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Relative Performance Contracts and Fund Managers' choice of Investment Strategies

CONTRATS DE PERFORMANCE RELATIVE ET CHOIX DES DIRECTEURS DES FONDS SUR LA STRATEGIE D'INVESTISSEMENT

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Abstract: This paper has constructed a programming model used to solve the optimal relative performance contracts. Under the contract that the constraints of the model can be satisfied, the strategy combination that the investors wish to see must form a Nash equilibrium in the managers' subgame. It is further indicated by the numerical example that more often than not this equilibrium constitutes a dominant strategic equilibrium as long as the managers do not collaborate with each other.
Key words: Relative Performance, Compensation Fee, Fund Manager, Investment Strategies

Résumé: Ce texte a conçu un modèle programmé pour résoudre les contrats de performance relative . Sous le contrat que les contraites de modèle peuvent être satisfaites , la combinaison stratégique que souhaitent les investisseurs doit former un équilibre Nash dans les sub-jeux des directeurs . L'exemple numérique montre davantage que cet équilibre constitue l'équilibre de stratégie dominante , pourvu que les directeurs ne se collaborent pas .
Mots-clés: performance relative, rénumération compensatoire, directeur de fonds, stratégie d'investissement

1. INTRODUCTION

With the development of Chinese securities market, the industry of investment funds has been growing up rapidly. By the end of first quarter of 2004, total investment in funds under management has reached more than ￥214 billion, which account for 11% of the market value of securities tradable volume in Shanghai and Shenzhen Stock Exchange. Undoubtedly, the industry is playing an increasing important role in Chinese securities market.

However, those funds managed by experts failed to achieve satisfying results. For instance, the closed-end fund has suffered loss added up to￥0.98 billion in 2003. Nevertheless, the fund companies still gain fat profits because of monopolization. This fact has not only given rise to much controversy over the reasonableness of fixed fee scheme practiced in investment fund industry, but also urged the necessity for designing a set of effective incentive schemes to rationalize the fee structure by which managers are compensated.

The investor- fund manager relationship can be characterized as a principle-agent relationship in which the investor (principle) who is endowed with capital but no information delegates their investment decision to a manager (agent) who is endowed with information-gathering abilities. Since it is prohibitively costly for the investor to monitor his fund manager, it is quite likely for the manager to ignore his responsibility of collecting information to save effort unless the investor offers a reasonable incentive contract that provides the proper incentives.

Most previous work (Starks, L., 1987; Grinold, R.

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and A.Rudd, 1987; Cohen, S. and Starks, L., 1988) on the incentives of fund managers compared the performance of a fund manager with that of a passive benchmark, few research has focused on the problem of designing a relative performance contract based on performance comparison between different managers. Since it is really a promising research subject, it appeals to many researchers recently, among which are Heinkel and Stoughton(1994), Steven Huddart (1999) and Jürgen Eichberger (1999).

Heinkel and Stoughton (1994) and Steven Huddart (1999) examine the effect of reputation and performance fee on fund manager's portfolio choice, under the condition that one manager is strictly better informed than the other.

On the assumption that each manager's signal is informative and their signals are relative, Jürgen Eichberger et al (1999) design a relative performance contract (abbreviated to EGK model) based on comparison of two managers' performance. They conclude that if two managers of different funds both accept contracts that depend on their relative as well as absolute performances, then there may exist equilibria in the managers' subgame that result in undesirable outcomes for the owners. The fund owner who offers relative performance contracts can exploit the extra benefits only if the other owner offers non-relative performance contracts. But the reason why such conclusion is drawn is that the constraints of their model are not thorough, which cause a flaw in the contract that the managers can exploit. What's more, they didn't show how managers' compensation is related to asset size, it seems that the optimal contract is independent of asset size, the manager will receive the same payment no matter how many money has been under their control. It is obviously unreasonable. First, it is contrary to the business practice where the compensations of fund managers are often stipulated as a percentage of assets under management. Second, managing assets of different size often require different effort exertion. Therefore, the remuneration of fund managers should be relevant to asset size.

In view of above reasons, this paper constructs a programming model used to solve the optimal relative performance contract, under which the strategy combination that the investors wish to see must form a Nash equilibrium in the managers' subgame. Like EGK model, our model characterizes the interaction among two fund owners and their respective managers. It is common knowledge that the two managers are equally informed about the future returns of a risky asset. But our model differs from those of Eichberger et al at least in two ways. First, our model chooses not the absolute payment but the compensation fee as the endogenous variable. As discussed above, this arrangement conforms to business practice. Second, while EGK model assumes investors are risk-neutral and managers are risk- averse described by a special utility function, in our model both investors and managers are considered risk-averse denoted by a general CRRA utility function, which have more extensive implications without loss of tractable.

The paper is organized as follows. Section 2 presents the basic hypothesis. Section 3 develops the model. The information structure and the form of relative compensation contract as well as the investment strategy set have been described in that section. It also demonstrates and discusses the property of the solution. Section 4 is a numerical example. By using of the example, we conduct a sensitive analysis with regard to the main arguments in the model advanced in Section 3 and categorize the equilibria in the managers' subgame into three groups. Section 5 concludes the paper.

2. BASIC HYPOTHESES

The main purpose of this paper is to illustrate how investors can ensure the preferred investment policies must form a Nash equilibrium in the managers' subgame by using of a relative performance contract. To focus on the problem and keep the analysis tractable, several hypotheses has to be made below.

(H1) There are two independent funds, fund 1 and 2, which are delegated to fund manager 1 and 2 respectively. The difference of the two managers in information gathering ability is insignificance.

(H2) Investors who hold shares of a fund are treated as a representative investor. All the investors and managers exhibit the same preferences, described by the constant relative risk-aversion (CRRA) utility function for wealth $U(W)=W^{\alpha}/\alpha$ for some α <1.The investors and managers have no disposable wealth other than the amounts they will receive from their funds at dissolution.

(H3) There are only two assets available as investment opportunities, *Q* and *M*. The riskless asset *M* has a state-independent return rate R_m, while risky asset *Q* has a state-dependent return rate R_q. There exist just two equally likely natural status, which correspond to 'nice' (N) and 'coarse' (C) environments for the risky investment opportunity. Without loss of generality, we normalize return rates so that the return on the riskless asset is zero, whereas the risky asset returns 1 in the event nice environment occurs and −1 in the event coarse environment takes place. Thus the return rate of the risky asset can be described as the following step function.

$$R_q(\omega)=\begin{cases} 1 & \omega\in N \\ -1 & \omega\in C \end{cases} \tag{1}$$

If we denote by $\delta_i\in[0,1]$ the proportion of the funds invested in the risky asset *Q*, then the return rate from a portfolio *i* in state ω may be expressed as

$$R_i(\omega,\delta_i)=\delta_i R_q(\omega)+(1-\delta_i)R_m=\delta_i R_q(\omega) = \begin{cases}\delta & \omega\in N\\ -\delta & \omega\in C\end{cases} \quad (2)$$

(H4) Fund managers carry on investment activities independently, they do not pool their information and there is no collusion.

(H5) The sequence of actions of the investors and the managers may be outlined as follows:

The two groups of investors simultaneously offer contracts to their respective managers.

The managers simultaneously decide whether to accept or reject their respective offers. If a manager rejects the contract offered to him, then he can only earn his reservation utility $\underline{U}$ from her outside opportunities.

If a manager accepts the contract offered to him, he chooses first whether to gather information and then how to act upon his private information. Undertake information gathering activities requires a manager to expend a fixed effort ***e*** in terms of the manager's disutility.

Each manager makes a portfolio decision according to his information.

Uncertainty is revealed, returns are realized, and the contracted payments to the managers are made.

3. DEVELOPMENT OF THE MODEL

3.1 Information Structure

Definition 1 Information reflects the nature status including social economic environment and fluctuation of security market that has an impact on return of risky assets is called a **signal**.

We will denote τ_i as the status of the signal that manager *i* received. Throughout this paper, we will assume that there are just two kinds of signals a manager can obtain from his information-gathering activity, which correspond to a 'good' (G) signal and a 'bad' (B) signal. A 'good' signal indicates that the natural status will be nice, so the risky assets will have a high return, vice versa, a 'bad' signal indicates that the natural status will be coarse, so the risky assets will have a low return. Let G_i and B_i express the case that the signal manager *i* obtained is a 'good' signal and a 'bad' signal respectively.

Definition 2 If given a signal 'good', the posterior probability of the event that status *N* realizes is greater than the prior probability that status *N* occurs, or given a signal 'bad', the posterior probability of the event that status *C* realizes is greater than the prior probability that status *C* occurs, then we say the signal possesses **information value**.

If the signal manager *i* obtained have information value, then

$$P(N|G_i) > \mathrm{P(N)} = 1/2 = \mathrm{P(C)} > P(C|G_i)$$
$$P(C|B_i) > \mathrm{P(C)} = 1/2 = \mathrm{P(N)} > P(N|B_i).$$

Definition 3 If the likelihoods of receiving a good signal and receiving a bad signal is equal, and $P(N|G_i)=P(C|B_i)$, then the signal is called a **uniform signal**.

If both managers receive uniform signals, then $P(N|G_1G_2)=P(C|B_1B_2)$.

Definition 4 If both managers receive uniform signals, the probability that the signals reflect the true status is called **precision of a signal**, which is denoted by $\theta=P(N|G_1G_2)=P(C|B_1B_2)$.

Definition 5 If signals the managers receive satisfy $P(G_2|G_1)=P(B_2|B_1)$, then the signals are called **balanced signals**.

Definition 6 If two manager receive balanced signals, given a manager receives a signal of some kind, the conditional probability that the other manager receive the same kind of signal is called relevance of signals, denotes by $\rho = P(G_2|G_1) = P(B_2|B_1)$. If ρ greater than 1/2, we say the signal are correlated。

Definition 7 If the joint probability distribution over each manager's signal and the natural status satisfies

$P(N\cap G_1)=P(N\cap G_2)$, $P(N\cap G_1\cap B_2)=P(N\cap B_1\cap G_2)$

$P(C\cap B_1)=P(C\cap B_2)$, $P(C\cap B_1\cap G_2)=P(C\cap G_1\cap B_2)$

the signals are called **symmetrical signals**.

Definition 8 If the signals each manager receives possesses information value, and the signals are uniform, symmetrical and correlated with each other, the signals are called **effective signal**.

Theorem 1 If both managers receive effective signals, then the joint probability over the signals and natural status can be showed in table 1.

[Insert table 1 about here]

Proof From definition 2 and definition 3, we have $P(N|G_i)=P(C|B_i)>1/2$

It follows that $\theta = P(N|G_1G_2) = P(C|B_1B_2) >$ 1/2

From definition 6, $\rho = P(G_2|G_1) = P(B_2|B_1) >$ 1/2

Then according to definition 3, we derive

$P(N \cap G_1 \cap G_2) = P(G_1) \times P(G_2|G_1) \times P(N|G_1G_2) = \rho\theta/2 = P(C \cap B_1 \cap B_2)$

$P(N \cap B_1 \cap B_2) = P(B_1) \times P(B_2|B_1) \times P(N|B_1B_2) = \rho(1-\theta)/2 = P(C \cap G_1 \cap G_2)$

$P(N \cap G_1 \cap B_2) + P(N \cap B_1 \cap G_2) + P(C \cap B_1 \cap G_2) + P(C \cap G_1 \cap B) = 1-\rho$

Then it is easily proved from definition 7 that

$P(N \cap G_1 \cap B_2) = P(N \cap B_1 \cap G_2) = P(C \cap B_1 \cap G_2) = P(C \cap G_1 \cap B) = (1-\rho)/4$

3.2 Relative performance contract and investment strategy of managers

Investors of both funds wish their managers to choose a δ that will maximize investors' expected utility. unfortunately, the investor can neither observe the information gathering activity of his own manager, nor the δ the manager choose, let alone the other manager's choice, the only thing he can observe is the return of fund managed by each manager, therefore the payment schedule can be only based on the returns of both funds. If the signals managers receive are correlated, it is worth to use relative performance contract.

Theorem 2 Denote $f_i(R_i,\ R_i - R_j)$ as the fee contract offered by investors of fund *i*, *W* is his initial investment in the fund, the remuneration the investor paid to manager *i* can be described as

$$\Gamma_i(R_i, R_i - R_j) = \begin{cases} f_i(\delta_i, \delta_i - \delta_j)\cdot W & \omega \in N \\ f_i(-\delta_i, -\delta_i + \delta_j)\cdot W & \omega \in C \end{cases} \tag{3}$$

Prove From equation (2)

$$R_i(\omega, \sigma_i) = \begin{cases} \delta_i & \omega \in N \\ -\delta_i & \omega \in C \end{cases},\quad R_i(\omega, \delta_i) - R_j(\omega, \delta_j)$$

$$= \begin{cases} \delta_i - \delta_j & \omega \in N \\ -\delta_i + \delta_j & \omega \in C \end{cases}$$

Hence, $f_i(R_i,\ R_i - R_j)$

$$= \begin{cases} f_i(\delta_i, \delta_i - \delta_j) & \omega \in N \\ f_i(-\delta_i, -\delta_i + \delta_j) & \omega \in C \end{cases} \tag{4}$$

It is obviously that $\Gamma_i(R_i, R_i - R_j) = f_i(R_i, R_i - R_j)W$ (5)

Subsititute (5) into (4), it is immediately equation (3) holds.

Denote $(\gamma_i, \eta_i) \in [0,1] \times [0,1]$ as the investment policy that manager *i* adopts, which means investing a portion γ_i (respectively, η_i) of the funds into the risky asset if the status of the signal received by manager *i* is good (respectively, bad).

Given a relative performance contract $f_i(R_i,\ R_i - R_j)$, and given that the manager *j* is following the investment policy (γ_j, η_j), according to theorem 1 and 2, the expected utility of the manager *i* who adopts the investment policy (γ_i, η_i) will be denoted by

$$\begin{aligned}
& U_{Ai}((\gamma_i, \eta_i), \Gamma_i | (\gamma_j, \eta_j)) \\
&= P(G_i) \times U_{Ai}(\gamma_i, \Gamma_i | (\gamma_j, \eta_j)) \\
&+ P(B_i) \times U_{Ai}(\eta_i, \Gamma_i | (\gamma_j, \eta_j)) \\
&= P(G_i) \times \\
&\{P(CG_j|G_i) \cdot U_{Ai}(\Gamma_i(-\gamma_i, -\gamma_i + \gamma_j)) \\
&+ P(CB_j|G_i) \cdot U_{Ai}(\Gamma_i(-\gamma_i, -\gamma_i + \eta_j)) \\
&+ P(NG_j|G_i) \cdot U_{Ai}(\Gamma_i(\gamma_i, \gamma_i - \gamma_j)) \\
&+ P(NB_j|G_i) \cdot U_{Ai} \cdot (\Gamma_i(\gamma_i, \gamma_i - \eta_j))\} \\
&+ P(B_i) \times \\
&\{P(CG_j|B_i) \cdot U_{Ai}(\Gamma_i(-\eta_i, -\eta_i + \gamma_j)) \\
&+ P(CB_j|B_i) \cdot U_{Ai}(\Gamma_i(-\eta_i, -\eta_i + \eta_j)) \\
&+ P(NG_j|B_i) \cdot U_{Ai}(\Gamma_i(\eta_i, \eta_i - \gamma_j)) \\
&+ P(NB_j|B_i) \cdot U_{Ai}(\Gamma_i(\eta_i, \eta_i - \eta_j))\}
\end{aligned}$$

If the information structure described in section 3.1

present, the expected utility of the manager i can be transformed into the following expression,

$$\rho(1-\theta)/2\cdot U_{Ai}\left(f_i\left(-\gamma_i,-\gamma_i+\gamma_j\right)W\right)$$
$$+(1-\rho)/4\cdot U_{Ai}\left(f_i\left(-\gamma_i,-\gamma_i+\eta_j\right)W\right)$$
$$+\rho\theta/2\cdot U_{Ai}\left(f_i\left(\gamma_i,\gamma_i-\gamma_j\right)W\right)$$
$$+(1-\rho)/4\cdot U_{Ai}\left(f_i\left(\gamma_i,\gamma_i-\eta_j\right)W\right)$$
$$+(1-\rho)/4\cdot U_{Ai}\left(f_i\left(-\eta_i,-\eta_i+\gamma_j\right)W\right)$$
$$+\rho\theta/2\cdot U_{Ai}\left(f_i\left(-\eta_i,-\eta_i+\eta_j\right)W\right)$$
$$+(1-\rho)/4\cdot U_{Ai}\left(f_i\left(\eta_i,\eta_i-\gamma_j\right)W\right)$$
$$+\rho(1-\theta)/2\cdot U_{Ai}\left(f_i\left(\eta_i,\eta_i-\eta_j\right)W\right) \quad (6)$$

Under the information structure described in section 3.1, it is obvious that both investors wish their respective managers to gather information, to invest all funds into the risky asset if the signal status is good, and to invest exclusively in the riskless asset if the signal 'bad' realizes. In a word, the investment policy the investors prefer is (1, 0).

When both manager can only choose $\delta\in\{0,1\}$, it can be seen from equation (2) that only 3 different return rate can be achieved, which are 1, 0 and -1. Thus there are only 7 possible different groups of return combination of two funds, which are (-1,0), (-1,-1), (1,0), (1,1), (0,1), (0,0), (0,-1). Therefore if investors of either fund include in the contract a provision that the manager will get a reward worse than the worst reward for the 7 return combinations mentioned above if other return combinations are observed, neither manager will choose a portfolio weight other than $\delta\in\{0,1\}$.

However, since the natural status is unobservable, managers still have three kinds of strategies that deviate from the desired strategy to choose. Denote S_{ik} (i=1,2; k=1,2,3,4) as the investment strategies the fund manager i adopts, the strategies are shown table 2. According to the table, S_{i1} is the desired strategy, S_{i2} is the reverse strategy of S_{i1}. Both the strategies require managers to undertake information-gathering activities, and so to expand an effort e. In contrast, the last two strategies do not require manager to collect information, the managers actually invest exclusively in risky or riskless asset according to their prior belief, hence no effort is expend.

[Insert table 2 about here]

3.3 The model

If investors of both funds offer the relative performance contracts $f_i\left(R_i,\ R_i-R_j\right)$ described in 3.2, and both signals the managers receive are effective signals, the problem facing the investor i can be described as

$$(\mathrm{M})\begin{cases}\max\limits_{f_i\left(R_i,R_i-R_j\right)} EU_{oi}\\
=\max\limits_{f_i\left(R_i,R_i-R_j\right)} P\left(G_i\right)\times\left[P\left(CG_j\middle|G_i\right)\cdot U_{oi}\left(\left(R_i\left(C,1\right)-f_i\left(-1,0\right)\right)W\right)\right.\\
+P\left(CB_j\middle|G_i\right)\cdot U_{oi}\left(\left(R_i\left(C,1\right)-f_i\left(-1,-1\right)\right)W\right)+P\left(NG_j\middle|G_i\right)\cdot U_{oi}\left(\left(R_i\left(N,1\right)-f_i\left(1,0\right)\right)W\right)\\
\left.+P\left(NB_j\middle|G_i\right)\cdot U_{oi}\left(\left(R_i\left(N,1\right)-f_i\left(1,1\right)\right)W\right)\right]+P\left(B_i\right)\times\left[P\left(CG_j\middle|B_i\right)\cdot U_{oi}\left(\left(R_i\left(C,0\right)-f_i\left(0,1\right)\right)W\right)\right.\\
\left.+P\left(B_j\middle|B_i\right)\cdot U_{oi}\left(\left(R_i\left(\omega,0\right)-f_i\left(0,0\right)\right)W\right)+P\left(NG_j\middle|B_i\right)\cdot U_{oi}\left(\left(R_i\left(N,0\right)-f_i\left(0,-1\right)\right)W\right)\right]\\
\text{s.t.}\\
U_{Ai}\left(S_{i1},X_i\middle|S_{j1}\right)\geq\underline{U}+e \qquad (\text{IR})\\
U_{Ai}\left(S_{i1},X_i\middle|S_{jl}\right)-e\geq U_{Ai}\left(S_{i2},X_i\middle|S_{jl}\right)-e \quad (l=1,2,3,4) \qquad (\text{IC1})\\
U_{Ai}\left(S_{i1},X_i\middle|S_{jl}\right)-e\geq U_{Ai}\left(S_{ik},X_i\middle|S_{jl}\right) \quad (k=3,4;\ l=1,2,3,4) \qquad (\text{IC2})
\end{cases}$$

Where EU_{oi} denotes the expected utility of investor i and under the information structure described in section 3.1, EU_{oi} can be reduced to the following expression,

$$\rho(1-\theta)/2\cdot U_{oi}\left(\left(-1-f_i(-1,0)\right)W\right)$$
$$+(1-\rho)/4\cdot U_{oi}\left(\left(-1-f_i(-1,-1)\right)W\right)$$
$$+\rho\theta/2\cdot U_{oi}\left(\left(1-f_i(1,0)\right)W\right)$$
$$+(1-\rho)/4\cdot U_{oi}\left(\left(1-f_i(1,1)\right)W\right)$$
$$+(1-\rho)/4\cdot U_{oi}\left(\left(1-f_i(0,1)\right)W\right)$$

$$+\rho/2\cdot U_{oi}\left(-f_i(1,0)W\right)$$

$$+(1-\rho)/4\cdot U_{oi}\left(-f_i(0,-1)W\right) \quad (7)$$

The objective of the problem is to maximize the expected utility that investor *i* gain when both managers choose the desired strategies S_{i1}. Individual rationality constraint (IR) ensure that manager *i* is willing to accept the contract offered to him. Incentive compatibility constraints (IC1) and (IC2) guarantee that manager *i* will have no incentive to deviate from the desired investment strategy S_{i1} whatever strategy the manager *j* implement, because he will gain the maximum expected utility with the adoption of S_{i1}. As the structures of both contracts are identical and the preferences of investors and managers are all alike, the optimal contracts of both funds are the same.

General properties of the solution are hard to demonstrate in theory, in this case we will discuss those properties by using of a numerical example in the next section, but there is one certain conclusion related to the optimal contract we can safely make below.

Theorem 3 Strategy combination（S_{11}, S_{21}）must form a Nash equilibrium in the managers' subgame that follows their simultaneous acceptance of their respective relative performance contracts.

Prove From (IC1) and (IC2), S_{11} will always be the (weak) dominant strategy over other strategies S_{1k}(k=2,3,4) for manager 1 whatever strategy manager 2 bring to effect, similarly, S_{21} must be the (weak) dominant strategy over other strategies S_{2l} (l=2,3,4) for manager 2 whatever strategy manager 1 choose *to* put into execution. Therefore Strategy combination（S_{11}, S_{21}）must form a Nash equilibrium in the managers' subgame that follows their simultaneous acceptance of their respective relative performance contracts.

However, there is still another issue needs to be addressed that whether there are other strategy combinations for the two managers that form a Nash equilibrium in this subgame. We will discuss the problem in the next section.

4. NUMERICAL EXAMPLES

We have considered a wide range of parameter values to investigate some properties of the solution to model M. We choose a representative set of parameters to illustrate the relevant problems. The second column of table 3 presents numerically-determined values of the fee contracts $f_i(R_i, R_i\text{-}R_j)$, the respective payoff that the investors and the managers will gain when the desired strategies were chosen, which are implied by formula (6) and (7) for α =3/5, $\underline{U}$ =10, e=1, ρ =0.6, θ =0.7 and W=5000.

4.1 Sensitivity analysis

We have performed sensitivity analysis with respect to parameters ρ , θ and W. The main results are shown in table 3.

[Insert table 3 about here]

We summarize the results as follows,

From the last line of table 3, it can be seen that the payoff that a investor gain when he chooses the desired strategy is slightly greater than or equal to reserved utility, which means the investor wish to pay as little as possible to his manager after the IR constraint is satisfied.

It is generally regarded fair that the better performance result in higher compensation. From the point of view of fair, there should exist the following relationship among the fees,

$f_i(1,1) > f_i(1,0) > f_i(-1,0) > f_i(-1,-1)$;

$f_i(1,1) > f_i(0,1)\ f_i(0,0) > f_i(0,-1) > f_i(-1,-1)$;

$f_i(1,0) > f_i(0,0) > f_i(-1,0)$; $f_i(1,0) > f_i(0,0) > f_i(0,-1)$;

$f_i(0,1) > f_i(0,0) > f_i(-1,0)$;

In our examples, the fees satisfy all above relationships except that $f_i(0,0) < f_i(-1,0)$. As a matter of fact, too large the fee $f_i(0,0)$ will encourage managers to evade responsibility and avoid the difficulty to collect information since by choosing strategy S_{i3}, he will also gain a lot without any risk. It is therefore sensible to set $f_i(0,0)$ small.

Compare column 2, 3 and 4, we find out that keep the relevance of signals ρ and investment scale W fixed on the initial value, the higher the precision of a signal θ is, the more expected utility and a investor will gain, the same is true of the utility gained per unit of asset However, change the value of θ have few influence on the utility a manager will gain. It shows that the benefits from enhancement of precision of a signal are almost entirely occupied by the investor.

Compare column 2, 5 and 6, we discover that hold θ and W constant, the higher the relevance of signals ρ is, the more expected utility a investor will gain, and so does the utility gained from per unit of asset. Nevertheless, the influence of relevance of signals is less than that of precision of a signal. It is obvious from table 3 that one percent gain in precision of a signal will 60 percent increase in expected utility, whereas one percent gain in relevance of signals will result in 25% increase in expected utility.

Compare column 2, 7and 8, we learn that hold ρ and θ constant, the larger the investment scale is, the more expected utility a investor will gain, but unlike the above two cases, the utility gained from per unit of asset is decreasing in W.

4.2 The choice of investment strategies made by managers

According to theorem 3, Strategy combination （S_{11}, S_{21}） must form a Nash equilibrium in the managers' subgame, but whether this strategy combination will come up depends on the payoffs the managers obtain from all possible strategy combinations. In this section we will analysis the managers' subgame that follows their simultaneous acceptance of their respective relative performance contracts.

The payoffs the managers obtain from any strategy combination can be computed by using of formula (6). In our example, the payoffs the managers obtain can be classified into three categories, which are category 1 represented by scenario Ⅲ (including scenario Ⅲ and Ⅵ), category 2 represented by scenario Ⅰ (including scenario Ⅰ, Ⅱ, Ⅳ, Ⅴ) and category 3 represented by scenario Ⅶ. Table 4 to 6 in turn list the Payoff matrixes of managers under scenario Ⅲ, Ⅰ and Ⅶ separately.

[Insert table 4 about here]

As can be seen from table 4, S_{11} and S_{21} are the dominant strategies of manager 1 and 2 under scenario Ⅲ respectively, therefore （S_{11}, S_{21}） forms the unique dominant strategy equilibrium, which is also strong Nash equilibrium. On condition that the two managers are rational and do not collaborate, none of them would have the incentive to deviate from the equilibrium. But take a further look at table 4, we will find that both managers will obtain more payoff from strategy combination (S_{14}, S_{24}) than the equilibria payoff. It seems to us that if both managers agree on implementing the investment policy (1, 1), the situation is certain adverse to investors. The results of games under scenarios in category 1 are analogous to scenario Ⅲ.

[Insert table 5 about here]

It is can be seen from table 5 that the results of games under scenarios in category 2 are similar to that of games under scenarios in category 1. Take the scenario Ⅰ as an example. Strictly speaking, (S_{11}, S_{21}) still constitute dominant strategy equilibrium, but unlike scenario Ⅲ, manager 1 is indifferent to payoffs gained from (S_{14}, S_{24}) and (S_{11}, S_{24}), at the same time, manager 2 is indifferent to payoffs gained from (S_{14}, S_{24}) and (S_{14}, S_{21}). Thus if we ignore the remote difference, (S_{14}, S_{24}) can also form a Nash equilibrium. Since neither (S_{11}, S_{21}) nor (S_{14}, S_{24}) constitutes a strong Nash equilibrium, it is likely that a mixed strategy equilibrium appears, as is the case with category 3.

[Insert table 6 about here]

As illustrated in table 6, there will be four Nash equilibria in the subgame of fund managers under scenarios in category 3, which are (S_{11}, S_{21}), (S_{11}, S_{24}), (S_{14}, S_{21}) and (S_{14}, S_{24}). Since there is no difference between S_{14} and S_{11} to manager 1, and there is no difference between S_{24} and S_{21} to manager 2, no dominant strategy equilibrium exists. It is possible that a mixed strategy constituted by combination of S_{14} and S_{11} and that of S_{24}与S_{21}.

5. CONCLUSIONS

With the development of Chinese mutual fund industry, trades made by funds have an increasing prominent influence on securities markets. It is natural for investors and regulatory institutions to focus on the problem of monitoring the behavior of managers who manipulate those trades. Fair and reasonable compensation contract is an effective means to control managers' behavior and thus alleviate conflicts of interests between investors and fund managers.

This paper has constructed a programming model used to solve the optimal relative performance contract. It can be proved that when the constraints of the model can be satisfied, the strategy combination that the investors wish to see must form a Nash equilibrium in the managers' subgame. As can be seen from the numerical example, the intended equilibrium will always constitute a dominant strategy equilibrium unless the two managers simultaneously deviate from the contract-consistent behavior to increase their payoffs. The study provides a feasible scheme for rationalization of compensation for asset management.

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Table 1 The joint probability over the signals and natural status

ω / τ_1 τ_2	(1) *N*		(2) C	
	G_2	B_2	G_2	B_2
G_1	$\rho\theta/2$	$(1-\rho)/4$	$\rho(1-\theta)/2$	$(1-\rho)/4$
B_1	$(1-\rho)/4$	$\rho(1-\theta)/2$	$(1-\rho)/4$	$\rho\theta/2$

$$\rho,\theta \in (1/2,1]$$

Table 2 Investment strategies of fund managers

	S_{i1}	S_{i2}	S_{i3}	S_{i4}
γ_i	1	0	0	1
η_i	0	1	0	1

Table 3 The main results of the numerical example

Scenario / Parameter	I	II	III	IV	V	VI	VII
ρ	0.6	0.6	0.6	0.7	0.8	0.6	0.6
θ	0.7	0.8	0.9	0.7	0.7	0.7	0.7
W	5000	5000	5000	5000	5000	10000	15000
$f_i(-1,0)$	0.0017	0.0012	0.0021	0.0021	0.0023	9.11E-05	-6.05E-09
$f_i(-1,-1)$	-0.0102	-0.0051	-0.0062	-0.0082	-0.0081	-0.0153	-0.0206
$f_i(1,0)$	0.0216	0.0188	0.0202	0.0202	0.0203	0.0168	0.0201
$f_i(1,1)$	0.025	0.0199	0.0239	0.0236	0.0222	0.0212	0.0219
$f_i(0,1)$	0.0198	0.0125	0.0213	0.0180	0.0170	0.0152	0.0146
$f_i(0,-1)$	3.02E-08	3.49E-08	-2.51E-08	-4.98E-11	-9.05E-12	-0.0016	-0.0019
$f_i(0,0)$	0.0009	0.0008	2.56 E-08	5.09 E-04	2.94 E-04	2.33E-08	-1.26E-03
max EU_{oi}	28.2851	45.5885	63.0916	34.7337	41.0471	46.6588	62.621
max EU_{oi}/W	0.00566	0.00912	0.01262	0.00695	0.00821	0.00467	0.00418
$U_{Ai}-e$	10.4453	10.0009	10.0006	10.1258	10.0001	10.0009	10.00

Table 4 Payoff matrix of managers under scenario III

		Fund manager 2			
		S_{21}	S_{22}	S_{23}	S_{24}
Fund Manager 1	S_{11}	10.001, 10.001	17.054, -1.531	8.190, 0.754	18.865, 9.715
	S_{12}	-1.531, 17.054	5.238, 5.238	-1.993, 9.157	5.701, 15.136
	S_{13}	0.754, 8.190	9.157, -1.993	0.008, 0.008	9.904, 8.189
	S_{14}	9.715, 18.865	15.136,5.701	8.189, 9.904	16.662, 16.662

Table 5 Payoff matrix of managers under scenario Ⅰ

		Fund manager 2			
		S_{21}	S_{22}	S_{23}	S_{24}
Fund Manager 1	S_{11}	10.445, 10.445	13.452, 4.567	7.028, 7.001	16.870, 10.011
	S_{12}	4.567, 13.452	7.862, 7.862	1.293, 10.152	11.136, 13.163
	S_{13}	7.001, 7.028	10.151, 1.293	4.016, 4.016	13.137, 6.305
	S_{14}	10.011, 16.870	13.163, 11.136	6.305, 13.137	16.869, 16.869

Table 6 Payoff matrix of managers under scenario Ⅶ

		Fund manager 2			
		S_{21}	S_{22}	S_{23}	S_{24}
Fund Manager 1	S_{11}	10, 10	16.566, -2.711	0.968, -0.711	25.598, 10
	S_{12}	-2.711, 16.566	3.855, 3.855	-11.743, 5.855	12.887, 16.566
	S_{13}	-0.711, 0.968	5.855, -11.743	-9.743, -9.743	14.887, 0.968
	S_{14}	10, 25.6	16.566, 12.89	0.968, 14.89	25.598, 25.598

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