hat{\beta}^2 \omega_2 \omega_3 \omega_1 \frac{3\omega_1}{3\omega_1}.$$ 

Press’ estimates often carried wrong-sign and were not plausible. Beckers (1981) adopted the same method as Press, however, setting $\beta$, instead of $\mu$, to zero (Cumulants three). Using sixth order cumulants, his cumulant equations yielded the following system:

$$\hat{\mu} = \omega_1, \quad \hat{\lambda} = \frac{25\omega_2^2}{3\omega_1^2}, \quad \hat{\delta}^2 = \frac{\omega_2}{5\omega_1}, \quad \hat{\sigma}^2 = \omega_2 - \frac{5\omega_1^2}{3\omega_1}.$$ 

Beckers’ estimates improved those of Press, yet they were not free of anomalies. Ball and Torous (1985), using a Bernoulli, instead of a Poisson, jump process and maintaining Beckers’ restriction $\beta = 0$, derived the following cumulant equations (Cumulants four):

$$\omega_1 = \mu, \quad \omega_2 = \sigma^2 + \lambda \delta^2, \quad \omega_3 = 0, \quad \omega_4 = 3\delta^3 \lambda (1 - \lambda),$$

$$\omega_5 = 0, \quad \omega_6 = 156 \lambda (1 - \lambda) (1 - 2\lambda).$$

Again by equating with population cumulants, they obtained these estimators $\hat{\mu}, \hat{\lambda}, \hat{\sigma}^2$ and $\hat{\delta}^2$ given by:
\[ \hat{\mu} = \omega_a, \quad \hat{\lambda} = \left(1 \pm \sqrt{3\hat{\lambda}_a^2 / (3\omega_a^2 + 100\omega_a^4)}\right) / 2, \]
\[ \hat{\sigma}^2 = \omega_b / (\omega_a(5(1 - 2\lambda))), \quad \hat{\sigma}^2 = \omega_2 - \lambda \hat{\sigma}^2. \]  

\[ (12) \]

3. EMPIRICAL ANALYSIS

Based on a sample of weekly prices for Datong mix coal described in Section 2, jump diffusion process was estimated using cumulants method both unrestrictedly and under the assumption of a Bernoulli jump-diffusion process. The estimated parameters for the jump diffusion model are given in Table 2. Alternative estimations of jump diffusion model, except Cumulants two, yielded parameters estimates that were consistent with empirical features of coal prices discussed in Section 2. They showed pointedly that the dynamics of the coal prices process were influenced by both diffusion and jump component: however the jump component was dominant. Besides having high intensity, the jump components had a much higher variance than the diffusion component. The high variance of the jump component illustrated the presence of jumps of large magnitude and was in conformity with excess kurtosis in the empirical distribution of coal prices log returns. The mean of the jump size tended to be positive, in line with positive skewness in the empirical distribution. This was due to the fact that coal prices were not monotonic; they leapt forward, than retreated back in smaller movements before taking a new jump. The drift of the diffusion component was high, in conformity with the observed upward trend in coal prices; it illustrated the presence of a force that kept pushing coal prices upward.

By using of Cumulants one in J-D model, intensity of the jump process, estimated at \( \hat{\lambda} = 0.14 \), was high and significant, indicating that the coal prices processes were characterized by frequent jumps. Drift of the diffusion component, estimated at \( \hat{\mu} = 0.20 \), was high, implying that coal prices were constantly under pressure to move upward. The variances of the diffusion and jump components were respectively estimated at \( \hat{\sigma}^2 = 3.04 \) and \( \hat{\sigma}^2 = 7.46 \), indicating that the jump component tended to dominate the dynamics of the coal prices process. The mean of the jump component, estimated at \( \hat{\beta} = 1.17 \), was positive and consistent with the positive skewness in coal prices returns. Application of Cumulants two, with restriction \( \mu = 0 \), yielded implausible results for the variance of the jump component, namely \( \hat{\beta} = -10.84 \). Such an anomaly was not unexpected, indicating that the restriction \( \mu = 0 \), could not be borne by the data, and was in sharp contrast with the strong upward trend in coal prices.

In contrast, Cumulants three, with restriction \( \beta = 0 \), yielded results which were highly plausible. The drift component of the diffusion, estimated at \( \mu = 0.34 \), was larger than in the Cumulants one case, since \( \beta = 0 \) implied less influence for the drift of the diffusion, compared to the case when \( \beta \) was positive, to maintain an upward trend in coal prices. The variances of the diffusion and jump components were high, \( \hat{\sigma}^2 = 3.08 \) and \( \hat{\beta} = 7.39 \), respectively. The variance of the jump component, however, dominated that of the diffusion component. Noticeably, jump intensity, estimated at \( \hat{\lambda} = 0.18 \), is the frequency of jumps in coal prices exceeding \( \pm 3\% \) computed from the data set.

Assuming a Bernoulli jump-diffusion process, Cumulants four estimates were significant. Drift of the diffusion component, estimated at \( \hat{\mu} = 0.22 \), was high and significant, showing that coal prices were constantly under upward pressure. The variances of the diffusion and jump components were high and significant, \( \hat{\sigma}^2 = 3.07 \) and \( \hat{\beta} = 6.95 \) respectively. The probability of a jump computed at \( \hat{\lambda} = 0.15 \) was significant. The mean of the jump component, estimated at \( \hat{\beta} = 1.16 \), was positive and consistent with positive skewness observed in the data.

By using of Cumulants one in J-D model, intensity of the jump process, estimated at \( \hat{\lambda} = 0.14 \), was high and significant, indicating that the coal prices processes were characterized by frequent jumps. Drift of the diffusion component, estimated at \( \hat{\mu} = 0.20 \), was high, implying that coal prices were constantly under pressure to move upward. The variances of the diffusion and jump components were respectively estimated at \( \hat{\sigma}^2 = 3.04 \) and \( \hat{\sigma}^2 = 7.46 \), indicating that the jump component tended to dominate the dynamics of the coal prices process. The mean of the jump component, estimated at \( \hat{\beta} = 1.17 \), was positive and consistent with the positive skewness in coal prices returns. Application of Cumulants two, with restriction \( \mu = 0 \), yielded implausible results for the variance of the jump component, namely \( \hat{\beta} = -10.84 \). Such an anomaly was not unexpected, indicating that the restriction \( \mu = 0 \), could not be borne by the data, and was in sharp contrast with the strong upward trend in coal prices.

In contrast, Cumulants three, with restriction \( \beta = 0 \), yielded results which were highly plausible. The drift component of the diffusion, estimated at \( \mu = 0.34 \), was larger than in the Cumulants one case, since \( \beta = 0 \) implied less influence for the drift of the diffusion, compared to the case when \( \beta \) was positive, to maintain an upward trend in coal prices. The variances of the diffusion and jump components were high, \( \hat{\sigma}^2 = 3.08 \) and \( \hat{\beta} = 7.39 \), respectively. The variance of the jump component, however, dominated that of the diffusion component. Noticeably, jump intensity, estimated at \( \hat{\lambda} = 0.18 \), is the frequency of jumps in coal prices exceeding \( \pm 3\% \) computed from the data set.

Assuming a Bernoulli jump-diffusion process, Cumulants four estimates were significant. Drift of the diffusion component, estimated at \( \hat{\mu} = 0.22 \), was high and significant, showing that coal prices were constantly under upward pressure. The variances of the diffusion and jump components were high and significant, \( \hat{\sigma}^2 = 3.07 \) and \( \hat{\beta} = 6.95 \) respectively. The probability of a jump computed at \( \hat{\lambda} = 0.15 \) was significant. The mean of the jump component, estimated at \( \hat{\beta} = 1.16 \), was positive and consistent with positive skewness observed in the data.

\begin{table}[ht]
\centering
\begin{tabular}{|c|cccccc|}
\hline
Method & Drift & Variance & Intensity & Mean & Variance & \hline
Cumulant one & 0.20 & 3.04 & 0.14 & 1.17 & 7.46 \\
Cumulant two & 0.00 & 5.23 & 0.08 & 1.89 & -10.84 \\
Cumulant three & 0.34 & 3.08 & 0.18 & 0.00 & 7.39 \\
Cumulant four & 0.22 & 3.07 & 0.15 & 1.16 & 6.95 \\
\hline
\end{tabular}
\end{table}

Figure 4 Normal Probability Plots for the Jump Diffusion Model

In sum, except for Cumulants one method, parameter estimates of the jump diffusion were fully concordant with the data. They established that the drift component of the diffusion process was very high for weekly data, indicating that coal demand was pushed up by a strong income effect; consequently, coal prices were under a constant pressure to move upward. However, the coal prices process was dominated by a jump process, with large discontinuities occurring at high intensity.

Finally, a useful model diagnostic is provided by the residuals obtained from the discrete model (Eq. (2)) which implies that

\[ \epsilon = \frac{Y_t - \mu - J_t}{\sigma} \sim N(0,1). \]  

where \( Y_t = \ln P_t - \ln P_{t-1} \). The estimated residuals should therefore be approximately \( N(0,1) \). Figure 4 shows the
normal probability plots for the jump diffusion model. It is clear that the residuals for the J-D model show no strong signs of being non-normal. We also tested the normality of the estimated residuals formally using the Jarque-Bera test. The test could not reject the null hypothesis of normality for the jump diffusion at 5% standard confidence level; the $p$-value of the test was 0.092.

**CONCLUSION**

We have analyzed coal prices dynamics as represented by the Datong mix weekly prices during the period 2004-2013. Estimates of parameters and latent variables were obtained using the Cumulants method. Our main findings are that these dynamics are dominated by strong upward drift and frequent jumps, causing coal markets not to settle around a mean. While coal prices attempted to retreat following major upward jumps, there was a strong positive drift which kept pushing these prices upward. Volatility was high, making coal prices very sensitive to small shocks and to news. The findings for the jump diffusion specification were fully consistent with underlying fundamentals for coal markets and domestic economy. More specifically, faster world economic growth during the sample period and highly expansionary monetary policies caused demand for coal to expand at a similar pace. Given prices inelastic coal demand and supply, any small excess demand (supply) would require a large prices increase (decrease) to clear coal markets; hence, the observed high intensity of jumps and the strong stimulus for coal prices to rise.

Our findings are relevant for policymakers and industry analysts. The results establish the nature of the stochastic process underlying coal prices and the importance of the components driving this process. Process parameter estimates could be seen to convey the effect of expansionary macroeconomic policies on coal prices during the sample period. Moreover, our modeling approaches are highly relevant for forecasting, risk management, derivatives pricing, and gauging the market’s sentiment. Our findings should be also helpful for developing and monitoring these policies for stabilizing Chinese coal markets.

**REFERENCES**


