Industrial Cluster, Similarity, and External Economies

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Abstract
This paper develops a simple model in which external economies are caused by interconnectedness among firms in intracluster and intercluster. Two main results are developed. First, the larger the elasticity of substitution between any two products produced within the cluster, the larger the cluster’s output; second, a cluster with larger elasticity of substitution yields larger output among different clusters other things being equal.

Key words: Cluster, Similarity, External economies

INTRODUCTION
It has been widely known that industrial clusters provide an explanation of scale economics at the level of the industries (Porter, 1998). The analysis of external economies goes back to Marshall (1920). Along this line of argument, Fujita et al. (2001) focus that a cluster is a key feature of urbanization. Indeed, the industrialization process in Italy and East Asian countries and regions nearly follow the Marshallian industrial district model (Becattini, 1990). Another pattern of industrialization in the U.S and UK fits into the hub-and-spoke model (Chander, 1977; Landes, 1998). Cheryl et al. (2012) propose that the industrialization process in China is largely cluster-based phenomenon in which numerous highly interconnected firms are located within a well-defined geographic region.

However the mechanisms through which clustering affects firm growth remain to be explored (Cheryl et al., 2012). In particular, this paper is concerned with product similarity which leads to cluster growth.

Rainer et al. (2000) propose that when two countries start from the same initial conditions, the one with the higher elasticity of substitution will experience a higher income per bead, other things being equal. So in the paper product similarity is measured with the elasticity of substitution.

In this paper, there are two important points to be emphasized. Firstly, I shall adopt the assumption proposed by Porter (1998) that clusters are critical to competition. So it is natural to use monopolistic competition model to solve the cluster issue, as Krugman (1991) did. Secondly, the similarity of products is used as a proxy of the interconnectedness of firms.

This paper develops a simple model in which external economies are caused by interconnectedness among firms in a cluster(s) which is defined as geographic concentrations of interconnected companies and institutions in a particular field (Porter, 1998). The approach differs from that of most formal treatments of scale economics, which assume that scale economies are external to firms. Instead, external economies are here assumed to be interconnectedness among firms in a cluster(s), with the market structure that comes forth being one of Chamberlinian monopolistic competition (Negishi et al., 1969). The formal treatment of monopolistic competition is borrowed with slight modifications from the work by Dixit and Stiglitz (1977).

1. CLUSTER, INTERCONNECTEDNESS, AND SIMILARITY

1.1. Concept of Similarity
There has been much discussion of nature of the externalities. Marshall (1920) proposed that there are
three main reasons, which continue to be valid today, why a cluster of firms might be more efficient than an individual firm in isolation: specialized suppliers, labor market pooling, and knowledge spillovers. However, in this paper, instead of asking why a particular cluster is concentrated in a particular area, I shall ask why some clusters are larger than other clusters. The proposed explanation correspondingly concentrates on generalized external economies rather than those specific to a particular cluster. Especially, this paper is concerned with cluster similarity which leads to cluster growth.

This paper uses cluster similarity to measure interconnectedness among firms.

Porter (1998) proposes that other things being equal, the larger cluster in production will be those with relatively large interconnectedness, since the interconnectedness promotes both competition and cooperation. On the issue,

Porter (2000) states that firms in a cluster share the same providers of specialized infrastructure as well as suppliers of specialized inputs such as machinery, components, and services. Moreover, Porter suggests that geographically concentrated clusters cause one main benefit which industries in the same cluster share common inputs, institutions, skills, technologies, and knowledge.

### 1.2 Intra-Cluster Relation

One follows the above train of thought. If firms and industries produce similar commodities, it is more possible for them to rely on the same set of clients and suppliers, and for them to use similar combinations of inputs in their production, and thus they are more likely to be interconnected through technologies, skills, and other common inputs. So according to Porter’s definition, the similarity among products can be used as a measure for interconnectedness among firms.

Especially, in this paper the elasticity of substitution between any two products within a cluster is used as a measure of interconnectedness for cluster similarity. The interconnectedness is concerned with the relation among firms in a cluster (intra-cluster relation). Cheryl et al. (2012) also uses a similar concept to measure the interconnectedness in a cluster. Another kind of interconnectedness is inter-cluster relation.

Consider, a cluster in an economy is assumed to be able to produce any of amounts of commodities, with the commodities indexed by \( i \). This paper orders the commodities so that those actually produced range from 1 to \( n \), where \( n \) is assumed to be amounts number, though small relative to the amounts of potential products. This paper will aggregate the rest of the economy into one commodity labeled 0, chosen as the numeraire and normalized at unity. Writing the amounts of the various commodities as \( x_i \) and \( x=\{x_2, x_3, \ldots \} \), all residents are assumed to share the same utility function with convex indifference surfaces. The utility function in this section is

\[
U = U(x_0, \{\sum_i x_i^\rho \})^{1/\rho}.
\]  

Where \( 0<\rho<1 \).

The budget constraint is

\[
x_0 + \sum_{i=1}^n p_i x_i = I,
\]

where \( I \) is income in terms of the numeraire and \( p_i \) are prices of the commodities being produced.

Define both quantity induces and price indices

\[
y = \{\sum_{i=1}^n x_i^\rho\}^{1/\rho}, \quad q = \{\sum_{i=1}^n p_i^{-1/\beta}\}^{-\beta}.
\]

Where \( \beta=(1-\rho)/\rho \), which is positive because \( 0<\rho<1 \). Then in the first stage of a two-stage budgeting procedure

\[
Y = Is(q)/q, \quad x = ((1-s)(q)).
\]

In the second stage of the problem, for each \( i \)

\[
x_i = y_i q / p_i \]^{(1-\rho)}
\]

where \( y \) is defined by (3). From (3) one has the elasticity

\[
\frac{\partial \log q}{\partial \log p_i} = \left( \frac{q}{p_i} \right)^{1/\beta},
\]

since this paper assumes \( n \) fairly large, the effect of each \( p_i \) on \( q \) is neglected. This leaves the elasticity

\[
\frac{\partial \log x_i}{\partial \log p_j} = \frac{-1}{1-\rho} = \frac{-(1+\beta)}{\beta}.
\]

From Equation (5), one obverses that for \( i \neq j \),

\[
\frac{x_i}{x_j} = \left[\frac{p_j}{p_i}\right]^{(1-\rho)}.
\]

Therefore the elasticity of substitution between any two products within the cluster is

\[
\gamma = \frac{1}{(1-\rho)}>1 \quad (0<\rho<1).
\]

Consider a symmetric situation where \( p_i=p \) and \( x_i=x \) for all \( i \). Then

\[
y = \sum_i x_i^{1/\rho} = x^{1/\rho}, \quad q = \sum_i p_i^{-1/\beta} = p \cdot x^{-\beta}.
\]

Now consider profit-maximizing pricing behavior, since \( n>>1 \), each individual firm can ignore the effects of

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2. Monopsonistic Competition in a Closed Economy

This section develops the basic model of monopolistic competition which will be used in the next section. The model is a simplified version of the model proposed by Dixit and Stiglitz (1977). Instead of developing a general model, this paper will assume particular forms for cost and utility functions.
its decisions on the decisions of other firms. Therefore the $i$th firm will choose its price to maximize its profits,

$$\Pi_i = p_i x_i - (c + \alpha x_i) x_i,$$

where $c$ is the common marginal cost. Profits will be driven to zero by entry of new firms.

3. INDUSTRY PROXIMITY AND SCALE OF CLUSTER

3.1 Intra-Cluster Relation

The profit-maximization condition for each firm to act on its own is the equality of marginal cost and marginal revenue.

Writing $p_e$ for the common equilibrium price for each variety being produced, and from Equation (7)

$$p_e[1-(1-\rho)] = c.$$

From Equation (9)

$$p_e = \frac{c}{\gamma - 1},$$

then

$$\frac{dp_e}{d\gamma} = -\frac{c}{(\gamma - 1)^2} < 0.$$

The theory of external economies (Krugman, 2009) suggests that when external economies are important, a country with a large cluster will be more efficient in that cluster than a country with a small cluster other things being equal. Therefore external economies can be represented by assuming that when a cluster’s costs are lower, the larger the cluster. This means that the cluster will have a forward-falling supply curve: The larger the cluster’s output, the lower the price at which firms are ready to sell their output. Then

$$\frac{dp}{dy} < 0.$$

From Equations (12) and (13)

$$\left.\frac{dy}{dx}\right|_{x^*} = \frac{dy}{dp}\frac{dp}{dx}\left|_{x^*} = \frac{0! \frac{dp}{dy}}{0! \frac{dp}{dy}}\right|_{x^*} > 0.$$

The larger the elasticity of substitution between any two products produced within the cluster, the larger the cluster’s output other things being equal.

3.2 Inter-Cluster Relation

Assume that there are two clusters of commodities beside the numeraire, the two being perfect substitutes for each other. Each cluster consists of a large number of products produced by firms among the clusters-cluster 1 and cluster 2-themselves playing symmetric roles. All individuals will have the convenient utility function

$$u = x_1^{\frac{1}{\gamma_1}} \left( \left[ \sum_{i=1}^n x_i^{\frac{1}{\gamma_i}} \right]^{\gamma_1} + \left[ \sum_{i=1}^n x_i^{\frac{1}{\gamma_i}} \right]^{\gamma_i} \right)^{\frac{1}{\gamma_1}}.$$

This paper assumes that each firm in group $i$ has the same fixed cost $a$ and the same constant marginal cost $c$. Specially, consider only one good group being produced in each. The equilibria are given by

$$\frac{\alpha(y_1 - 1)}{c}, x_2 = 0$$

$$P_1 = c, \frac{y_1}{y_1 - 1}$$

$$\frac{s}{a y_1}$$

$$q_1 = \frac{P_1 n_i}{r^{\gamma_1}} = c\left(\frac{y_1}{y_1 - 1}\right)^{\gamma_1}, \frac{1}{s}$$

$$y_1 = \frac{\alpha(y_1 - 1)}{c} \left[ \frac{s}{a y_1} \right]^{\gamma_1}.$$

If and only if one does not pay a firm to produce a good of the second cluster, Equation (16) is a Nash equilibrium. The demand for such a good is

$$x_2 = \begin{cases} 0 & \text{for } P_2 \geq q_2 \\ s / p_2 & \text{for } P_2 < q_2 \end{cases}.$$

Conditions:

$$\max (p_2 - c)x_2 = s(1 - \frac{c}{q_2}) < a$$

or

$$q_1 < \frac{sc}{s - a}.$$
Similarly, Equation (17) is a Nash equilibrium if and only if

$$q_2 < \frac{sc}{s-a}. \quad (19)$$

Figure 1 is drawn to depict the possible equilibria. Then Equations (18) and (19) tell us whether either or both of the situations are possible equilibria. Now consider the region or region B, both Equation (16) and Equation (17) are Nash equilibria,

$$\frac{y_1}{y_2} = \frac{\gamma_1}{\gamma_2} - 1 \left[ \frac{c_1}{c_2} \right].$$

From Equation (14),

$$\frac{dy}{dy} > a.$$  

If $y_1 > y_2$, then

$$\frac{y_1}{y_2} > 1, \quad (20)$$

CONCLUSION

This paper has constructed some models to study some aspects of the relationship between interconnectedness among firms in intracluster and intercluster and the externalities of clusters. The following general conclusions seem worth pointing out. First, the larger the elasticity of substitution between any two products produced within the cluster, the larger the cluster’s output other things being equal; second, a cluster with larger elasticity of substitution yields larger output among different clusters other things being equal. That is, the more similar products of a cluster are, the larger its output will be.

While the model depends on restrictive assumption that the similarity among a cluster(s) is used as a measure for clustering, it does show that it is possible for the theory of external economies to make at least some progress into this virtually unexplored territory.

REFERENCES

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