Friction Coefficient Prediction Method for Extended-Reach Well Based on Grey Prediction

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Abstract
Torque and drag prediction is very crucial in the design and drilling of extended-reach well, which has relation to the success or failure of drilling, while the torque and drag prediction result is greatly affected by the value of friction coefficient. Firstly, using the modified 3D soft-string calculation model, the friction coefficient of upper wellbore section was obtained. Then referring the related mathematical methods of system theory, on the basis of the friction coefficient of upper hole section, the friction coefficient prediction model for impending drilling well segment was established based on grey prediction, the prediction results show the error of friction coefficient and hook load is less than 10%. The method could satisfy the engineering requirement, which provides a theoretical guidance for real-time supervision on torque and drag in the drilling of extended-reach well.

Key words: Extended-reach well; Friction coefficient; System theory; Grey prediction; Real-time supervision

INTRODUCTION
The friction phenomenon in drilling operation is a different concept compared with sliding friction in physics, the friction coefficient in drilling operation is a complex friction coefficient, which mainly includes the boundary friction, dry friction and fluid friction, therefore, it is affected by drilling fluid property, string structure, and rock property and so on[1]. Its characteristics are uncertainty, fuzziness and non-stationary. At present, there are mainly two methods to determine the friction coefficient, one is determined by experiment, the other is obtained by choosing appropriate calculation model of torque and drag, establishing friction coefficient inversing model under different working condition based on the field drilling data. There is big difference between indoor simulation conditions and actual drilling condition, the friction coefficient obtained by experiment has poor reproducibility and poor stability, drilling fluid and lubricant can only be evaluated and optimized from one side, there is still big difference between friction coefficient measured by indoor and actual drilling friction coefficient[2]. Nowadays, inversing method is still the most accurate method to determine the friction coefficient based on the field data. According to field data, referring the related mathematical methods of system theory, the new friction coefficient prediction method for undrilled interval were established in this paper, it provides a new solution for torque and drag research on extended-reach well.

1. THEORY OF GREY SYSTEM

1.1 Background
The grey system theory, which was created by Deng[3], provides an approach to investigate the relationships of input–output process with unclear inner relationships, uncertain mechanisms and insufficient information. Without restoring to forming a knowledge base, the grey modeling scheme constructs a differential equation to characterize the controlled system; therefore, the next output from the model can be obtained by simply solving the differential equation. In view of the grey characteristics widely existing in an injection-production system, grey
system theory can be applied by regarding the partially unknown process in the system as a grey process. This data-driven technique can minimize the noise in the model because it does not consider the intermediate hydrological processes while predicting the lumped model output\(^4\)\(^5\). Consequently, an attempt has been made in the present study to develop dynamic prediction model on a chemical injection process. A grey system is a partially unknown process in the system as a grey process. This system theory can be applied by regarding the partially unknown system as a grey system, such as the human body, agriculture, hydrology, economy, and so forth\(^6\)\(^7\). The objective of grey system theory and its application is to bridge the gap existing between the social and natural sciences. The raw data time series from an unknown system may be random; however, its degree of randomness may be reduced after subjecting it to accumulated generation operation. A once or twice accumulation of raw time series is normally enough to support the differential equation. According to Deng, the analysis of system characteristics is normally based on statistical models, which find the statistical properties between data in large sample sets\(^8\). However, the aims of grey system theory are to provide theory, techniques, notions and ideas for analyzing latent and intricate systems of grey system theory are to provide theory, techniques, notions and ideas for analyzing latent and intricate systems.

### 1.2 Grey Prediction Model GM(1,1)

This section reviews the operation of grey prediction in details. The grey model GM(1,1) is a time series prediction model. It has three basic operations: (a) accumulated generation, (b) inverse accumulated generation, and (c) grey modeling. The grey prediction model uses the operations of accumulated to construct differential equations. Intrinsically speaking, it has the characteristics of requiring less data.

The grey model GM(1,1), that is, a single variable first-order grey model, is summarized as follows:

**Step 1 for an initial time sequence**

\[
X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(i), \ldots, x^{(0)}(n)\}. \quad (1)
\]

Where \(x^{(0)}(i)\) the time series data at time \(i\), \(n\) must be equal to or larger than 4.

**Step 2 on the basis of the initial sequence \(X^{(0)}\), a new sequence \(X^{(1)}\) is set up through the accumulated generating operation in order to provide the middle message of building a model and to weaken the variation tendency, that is**

\[
X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(i), \ldots, x^{(1)}(n)\}. \quad (2)
\]

Where

\[
x^{(1)}(k) = \sum_{i=k}^{k} x^{(0)}(i) \quad k=1, 2, \ldots, n. \quad (3)
\]

**Step 3 the first-order differential equation of grey model GM(1,1) is then the following**

\[
\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \quad (4)
\]

And its difference equation is

\[
X^{(0)}(k) + aZ^{(1)}(k) = b \quad k = 2, 3, \ldots, n. \quad (5)
\]

And from Equation (5), it is easy to get

\[
\begin{align*}
X^{(0)}(2) & = x^{(0)}(1) + a1 \\
X^{(0)}(3) & = x^{(0)}(1) + a2 \\
& \vdots \\
X^{(0)}(n) & = x^{(0)}(1) + an \\
\end{align*}
\]

**Where \(a\) and \(b\) are the coefficients to be identified. Let**

\[
Y_n = [x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)]^T. \quad (7)
\]

\[
B = \begin{bmatrix}
-z^{(1)}(2) & 1 \\
-z^{(1)}(3) & 1 \\
& \vdots \\
-z^{(1)}(n) & 1
\end{bmatrix}. \quad (8)
\]

Also take

\[
Z^{(1)}(k+1) = \frac{1}{2}(x^{(0)}(k) + x^{(0)}(k+1)) \quad k = 1, 2, \ldots, (n-1). \quad (9)
\]

And

\[
A = [a, b]^T. \quad (10)
\]

Where \(Y_n\) and \(B\) are the constant vector and the accumulated matrix respectively. \(Z^{(1)}(k+1)\) is the \((k+1)^{\text{th}}\) background value. Applying ordinary least-square to Equation (6) on the basis of Equations (7)-(10), coefficient becomes

\[
A = (B^T B)^{-1}B^T Y_n. \quad (11)
\]

Step 4 substituting \(A\) in Equation (5) with Equation (11), the approximate equation becomes the following

\[
\hat{x}^{(1)}(k+1) = (x^{(0)}(1)-b/a) e^{-a} + b/a. \quad (12)
\]

Where \(\hat{x}^{(1)}(k+1)\) is the predicted value of \(x^{(1)}(k+1)\) at time \((k+1)\). After the completion of an inverse-accumulated generation operation on Equation (12), \(\dot{Z}^{(0)}(k+1)\), the predicted value of \(x^{(0)}(k+1)\) at time \((k+1)\) becomes available and therefore,

\[
\dot{Z}^{(0)}(k+1) = (\hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)). \quad (13)
\]

Where \(k = 0, 1, 2, 3, \ldots\)

Specific grey prediction process is shown in Figure 1.
2. FRICTION COEFFICIENT INVERSION

To take tripping out for example, when hook load was given, axial force of bit was zero, value range of friction coefficient was 0~1, permissible error was 0.02, the calculation procedures are as follows:

(a) The range of friction coefficient was assumed: \( u_{\text{min}} = 0, u_{\text{max}} = 1 \);

(b) The calculation of hook load: let be \( u_a = (u_{\text{min}} + u_{\text{max}})/2 \), calculate hook load \( F_{bc} \), if \( F_{bc} < F_h \), then \( u_{\text{min}} = u_a \), otherwise \( u_{\text{max}} = u_a \);

(c) Cyclic iteration, inverse friction coefficient, if \( (u_{\text{max}} - u_{\text{min}})/u_a > \text{EER} \), return to step (2), otherwise \( u_a \) is the friction coefficient;

When the related field data of extended-reach well was collected, the friction coefficient was inversed under different conditions, the inversion results are shown in Table 1.

![Figure 1](image-url)  
**Figure 1**  
Grey Prediction Process

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Inversion Results of Extended Reach Well Friction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well number</td>
<td>TD (m)</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
</tr>
<tr>
<td>Bin 173-1HF</td>
<td>3,983</td>
</tr>
<tr>
<td>Bin 435-4HF</td>
<td>5,097</td>
</tr>
<tr>
<td>Boye-ping 2</td>
<td>3,645</td>
</tr>
<tr>
<td>Fan 116-1HF</td>
<td>4,002</td>
</tr>
<tr>
<td>Fan 116-2HF</td>
<td>3,912</td>
</tr>
<tr>
<td>Fan 116-3HF</td>
<td>4,096</td>
</tr>
<tr>
<td>Fan 116-4HF</td>
<td>3,818</td>
</tr>
<tr>
<td>Fan 154-7HF</td>
<td>4,260</td>
</tr>
<tr>
<td>Fan 154-8HF</td>
<td>4,158</td>
</tr>
<tr>
<td>Fan 162-2HF</td>
<td>4,522</td>
</tr>
<tr>
<td>Gaoping 1</td>
<td>4,535</td>
</tr>
<tr>
<td>Niu 871-X7</td>
<td>3,752</td>
</tr>
<tr>
<td>Xin 14-8HF</td>
<td>4,069</td>
</tr>
<tr>
<td>Yan 227-2HF</td>
<td>4,522</td>
</tr>
<tr>
<td>Yan 227-5HF</td>
<td>4,740</td>
</tr>
<tr>
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<td>4,206</td>
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<td>Yan 227-4HF</td>
<td>4,323</td>
</tr>
<tr>
<td>Yan 227-9HF</td>
<td>4,925</td>
</tr>
<tr>
<td>Zhuang 129-1HF</td>
<td>5,341</td>
</tr>
<tr>
<td>Liangye 1HF</td>
<td>3,969</td>
</tr>
<tr>
<td>Boye-ping 1</td>
<td>4,335</td>
</tr>
</tbody>
</table>
3. FRICTION COEFFICIENT PREDICTION RESULTS

According to the inversion results of friction coefficient, the upper friction coefficient was used as input (the step size is 1m during the process of inversion), GM prediction model of friction coefficient was established by MATLAB software, which is used to predict friction coefficient of undrilled interval, the hook load can be calculated by the combination of prediction friction coefficient and the 3-d soft string model, and which was compared with the actual hook load, the results were shown in Table 2. It is concluded from Table 2 that the friction coefficient has high precision by the prediction model, the error of friction coefficient and hook load is less than 10%, the model has a high practical value, it can be used to real-time supervision on Torque and drag in extended-reach well, which provides a guidance for adjusting drilling fluid, optimizing drilling parameters and bottom hole assembly.

Table 2
Prediction Results of Friction Coefficient and Hook Load

<table>
<thead>
<tr>
<th>Well number</th>
<th>Hole section (m)</th>
<th>Average prediction error of friction coefficient (%)</th>
<th>Average prediction error of hook load (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bin 173-1HF</td>
<td>3,492-3,979</td>
<td>1.5</td>
<td>7.8</td>
</tr>
<tr>
<td>Bin 435-4HF</td>
<td>4,900-5,097</td>
<td>8.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Boye-ping 2</td>
<td>3,200-3,642</td>
<td>8.8</td>
<td>9.4</td>
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<tr>
<td>Fan 116-1HF</td>
<td>3,485-4,002</td>
<td>5.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Fan 116-2HF</td>
<td>3,200-3,912</td>
<td>4.5</td>
<td>2.4</td>
</tr>
<tr>
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<td>3,800-4,096</td>
<td>3.6</td>
<td>1.3</td>
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<td>2.1</td>
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<td>4,100-4,260</td>
<td>4.3</td>
<td>3.5</td>
</tr>
<tr>
<td>Fan 154-8HF</td>
<td>3,780-4,158</td>
<td>5.8</td>
<td>1.4</td>
</tr>
<tr>
<td>Fan 162-2HF</td>
<td>3,900-4,522</td>
<td>5.2</td>
<td>1.8</td>
</tr>
<tr>
<td>Gaoping 1</td>
<td>4,200-4,535</td>
<td>3.1</td>
<td>6.9</td>
</tr>
<tr>
<td>Niu 871-X7</td>
<td>3,330-3,719</td>
<td>8.2</td>
<td>1.5</td>
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<tr>
<td>Xin 14-8HF</td>
<td>3,800-4,069</td>
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<td>7.3</td>
</tr>
<tr>
<td>Yan 227-2HF</td>
<td>4,000-4,522</td>
<td>6.1</td>
<td>1.5</td>
</tr>
<tr>
<td>Yan 227-5HF</td>
<td>4,200-4,738</td>
<td>4.7</td>
<td>1.3</td>
</tr>
<tr>
<td>Yan 227-7HF</td>
<td>3,380-4,203</td>
<td>4.2</td>
<td>7.8</td>
</tr>
<tr>
<td>Yan 227-4HF</td>
<td>4,100-4,323</td>
<td>5.4</td>
<td>3.4</td>
</tr>
<tr>
<td>Yan 227-9HF</td>
<td>4,200-4,925</td>
<td>6.1</td>
<td>4.8</td>
</tr>
<tr>
<td>Zhuang 129-1HF</td>
<td>5,035-5,341</td>
<td>8.5</td>
<td>2.1</td>
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<tr>
<td>Liangye 1HF</td>
<td>3,500-3,969</td>
<td>6.2</td>
<td>2.8</td>
</tr>
<tr>
<td>Boye-ping 1</td>
<td>3,800-4,335</td>
<td>5.3</td>
<td>3.6</td>
</tr>
</tbody>
</table>

The friction coefficient prediction results are shown in Figure 2 of well Boye-ping 2, hook load prediction results of well Boye-ping 2 are shown in Figure 3.

Figure 2
Friction Coefficient Prediction Results of Well Boye-Ping 2

Figure 3
Hook Load Prediction Results of Well Boye-Ping 2

CONCLUSION

(a) The friction phenomenon in drilling operation is a different concept compared with sliding friction in physics; the friction coefficient in drilling operation is a complex friction coefficient.

(b) Grey system is a partially known and partially unknown system. In situations where a large sample set is not easily available, a system may be considered in the status of poor, uncertain and incomplete information and is known as a grey system, it is feasible that the gray system is used to treat friction coefficient prediction in impending drilling well segment.
The friction coefficient prediction for impending drilling well segment based on gray prediction model in this paper has high accuracy, the friction coefficient and hook load error are less than 10%, which provides theoretical guidance in real time monitoring, prediction for extended-reach well drilling process.

REFERENCES


