CSCanada
Canadian Social Science
Vol. 7, No. 5, 2011, pp. 176-181
DOI:10.3968/J.css.1923669720110705.502

ISSN 1712-8056[Print]
ISSN 1923-6697[Online]
www.cscanada.net
www.cscanada.org

Mathematical Thinking: Teachers Perceptions and Students Performance

UNE PENSEE MATHEMATIQUE: LES PERCEPTIONS DES ENSEIGNANTS ET LE RENDEMENT DES ELEVES

Mamoon. M. Mubark[1,*]

[1]Ph.D, Department of Curriculum and Instruction, Faculty of Educational Sciences, Al- Hussein Bin Talal University, Jordan, P. O. Box (20)
[*]Corresponding author.
Email: Mamoon_shannaq@hotmail.com

Received 25 June 2011; accepted 29 July 2011

Abstract
This paper was investigated the teachers rating of the six different aspects of mathematical thinking developed by the researcher: Searching for patterns , Induction, Deduction, symbolism, Logical thinking and Mathematical proof in relation to level of importance, level of difficulty, and time spent in teaching each aspect. This paper was also aimed to examine any possible consistencies and inconsistencies between teacher opinions about the level of importance of mathematical thinking aspects to mathematics achievement, level of difficulty and test data collected. Also, it was examined if the students were familiar with solving specific problems (such as rice problem) logical ways like searching for patterns rather than more traditional approaches and if they also applying the fourth step in problem solving according to Polya, (1990) (i.e., looking back (a checking the answer)).

Key words: Mathematical thinking; Teacher perceptions; Students performance

Résumé
Ce document a étudié la notation des six aspects différents de la pensée mathématique des enseignants développé par le chercheur: la recherche de modèles, à induction, déduction, le symbolisme, la pensée logique et mathématique la preuve par rapport au niveau d'importance, le niveau de difficulté et le temps passé dans l'enseignement de chaque aspect. Ce document visait également à examiner toute consistances et des incohérences éventuelles entre les opinions des enseignants sur le niveau d'importance des aspects la pensée mathématique à la réussite en mathématiques, niveau de difficulté et les données recueillies lors des essais. En outre, il a été examiné si les élèves ont été familiarisés avec la résolution de problèmes spécifiques (tels que les problèmes du riz) façons logiques, tels que la recherche de modèles plutôt que des approches plus traditionnelles, et si ils ont également l'application de la quatrième étape dans la résolution de problèmes en fonction de Polya, (1990) (à savoir, en regardant en arrière (une vérification de la réponse)).

Mots clés: Pensée mathématique; Les perceptions des enseignants et le rendement des étudiants

Mamoon. M. Mubark (2011). Mathematical Thinking: Teachers Perceptions and Students Performance. *Canadian Social Science*, *7*(5), 176-181. Available from: URL: http://www.cscanada.net/index.php/css/article/view/j.css.1923669720110705.502
DOI: http://dx.doi.org/10.3968/j.css.1923669720110705.502.

INTRODUCTION
Mathematics is considered an important branch of cognition and the development of mathematical thinking is a basic pillar in the orientation of educational development particularly within a new, advanced educational system. There are many possible aspects of mathematical thinking that have been identified: Symbolism, Logical analysis, Inference, Optimizations, and Abstraction (Schielack, Chancellor, and Childs, 2000), Specialization, Searching for patterns and Reasoning. However, mathematical thinking is fundamental rule to mathematics.

In Jordan mathematical thinking has tended to be expressed in terms of six fundamental aspects based on the views of a group of mathematics education specialists in the Jordanian Universities, and the Ministry of Education, because of their appropriateness to high level students and their possibility of measurement (Mubark, 2005). According to this scholar the six aspects of

Copyright © Canadian Research & Development Center of Sciences and Cultures

mathematical thinking include: Searching for a patterns, Induction, Deduction, symbolism, Logical thinking, and Mathematical proof.

This paper accepts these six aspects of mathematical thinking as appropriate, and is based on teacher views of the importance of each aspects for mathematics achievement, the difficulty and the time spent teaching each aspect, and on student performance on each aspect. The researcher questions addressed are:

1. What is the level of consistency between individual teachers for importance, difficulty and time spent teaching each of the aspects of mathematical thinking?

2. What are the relative levels of teachers' perceptions of impotence, difficulty and time spent teaching for each aspect of mathematical thinking?

3. What are the relationships between teacher perceptions of importance, difficulty and time spent for each aspect?

4. What are the relationships between teacher perceptions of difficulty and student performance on each aspect?

5. Are the students familiar with solving problems such as rice problem in logical ways? If they also applying the fourth step in problem solving according to Polya, (1990) (a checking the answer).

THE SAMPLE AND THE DATA

The level of education chosen for this study was Year 10 (almost 16 years- old). Mathematics is considered an important subject for all students, particularly in this Year because these students in the end of the compulsory stage. This paper was extracted from large study which includes quantitative and qualitative data that represented more than 400 students in 15 different schools. One class of Year 10 students at each of 15 schools in Ma'an[2] governorate was selected in the academic Year 2009/ 2010 to represent urban, rural, and badia schools. As schools in Jordan are normally single-sex, the schools were also selected to represent both genders. Among other things included in the study, these students did a 24-item test, with four items covering each of six aspects of mathematical thinking.

The 15 teachers of these students were invited to be interviewed about mathematical thinking, and 12 of them consented, six male and six female teachers participating in this part of the study. Information was obtained from each of these teachers individually, which included completing questionnaires. The researcher told the teachers to complete the questionnaire at school or at home, the male teachers provided the information at school. In contrast, the female teachers favourite to complete the questionnaire at home and returned it to the researcher.

For each of the six aspects of mathematical thinking, the teachers were asked to rank separately the aspects of importance and difficulty, and to indicate the proportion of time spent teaching it. A six-point Likert scale was employed for the Importance and difficulty dimensions, with responses ranging from the most important or difficult (6) to least important or difficult (1). These data are displayed for each teacher initially to indicate the range of responses, before being compared in order to respond to the research questions.

RESULTS

Relative Importance of the Six Aspects

As can be seen in Table 1, there was a wide of views with each of the six aspects being rated as most important and least important by at least one teacher in each case. On average Mathematical proof (MP), Looking for a pattern, and symbolism were seen as the most important aspects of mathematical thinking, while Deduction was seen as the least important, followed by Induction. Logical thinking (LT), with a mean of 3.42, was approximately on the mid-point of the importance scale.

Table 1-a
Level of Importance According to Teachers' Opinions

Teacher No	Looking for a pattern (1)	Induction (2)	Deduction (3)	Symbolism (4)	LT (5)	MP (6)
1	4	3	1	2	5	6
2	5	2	1	3	6	4
3	5	2	1	3	4	6
4	5	1	2	6	3	4
5	4	5	1	3	2	6
6	1	2	3	6	5	4
7	4	6	3	5	1	2
8	3	2	1	5	4	6
9	4	6	3	5	2	1
10	6	3	2	1	4	5
11	6	3	1	2	4	5
12	6	3	2	4	1	5

[2]Ma'an is the largest governorate in area in Jordan, Moreover, the researcher lives and teaches in this governorate.

 Copyright © Canadian Research & Development Center of Sciences and Cultures

Descriptive Statistics Table 1-b
looking for a Pattern, Induction, Deduction, Symbolism, Logical Thinking, and Mathematical Proof

Variable	Mean	SD	Min	Q1	Median	Q3	Max	Range	Mode	N of mode
1	4.42	1.44	1.00	4.00	4.50	5.75	6.00	5.00	4	4
2	3.17	1.69	1.00	2.00	3.00	4.50	6.00	5.00	2	4
3	1.75	0.87	1.00	1.00	1.50	2.75	3.00	5.00	1	6
4	3.75	1.66	1.00	2.25	3.50	5.00	6.00	5.00	3	3
5	3.42	1.62	1.00	2.00	4.00	4.75	6.00	5.00	4	4
6	4.50	1.62	1.00	4.00	5.00	6.00	6.00	5.00	6	4

Relative Difficulty of the Six Aspects

Perceived difficulty of each of the six aspects of mathematical thinking is shown for each teacher in Table 2. The range for each aspect is not as great as it was for importance, particularly because the teachers uniformly indicated that Mathematical proof was the most difficult aspect, followed by Looking for a pattern. Logical thinking was considered roughly at the mid-point of the difficulty scale, while symbolism and Deduction were perceived as progressively slightly less difficult. Induction, with a mean score of 1.5, was clearly seen as the least difficult aspect overall, although three teachers rated it as moderate level of difficulty.

Table 2-a
Level of Difficulty According to Teachers' Opinions

Teacher No	Looking for a pattern (1)	Induction (2)	Deduction (3)	Symbolism (4)	LT (5)	MP (6)
1	4	3	5	1	2	6
2	4	3	2	1	5	6
3	5	1	3	4	2	6
4	5	1	4	2	3	6
5	5	2	1	4	3	6
6	3	2	1	4	5	6
7	5	1	4	3	2	6
8	2	1	3	5	4	6
9	2	1	5	4	3	6
10	5	1	2	4	3	6
11	2	1	3	5	6	4
12	4	1	2	3	5	6

Descriptive Statistics Table 2-b
looking for a Pattern, Induction, Deduction, Symbolism, Logical Thinking, and Mathematical Proof

Variable	Mean	SD	Min	Q1	Median	Q3	Max	Range	Mode	N of mode
1	3.83	1.27	2.00	2.25	4.00	5.00	5.00	5.00	5	5
2	1.50	0.80	1.00	1.00	1.00	2.00	3.00	5.00	1	8
3	2.92	1.38	1.00	2.00	3.00	4.00	5.00	5.00	2	3
4	3.33	1.37	1.00	2.25	4.00	4.00	5.00	5.00	4	5
5	3.58	1.38	2.00	2.25	3.00	5.00	6.00	5.00	3	4
6	5.83	0.58	4.00	6.00	6.00	6.00	6.00	5.00	6	11

Percentage of Time Spent Teaching each Aspect of Mathematical Thinking

The percentages of time each teacher spent teaching each of the six aspects of mathematical thinking are shown in Table 3. Overall most time was spent on Mathematical proof. This was also seen as the most difficult and the most important aspect. However, the percentage of time spent teaching Mathematical proof ranged from 5 to 30% for different teachers. The teacher who spent least time on this aspect had agreed that it was both the most difficult and the most important aspect. The teacher who spent most time on Mathematical proof had agreed that it was the most difficult, but considered to be moderate importance aspect. Induction received the next highest percentage of time with great ranged from 0 to 40%. Followed by Deduction, Looking for a patterns, and Symbiosis were perceived as moderate percentage ranged from (14.83 to 14.17 respectively). Logical thinking was the aspect that received least time overall, with two teachers indicating they spent no time on it, and one teacher claiming to spend 25 % of their teaching time on this aspect.

Copyright © Canadian Research & Development Center of Sciences and Cultures

Table 3-a
Recentage of Time Spend Teaching the Aspects According to Teachers' Opinions

Teacher No	Looking for a pattern (1)	Induction (2)	Deduction (3)	Symbolism (4)	LT (5)	MP (6)
1	15	7	8	10	0	15
2	10	15	10	15	15	20
3	10	25	15	10	15	20
4	10	25	15	10	15	20
5	10	15	5	20	10	20
6	10	15	10	15	10	30
7	27	10	25	15	0	10
8	15	40	20	10	10	5
9	20	10	15	15	10	20
10	15	15	15	20	20	15
11	10	15	15	15	25	20
12	20	0	15	15	10	20

Descriptive Statistics Table 3-b
looking for a pattern, Induction, Deduction, Symbolism, Logical Thinking, and Mathematical proof

Variable	Mean	SD	Min	Q1	Median	Q3	Max	Range	Mode	N of mode
1	14.33	5.55	10.00	10.0	12.25	18.75	27	17	10	6
2	16.00	10.23	0.00	10.0	15.00	22.50	40	40	15	5
3	14.83	5.33	5.00	10.0	15.00	15.00	25	20	15	5
4	14.17	3.59	10.00	10.0	15.00	15.00	20	10	15	6
5	11.67	7.18	0.00	10.0	10.00	15.00	25	25	10	5
6	17.92	6.20	5.00	15.0	20.00	20.00	30	25	20	7

It will be noted that the total percentages of class mathematics time spent on these six aspects by most individual teachers do not add to 100 %. However, the lowest total was 55 % for teacher 1, and the mean total for these six aspects was approximately 89%. Clearly 9 of the 13 teachers were teaching something else in addition to the six aspects identified.

Relationships between Importance, Difficulty and Time Spent

There is relationship between the relative levels of importance and difficulty of the six aspects of mathematical thinking. In addition, we might expect to find relationships between importance and time spent in teaching each aspect and between difficulty of each and time spent.

Table 4
Level of Importance, Level of Difficulty, and Time Spend for all Teachers

Aspect of mathematical thinking	Level of importance	Level of difficulty	Spend time (%)
Looking for a pattern	4.42	3.83	14.33
Induction	3.17	1.50	16.00
Deduction	1.75	2.92	14.83
Symbolism	3.75	3.33	14.17
Logical thinking	3.42	3.58	11.67
Mathematical proof	4.50	5.83	17.92

The higher level of correlation between level of importance and level of difficulty (0.600), then the correlation between level of difficulty and time spend and correlation between level of importance and time spend (0.311, 0.223 respectively).

In terms of the relationship between importance and time spend, level of importance is sometimes reflected in the time spent in teaching each aspect. For example, mathematical proof was the considered both most important aspect and the highest time spent in teaching. In contrast, deduction was of less importance aspect and was rated the third highest time spent in teaching. In addition, logical thinking was approximately considered to be moderate level of importance and was the least time spent in teaching. However, Looking for pattern was rated the second most important aspect with recording the moderate time spent teaching aspect, along with of symbolism which considered to be the third important aspect.

Level of importance is sometimes reflecting the fundamental role with level of difficulty. For example, Mathematical proof was recording both the most difficult and the most important aspect; also Looking for a pattern was both the second most difficult the most important aspect. Logical thinking was of the same importance of

 Copyright © Canadian Research & Development Center of Sciences and Cultures

Induction with but nearly of mid-level of difficulty and least difficult for Induction. An aspect such as Symbolism was approximately midway in range of importance and difficulty. Finally, Deduction was the least important aspect and nearly was of the middle level of difficulty.

Comparing the six aspects of mathematical thinking in terms of level of importance, level of difficulty, and time spent of teaching each aspect. Mathematical proof was considered the most difficult aspect, the most time spent in teaching, and the most important aspect. Similarly, Looking for a pattern was considered the second important aspect, the second most difficult, and even thought it was considered the third more aspect in relation to time spent in teaching. Also, Deduction was considered the least important aspect, the second easiest aspect and moderate amount of time was spent in teaching it. In contrast, Logical thinking was recorded the least time in teaching and nearly was of moderate level of importance and difficulty. In addition, Induction was considered the easiest aspect and the second aspect in time spend in teaching and moderate in level of importance. Finally, Symbolism was rated to be moderate with regard to level of importance, level of difficulty, and time spent in teaching aspects. Table 4 are shown level of importance, level of difficulty, and time spent by teachers.

Relationships between importance and difficulty for teachers and student performance

It was also of interest to note the relationships between teacher views of importance and difficulty and actual student performance on each of the aspects of mathematical thinking. Teacher opinions of difficulty differed more from the student tests results then was the case for importance. Although mathematical proof was consistently the most difficult aspect, Looking for a pattern was rated the second most difficult according to teachers views and student performance. Further, symbolism was the fourth difficult. Induction was the easiest aspect according to teachers and the second easiest for the students. In contrast, Logical thinking was moderately difficult according to the teachers and the least difficult aspect according to student tests. Deduction was considered of moderate according to the student test results and the second least difficult according to teachers perceptions.

Table 5
Level of Importance, Level of Difficulty According to Teachers Perceptions and Students Performance

Aspects of Mathematical Thinking	Importance/Teachers	Importance/Students*1000	Difficulty/Teachers	Difficulty/ Students(out of 15)
Looking for a pattern	4.17	265	3.83	7.45
Induction	3.17	132	1.50	8.65
Deduction	1.75	158	2.91	7.77
Symbolism	3.75	191	3.33	8.00
Logical Thinking	3.17	175	3.58	9.40
Mathematical Proof	4.50	280	5.83	4.50

There was a high relationship between level of difficulty according to teacher perceptions and student performance (0.793), and the relationship between level of importance according to teacher opinion and student performance (0.793).

The researchers discussed and noted the similarities and differences between teacher interviews and the results that extract from students mathematical thinking test. The teachers' opinions about mathematical thinking aspects, with regard to level of importance, and level of difficulty are now discussed in terms of the students test results. Teachers' opinions of importance were almost the same as importance of the mathematical thinking aspects for mathematics achievement in the student tests. There was some change in the order of the last two aspects. The order for Induction Deduction (the two least important aspects) was reversed in each case.

In relation to the final question, are the students familiar with solving problems such as rice problem in logical approaches? Some of students were unfamiliar with solving problems like rice problem in logical ways. Interviewee students (8 out of 10) solved this problem using traditional ways, without looking for a pattern. They doubled the number in each square to find the number of rice in each square to find the total of rice grains in the last square. The fourth step in problem solving according to Polya, (1990) is looking back. Most of the students agreed that checking the answer is an important step in solving problem. However, the students did not activate this step, due to there is no enough time.

DISCUSSION AND CONCLUSION

This section will describe and discuss the consistencies and inconsistencies between interview data and test results of mathematical thinking, in relation to level of importance, and level of difficulty and the results derived from student responses. This will be followed by a discussion of the time spent in teaching mathematical thinking.

According to importance level, the consistencies and inconsistencies between the teachers' opinions about aspects of mathematical thinking, in terms of level of importance, and the results extracted from student responses were discussed. In respect of the relative levels of importance of the six aspects of mathematical thinking, the results for teachers' opinions and student responses were almost the same with some change in the order for

Copyright © Canadian Research & Development Center of Sciences and Cultures

the last two aspects (Induction and Deduction). There was generally consistency between teacher opinions and test results that Mathematical proof and Looking for a pattern were considered the most significant related to mathematics achievement. These consistencies between teacher opinions of importance and test results indicate that those teachers who participated in this study were relatively accurate about what were the most significant aspects among different aspects of mathematical thinking that lead to high mathematics achievement. These results were expected, because generally teacher opinions reflect student performance across the six aspects of mathematical thinking. This result also was consistent with Savas (2010) who found that Mathematical proof was considered as an important area in mathematics education.

With regard to level of difficulty, all teachers agreed that Mathematical proof was the most difficult aspect among the mathematical thinking aspects which is consistent with the test results. This result was expected, because of the nature of proof which is needed to understand concepts and procedures, and justification of each procedure and which also requires high ability in thinking, this indicates that many students faced a difficulty in constructing mathematical proof and proving (Baker & Campbell, 2004; Savas, 2010). Writing of the proofs is a process and can be gained by practice, and can cumbersome in some cases. This idea is supported, by Senk (1985) who stated that writing proofs is one of the most difficult processes for students to achieve. Also, this results is consist with (Wong, et al, 2011) that reported geometry proof as considered as difficult skill to learn. In contrast, the easiest aspect was Induction in teacher opinions, this result was nearly consistent with the test data collected results indicating that Induction was the second easiest, and logical thinking was the least difficult aspect. Teacher believes that Induction was the easiest aspect, because this aspect is the most fundamental aspect in mathematics and the student develops mathematical Induction skills in particular in mathematics and generally in other subjects. However, the test results found that logical thinking was considered the least difficult aspect, possibly because the nature of items that measure this aspect focused on the meaning of some of the logical relations concepts such as union, intersection, and negation the statements, which are also concepts familiar in other contexts.

In terms of time spent in teaching the different aspects of mathematical thinking, mathematical proof received the greatest time allocation. This result was expected, because the mathematics curriculum for each class includes one chapter of geometry, due to the importance of geometry in understanding the environment and the world and it plays a fundamental role in mathematics. Mathematical proof also was one of NCTM standards (Hynes, 1995, 1996). In addition, not every mathematical proof is geometry proofs. Induction received the second largest time allocation, perhaps because the teachers believed that Induction has a more general application in the curriculum than generalization. In contrast, logical thinking received the least time. This result was expected as well, because the Jordanian Ministry of Education omitted specific reference to this aspect from the curriculum. Other aspects received approximately the same moderate percentage of time class.

LIMITATIONS OF THE STUDY

• The researcher was considered of only six aspects of mathematical thinking (Looking for a pattern, induction, deduction, symbolism, logical thinking and mathematical proof. It is possible that the results would differ if a wider range of mathematical thinking aspects were included. Additional aspects that could also test in the future: logical analysis, abstraction, reasoning.

• The researcher applied this study for the Year 10 in Ma'an governorate (out of 12 governorates in Jordan). It is suggested that these results could be generalized for this population and similar populations.

REFERENCES

Baker, D., & Campbell. C. (2004). Fostering the Development of Mathematical Thinking: Observations from a Proof Course. *Primus: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 14*(4), 345-353.

Hynes, M. (1995). *Ideas: NCTM Standards-based Instruction: Grades K-4*. Reston, Va: National Council of Teachers of Mathematics.

Hynes, M. (1996). *Ideas: NCTM Standards-based Instruction: Grades 5-8*. Reston, Va: National Council of Teachers of Mathematics.

Mubark, M. (2005). *Mathematical Thinking and Mathematics Achievement of Students in the Year 11 Scientific Stream in Jordan*. Unpublished PhD thesis, The University of Newcastle, Australia.

Polya. G. (1990). *How to Solve it: A New Aspects of Mathematical Method.* Prefaced by Ian Stewart. London, Penguin.

Savas. B. (2010). First-year Secondary School Mathematics Students' Conceptions of Mathematical Proofs and Proving. *Educational Studies, 36*(3), 283-298.

Schielack. J., Chancellor. D. & Child, K. (2000). Designing Questions to Encourage Children's Mathematical Thinking. *Teaching Children Mathematics, 6*(6), 398-402.

Senk, S. (1985). How Well do Students Write Geometry Proofs? *Mathematics Teacher, 78*(6), 448-456.

Wong, W., Yin, S., Yang, H., & Chen, Y. (2011). Using Computer-Assisted Multiple Representations in Learning Geometry Proofs *Journal of Educational Technology & Society, 14*(3), 43-54.

 Copyright © Canadian Research & Development Center of Sciences and Cultures