The Effects of Non-darcy Flow on Hydraulic Fracturing Optimization Design

Abstract: In recent years, the petroleum industry has been aware of the potential for non-Darcy flow in propped fracture. In hydraulic fracture stimulation treatments, the effects of non-Darcy flow as one of the most critical factors in reducing the productivity of hydraulically fractured high rate wells have been studied widely with examples of field cases. In the hydraulic fracture design, the non-Darcy flow can have great impact on the reduction of a propped half-length, thus lowering the well’s productive capability. These non-Darcy flow effects in propped fractures have been typically associated with high flow rates in both oil and gas wells. This paper studied the effects of non-Darcy flow in fracture on the hydraulic fracturing design, studied the propped porosity and bottom-hole on hydraulic fracturing design and deliverability of fractured well taking into account non-Darcy flow.

Key words: Non-darcy flow; Hydraulic fracturing; Optimization design; Productive capability

NOMENCLATURE

\[ \frac{\Delta p}{\Delta L} \]  \quad \text{pressure gradient, atm/cm}[Pa/cm] \\
\[ \mu_g \]  \quad \text{gas viscosity, cp}[Pa\cdot s] \\
\[ v \]  \quad \text{gas velocity, ft/s}[m/s] \\
\[ k_f \]  \quad \text{Darcy permeability in fracture, darcies}[md] \\
\[ \rho_g \]  \quad \text{gas density, g/cm}^3 \\
\[ x_f \]  \quad \text{fracture half length, ft}[m] \\
\[ x_r \]  \quad \text{the length of reservoir, ft}[m] \\
\[ k \]  \quad \text{permeability in reservoir, darcies}[md] \\
\[ w \]  \quad \text{the propped fracture width, ft}[in.] \\
\[ h_p \]  \quad \text{the net pay thickness, ft} \\
\[ V_p \]  \quad \text{the volume of the proppant in the pay, ft}^3 \\
\[ V_f \]  \quad \text{the volume of one propped wing} \\
\[ k_{f,n} \]  \quad \text{nominal fracture permeability, darcies}[md] \\
\[ J_{D,max} \]  \quad \text{the maximum productivity index, dimensionless} \\

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1. INTRODUCTION

Hydraulic fracturing is today the mainly stimulation treatments in many producing wells all over the world. The petroleum engineers have been aware of the potential for non-Darcy flow in propped fracture for many years since the work of Cooke(1973). In recent years, non-Darcy flow has a significant increase in interest in the petroleum industry, especially in hydraulically fractures, and the non-Darcy flow effects have been studied widely with examples of field cases. In hydraulic fracture design, the non-Darcy flow can have great impact on the reduction of a propped half-length to a considerably shorter “effective” half-length, thus lowering the well’s productive capability. These effects within the propped fracture are mainly due to high velocity and higher pressure drop in the fracture. These non-Darcy flow effects in propped fractures have been typically associated with high flow rates in both oil and gas wells. The non-Darcy effects significantly influence gas production performance(2004).

2. THEORY OF NON-DARCY FLOW

Darcy’s law describes laminar flow through porous media. In this case the fluid velocity was very low, and the pressure gradient is directly proportional to fluid velocity.

\[ \frac{\Delta p}{\Delta L} = \frac{\mu v}{k_f} \]  

But when flow velocity increase, Equation (1) is not valid anymore because of the additional pressure drop caused by the frequent acceleration and deceleration of the particles of the moving fluid. Cornel and Katz(1953) described these inertial effects using equation (2).

\[ \frac{\Delta p}{\Delta L} = \frac{\mu v}{k_f} + \beta \rho_g v^2 \]  

When velocities are low, the second term in Equation (2) can be neglected. However, for higher velocities this term becomes more important. In order to compare Darcy and non-Darcy flow, we can obtain equation (3) from the equations (1) for Darcy flow and equation (4) from the equations (2) for non-Darcy flow.

\[ \frac{\Delta p}{\Delta L \mu_g v} = \frac{1}{k_f} \]  

\[ \frac{\Delta p}{\Delta L \mu_g v} = \frac{1}{k_f} + \beta \rho_g v = \frac{1}{k_f} \left(1 + \frac{\beta k_f \rho_g v}{\mu_g}\right) \]  

Geertsma(1974) defined the Reynolds number \((N_{Re})\) in a porous media as

\[ N_{Re} = \frac{\beta k_f \rho_g v}{\mu_g} \]  

From the equation (3) and (4) we can obtain the final expression of effective permeability \(k_{f\text{-eff}}\).

\[ k_{f\text{-eff}} = \frac{k_f}{1 + N_{Re}} \]  

When taking into account non-Darcy flow, the equivalent permeability should be calculated firstly to forecast the production of oil and gas wells.

3. CALCULATION OF NON-DARCY COEFFICIENT

Lopez-Hernandez et al.(2004) summarized many \(\beta\) coefficient equations, and all the equations are function of \(k_f\) and/or \(\phi_p\). So all equations can be summarized in a general expression(equation 7), where a, b and c parameters are different for each case.

\[ \beta = \frac{a}{k_f^b \phi^c} \]  

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If the unit of $k_f$ and $\beta$ is $m^2$ and $1/m$, respectively, then $a=0.143$, $b=0.5$ and $c=1.5$.

4. PHYSICAL OPTIMIZATION THEORY

Valko and Economides (1998, 2002) introduced a physical optimization technique to maximize the fractured-well PI under pseudo-steady state in a more realistic square reservoir. It is well understood that the well performance, in addition to the fracture conductivity, also depends on the x-direction penetration ratio, $I_x$ and the dimensionless fracture conductivity $C_{FD}$:

$$I_x = \frac{2x_f}{x_e} \quad \text{and} \quad C_{FD} = \frac{k_jw}{kx_f} \quad (8)$$

Because the penetration and the dimensionless conductivity, through width, compete for the same resource: the propped volume, the injected propped volume provides a constraint in the form, so they defined $I_x^2C_{FD}$ as proppant numbers, $N_{prop}$.

$$N_{prop} = I_x^2C_{FD} = \frac{4k_jx_fw}{kx_e^2} \quad \text{and} \quad \frac{4k_jx_fwh_p}{kx_e^2h_p} = \frac{2k_jV_p}{kV_f} \quad (9)$$

For a specific $N_{prop}$ the maximum $J_D$ occurs for a well defined value of $C_{FD}$. For all proppant numbers, the optimum fracture dimensions can be obtained from

$$x_{f, opt} = \sqrt[3]{\frac{k_fV_f}{C_{FD, opt}wh_p}} \quad \text{and} \quad \frac{w}{h_{p, opt}} = \sqrt[3]{\frac{C_{FD, opt}k_fV_f}{kwh_p}} \quad (10)$$

5. INCORPORATING NON-DARCY FLOW EFFECTS INTO OPTIMIZATION OF FRACTURE DIMENSIONS

We can incorporate the non-Darcy flow into the hydraulic fracturing design. the iterative procedure for calculating the optimal hydraulic fracture length and width is below.

a. Assume a Reynolds Number $N_{Re}$, calculate the effective fracture permeability using equation 6.

b. Using the calculated effective fracture permeability in step 1, the fixed volume of proppant injected, the volume of proppant reaching the pay is estimated from the ratio of pay to the fracture height. So the Proppant Number $N_{prop}$ can be calculated from equation 9.

c. Using $N_{prop}$, the maximum productivity index, $J_{D,max}$ and optimal dimensionless fracture conductivity $C_{F, opt}$ can be obtained.

d. With the optimal dimensionless fracture conductivity $C_{F, opt}$, the optimal optimum fracture length and width can be calculated from equation 10.

e. Calculate gas production and velocity in the fracture, then calculate the new Reynolds number.

f. Compare $N_{Re}$ calculated in step 5 and the assumed $N_{Re}$ in step 1. If they are close enough, the procedure can be ended. Otherwise, go back to step 1 until they are close enough.

6. PRACTICAL APPLICATION OF THE METHOD AND THEORY

To show the applicability of this method and theory, results of this method are validated using comparisons with the hydraulic fracturing design for Darcy flow in vertical fractured gas well. The characteristics needed in calculation are listed in table 1.
6.1 Effects of Propped Porosity on Fracture Dimension and Production

This work is to optimize hydraulic fracture dimension under non-Darcy flow effects in hydraulic fractured well, and to compare the calculated results with that under Darcy flow.

We firstly fixed the bottom-hole flow pressure \( p_{wf} = 300 \) psi, fixed reservoir permeability \( k_{res} = 0.1 \) md, changed the prop porosity in fracture from 0.15 to 0.35, and all calculation is only for vertical well. In table 2, the calculated results are listed for different propped porosity in fracture.

### Table 2: Calculated Results for Different Propped Porosity in Fracture

<table>
<thead>
<tr>
<th>( \phi_{res} )</th>
<th>flow state</th>
<th>( x_f ) (ft)</th>
<th>( w ) (in)</th>
<th>( k_{f-eff} ) (md)</th>
<th>( N_{prop} )</th>
<th>( J_{D,max} )</th>
<th>( C_{fDopt} )</th>
<th>( q_{gas} ) (mscf/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.325</td>
<td>1.843</td>
<td>21.45</td>
<td>11037.59</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>443.63</td>
<td>0.192</td>
<td>7552.96</td>
<td>1.233</td>
<td>0.928</td>
<td>2.729</td>
<td>5558.98</td>
</tr>
<tr>
<td>0.20</td>
<td>Darcy flow</td>
<td>577.84</td>
<td>0.157</td>
<td>100000</td>
<td>17.346</td>
<td>1.851</td>
<td>22.629</td>
<td>11084.12</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>473.47</td>
<td>0.192</td>
<td>10229.44</td>
<td>1.774</td>
<td>1.020</td>
<td>3.448</td>
<td>6190.11</td>
</tr>
<tr>
<td>0.25</td>
<td>Darcy flow</td>
<td>579.95</td>
<td>0.167</td>
<td>100000</td>
<td>18.502</td>
<td>1.859</td>
<td>23.962</td>
<td>11132.02</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>498.37</td>
<td>0.194</td>
<td>12811.68</td>
<td>2.370</td>
<td>1.120</td>
<td>4.157</td>
<td>6709.73</td>
</tr>
<tr>
<td>0.30</td>
<td>Darcy flow</td>
<td>582.13</td>
<td>0.178</td>
<td>100000</td>
<td>19.824</td>
<td>1.867</td>
<td>25.482</td>
<td>11181.38</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>517.04</td>
<td>0.200</td>
<td>15416.45</td>
<td>3.056</td>
<td>1.226</td>
<td>4.980</td>
<td>7343.868</td>
</tr>
<tr>
<td>0.35</td>
<td>Darcy flow</td>
<td>584.37</td>
<td>0.191</td>
<td>100000</td>
<td>21.349</td>
<td>1.875</td>
<td>27.232</td>
<td>11232.32</td>
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<tr>
<td></td>
<td>Non-Darcy</td>
<td>530.40</td>
<td>0.210</td>
<td>18133.59</td>
<td>3.871</td>
<td>1.335</td>
<td>5.994</td>
<td>7994.008</td>
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</table>

The results show that the optimal fracture length, the effective permeability in fracture, \( J_{D,max} \), \( C_{fDopt} \) and gas production \( q_{gas} \) under non-Darcy flow is less than under Darcy flow. The presence of non-Darcy flow in the hydraulic fracture significantly reduces the effective conductivity of the fracture, and adversely affects the...
productivity of a well. And under non-Darcy flow effect a shorter and wider fracture geometry provides better productivity than a longer and narrower fracture.

The Fig. 1 and Fig 2 is the effect of propped porosity on the reservoir pressure under the condition of Darcy flow and non-Darcy flow, respectively. From these figures we can see that the propped porosity nearly has no effects on reservoir pressure under Darcy flow, but has great effects on reservoir pressure when non-Darcy flow occurs in propped fracture.

6.2 Effects of Bottom-hole Flow Pressure on Fracture Dimension and Production

In this work, we fixed the propped porosity $\phi_p = 0.15$, fixed reservoir permeability $k_{res} = 0.1$ md, changed bottom-hole flow pressure from 300 to 1500 psi, other parameters are the same as above.

The calculated results are listed in the table 3. We can see that the optimal fracture length, the effective permeability in fracture, $J_{D,max}$, $C_{\text{Dopt}}$ and gas production $q_{gsc}$ under non-Darcy flow is less than under Darcy flow, too. These results show that the effects of non-Darcy flow in different bottom-hole flow pressure are the same the effects in different proppant porosity.

<table>
<thead>
<tr>
<th>$P_{wfr}$ (psi)</th>
<th>flow model</th>
<th>$x_f$ (ft)</th>
<th>$w$ (in)</th>
<th>$k_{f-eff}$ (md)</th>
<th>$N_{prop}$</th>
<th>$J_{D,max}$</th>
<th>$C_{\text{Dopt}}$</th>
<th>$q_{gsc}$ (mscf/d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.325</td>
<td>1.843</td>
<td>21.45</td>
<td>11037.59</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>443.63</td>
<td>0.192</td>
<td>7552.96</td>
<td>1.233</td>
<td>0.928</td>
<td>2.729</td>
<td>5558.98</td>
</tr>
<tr>
<td>600</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.326</td>
<td>1.843</td>
<td>21.45</td>
<td>10685.44</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>447.91</td>
<td>0.191</td>
<td>7994.38</td>
<td>1.305</td>
<td>0.940</td>
<td>2.83</td>
<td>5451.63</td>
</tr>
<tr>
<td>900</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.326</td>
<td>1.843</td>
<td>21.45</td>
<td>10114.94</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>453.42</td>
<td>0.188</td>
<td>8574.41</td>
<td>1.400</td>
<td>0.956</td>
<td>2.97</td>
<td>5248.06</td>
</tr>
<tr>
<td>1200</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.326</td>
<td>1.843</td>
<td>21.45</td>
<td>9338.73</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>460.32</td>
<td>0.185</td>
<td>9327.89</td>
<td>1.523</td>
<td>0.977</td>
<td>3.13</td>
<td>4951.01</td>
</tr>
<tr>
<td>1500</td>
<td>Darcy flow</td>
<td>575.79</td>
<td>0.148</td>
<td>100000</td>
<td>16.326</td>
<td>1.843</td>
<td>21.45</td>
<td>8369.87</td>
</tr>
<tr>
<td></td>
<td>Non-Darcy</td>
<td>468.91</td>
<td>0.182</td>
<td>10314.97</td>
<td>1.684</td>
<td>1.004</td>
<td>3.34</td>
<td>4562.30</td>
</tr>
</tbody>
</table>

![Fig. 3: The Curve of Time & Reservoir Pressure in Different pwf (Darcy Flow)](image1)

![Fig. 4: The Curve of Time & Cumulative Production in Different pwf (Darcy Flow)](image2)

The Fig. 3 and Fig. 4 is the curve for the reservoir pressure, cumulative production with time under different bottom-hole flow pressure, respectively, which didn’t take into account non-Darcy flow effects. From these two figures we can see that the bottom-hole flow pressure has great effects on whether reservoir pressure or cumulative production.

Tab. 3: Calculated Results for Different Bottomhole Flow Pressure
The Fig. 5 and Fig. 6 is the same curves taking into account non-Darcy effects, respectively. From these two figures we can see that the bottom-hole flow pressure has great effects on reservoir pressure and cumulative production whether taking into account non-Darcy flow effects or not.

Comparing the cumulative production in Darcy flow to that in non-Darcy flow in different bottom-hole flow pressure, we can see the cumulative production considering non-Darcy effects is less than that without considering non-Darcy effect.

Fig. 5: The Curve of Time & Reservoir Pressure in Different pwf (Non-darcy Flow)

Fig. 6: The Curve of Time & Cumulative Production in Different pwf (Non-darcy Flow)

7. CONCLUSION

a. Non-Darcy flow effects should be considered in hydraulic fracturing design in gas wells.

b. The optimal fracture length, the effective permeability in fracture, and gas production under non-Darcy flow is less than under Darcy flow. The calculated results show that the presence of non-Darcy flow in the hydraulic fracture significantly reduces the effective conductivity of the fracture, and adversely affect the productivity of a hydraulically fractured gas well.

c. If taking into account non-Darcy effects, the reservoir pressure drops less rapidly than that not taking into account non-Darcy effects, and the cumulative production is less than that under the condition of Darcy flow.

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REFERENCES


