

## An Analytic Hierarchy Process for Bit Optimization Based on the Fractal Crushing Work Ratio

# LIU Shanshan<sup>[a],\*</sup>; YAN Tie<sup>[a]</sup>; BI Xueliang<sup>[a]</sup>; YU Xiaowen<sup>[a]</sup>; XU Haisu<sup>[b]</sup>; ZHANG Yuenan<sup>[c]</sup>

<sup>[a]</sup> National Engineering Laboratory of Oil and Gas Drilling Technology, Northeast Petroleum University, Daqing, China.

<sup>[b]</sup> Key Laboratory of Ehanced Oil Recovery of the Ministry of Education, Northeast Petroleum University, Daqing, China.

<sup>[c]</sup> Drilling Group, Drilling Engineering Technology Research Institute, Daqing Oil Field Company, Daqing, China.

\*Corresponding author.

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### Abstract

The paper presents a new method of bit optimization by applying index scale AHP (Analytic Hierarchy Process). It constructs judgment matrix based on the complete consistency principle. For the matrix has the consistency of the importance and transitivity, it simplifies the process of calculation by cancelling the consistency checking process, which is used in the traditional AHP. The adjacent important ratio is selected by combining hierarchical thought and fractal dimension, and the effect of adjacent important ratio disturbance on the optimization result is reduced. The accuracy of the decision result of bit optimization hierarchical structure model is ensured effectively, and the stability and flexibility of the model is enhanced. The model can be applied in different regions based on actual needs, and to solve optimization problems of different types of drill bit.

**Key words:** Crushing work ratio; Index scale; Bit optimization

## INTRODUCTION

At present, the method of bit selecting can be roughly divided into three categories<sup>[1-2]</sup>. The first category is bit performance evaluation method. This method counts the use of the drill bit from different formations, according to the bit data of a certain area that has been drilled. Then we determine one or more indicators reflecting the effect of the bit performance as the selection basis of bit type. The second category is the rock mechanical parameter method. The method select the type of drill bit based on one or more rock mechanical parameters of the pre-drilling formations and combined with the instructions of the drill bit manufacturers. The third category is the synthesis method. It combines the bit performance and rock mechanical properties to select drill bit. We optimize bit by applying the index scale AHP in this paper.

## 1. INDEX SCALE AHP FOR BIT OPTIMIAZATION

# 1.1 Hierarchical Structure Model of Bit Optimization

According to the basic principle of index method<sup>[3-6]</sup> and considering the actual drilling situation, the hierarchical structure model of bit optimization is divided into three layers, they are respectively: The goal layer, criterion layer and plan layer. Each factors of the layer structure of the model play a dominant role on the factors of next hierarchy. The hierarchical structure model of bit selection is shown in Figure 1. For the multiple bit programs in the plan layers to be the optimization, the model optimizes the best bit type, according to the five indicators, mechanical footage, bit wear, average ROP, unit drill cost and crushing work ratio.

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#### Figure1

#### Hierarchical Structure Model for Bit Optimization

#### 1.2 Determination of the Adjacent Importance Ratio

Index scale is a new method to determine the weight, which establish the weight scale by "isometric classification, geometric assignment", according to the psychology law of Webb-Fechner. That is,  $S = k \cdot \log R$  or  $R = 10^{\frac{S}{k}}$  (S is the quantity of people's subjective feelings, R is the quantity of objective stimulus, K is the Webb constant), the formula is:

$$u = v^n \ (n = 0, 1, \cdots, \text{ natural number}). \tag{1}$$

Where, *n* is the grade of the importance,  $\gamma$  is undetermined coefficient.

#### 1.2.1 The Determination of the Importance Grade n

The degree of differences between 2 things include 9 levels at most in psychology, When more than 9, judgment is prone to appear to chaos and ambiguous phenomenon. Thus, it's appropriate to divide the grade of importance into nine, that is n = 8.

# **1.2.2** The Determination of the Adjacent Importance Ratio of the Judgment Matrix of Criterion Layer *b*

*b* value should be determined referring to the economic goal contribution of each criterion. According to the determining methods of the general economic problem, it can be determined by formula  $b^n = 9$  and  $b = \sqrt[8]{9} = 1.316$ .

# **1.2.3** The Determination of the Adjacent Importance Ratio of the Judgment Matrix of Plan Layer *a*

Considering the rock breaking is a process of statistical self-similar and from the initial effect of the rock breaked by the bit to form the cuttings returned to the ground with a certain fraction dimension, rock experienced a leap between two extreme state.

Then we can determine D as the ratio of the highest level of importance and the lowest level of the importance. It can be said that fractal dimension not only represents the structure characteristics of rocks, but also represents the fragmentation characteristics of formation, and it can be directly obtained by the analysis of upward cuttings in the drilling process. Therefore,  $a = \sqrt[8]{D}$  was selected as the adjacent importance the judgment matrix of plan layer.

#### 1.3 The Determination of the Bit Optimization Weight

First of all, the importance difference of each factor in criterion layer and plan layer is judged.  $c_{ij}$  ( $i, j = 1, 2, \dots, 5$ ) represents the importance difference between optimization criterion  $c_i$  and optimization criterion  $c_j$ .  $p_{gk}^i$  ( $g, k = 1, 2, \dots, m$ ,  $i = 1, 2, \dots, 5$ ) represents the importance difference between bit  $p_e$  and  $p_k$  under criterion  $c_i$ .

Then according to the average value of *D* in the block, the adjacent importance ratio is determined as  $a = \sqrt[8]{D}$ , taking b = 1.316, constructing the judgment matrix *C* in criterion layer and the judgment matrix  $P_i$  ( $i = 1, 2, \dots, 5$ ) in plan layer.

**1.3.1 The Weight of Criterion Layer Over Target Layer** The judgment matrix *C* is constructed according to the adjacent importance ratio *b* in criterion layer.

$$C = \begin{pmatrix} b^{c_{11}} & b^{c_{12}} & \cdots & b^{c_{15}} \\ b^{c_{21}} & b^{c_{22}} & \cdots & b^{c_{25}} \\ \vdots & \vdots & & \vdots \\ b^{c_{51}} & b^{c_{52}} & \cdots & b^{c_{55}} \end{pmatrix}.$$
 (2)

By using the maximum eigenvalue method, the feature vector corresponding to the maximum eigenvalue is computed. The weight  $w_c$  of criterion layer  $C_1, C_2, \dots, C_n$  over target layer O is obtained, set as  $w_c = (w_1, w_2, \dots, w_n)$ .

**1.3.2 The Weight of Plan Layer Over Criterion Layer** The judgment matrix  $P_i$  is constructed according to the of adjacent importance ratio *a* of criterion layer.

$$P_{i} = \begin{pmatrix} a^{p_{11}^{i_{11}}} & a^{p_{12}^{i_{22}}} & \cdots & a^{p_{1m}^{i_{m}}} \\ a^{p_{21}^{i_{21}}} & a^{p_{22}^{i_{22}}} & \cdots & a^{p_{2m}^{i_{2m}}} \\ \vdots & \vdots & & \vdots \\ a^{p_{m1}^{i_{m1}}} & a^{p_{m2}^{i_{m2}}} & \cdots & a^{p_{m4}^{i_{m4}}} \end{pmatrix}.$$
 (3)

After calculating the maximum eigenvalue and eigenvector and normalized, the weight is obtained  $w_{pi}$  (*i* = 1,2,..., *n*).

The synthetic weight of plan layer over criterion layer is  $w_{P-C} = w_C \times (w_{P_1}, w_{P_2}, \dots, w_{P_n})^T$ , it is recorded as  $W_{P-C}$  after normalized. According to the magnitude of the weight, the optimized order of bit program is obtained.

## 2. ANALYSIS EXAMPLE OF BIT OPTIMIZATION IN XUSHEN FORMATION DRILLING

In order to verify the model's accuracy, the field measured data and calculated data of four group of cone bits (HJT617GH, HJT637GH, HJT737GH, HF647GHMY) is collected, which are used in drilling construction of Xushen area. Through calculating and organizing data, the optimization criterion data of the bit to be optimization is obtained, as shown in Table 1.

#### Table 1 Criterion Data of Bits

Drill type	Mechanical footage(m)	Wear grade	Average ROP (m/h)	Unit drilling cost (Yuan/m)	Crushing work ratio (kJ/m³)
HJT637GH	100.4	Y2	2.0	672.7	1,371.5
HJT737GH	103.5	Y3	1.9	772.4	1,284.4
HJT617GH	115.7	Y5	2.2	714.8	1,366.2
HF647GHMY	94.0	Y4	1.8	779.9	1,196.3

According to the existing research results<sup>[7]</sup>, there is a good linear function relationship between fractal dimension and rock drill ability. It can be obtained by correlating, and the fractal dimension database which has been established for requiring. By requiring  $K_d$ corresponding to D through the database. the cuttings fractal dimension of single bit within the limits of use in this well is obtained, as shown in Table 2.

 Table 2

 Cuttings Fractal Dimension of Each Drilling Sections

Serial number	Rock drillability average (K <sub>d</sub> )	Cuttings fractal dimension (D)
1	7.67	2.15
2	8.10	2.24
3	9.08	2.35
4	8.26	2.27

The average fractal dimension of the cuttings D = 2.29,  $a = \sqrt[8]{D} = \sqrt[8]{2.29} = 1.11$ . The importances of each criterion are 5, 0, 3, 7, 2 under the control of objective factors and the adjacent importance ratio b = 1.316. The judgement matrix *C* of criterion layer over target layer is constructed.

	1	1.7321	3.9471	0.5774	2.2791		
	0.5774	1	2.2791	0.3334	1.316		
C =	0.2534	0.4388	1	0.1463	0.5774	(4)	
	1.7321	3	6.8358	1	3.9471		
	0.4388	0.7599	1.7321	0.2534	1		

After calculating maximum eigenvalue and eigenvector normalizing, the weight  $w_c = (0.3916, 0.1220, 0.2781, 0.8340, 0.2425)$ .

Considering the five optimization criterion, the importance of four groups of bits in plan layer are as follows: (1) 3, 5, 8, 0; (2) 6, 5, 3, 4; (3) 6, 5, 8, 0; (4) 7, 1, 5, 0; (5) 5, 8, 6, 0. Adjacent importance ratio a = 1.11.

Constructing the subjective judgement matrix  $P_i$ of plan layer  $C_i$  over criterion layer and calculating maximum eigenvalue and eigenvector normalizing, the weight,

$w_{p1} = (0.4120, 0.5076, 0.6942, 0.3012),$	(5)
$w_{p2} = (0.5731, 0.5447, 0.3982, 0.4651),$	(6)
$w_{p3} = (0.5259, 0.4738, 0.6480, 0.2812),$	(7)
$w_{p4} = (0.6778, 0.3624, 0.5501, 0.3265),$	(8)
$w_{p5} = (0.4738, 0.6480, 0.5259, 0.2812).$	(9)
synthetic weight of plan layer over target laye	r is

$W_{P_C} = W_C \times W_P^T$		
$I = C  C  I_i$		(10)
		 1101

The

=(0.5748, 0.4654, 0.5907, 0.3225).

The optimization weight vector for the four groups of bits (HJT637GH, HJT737GH, HJT617GH, HF647GHMY), is  $W_{P,C}$  = (0.5748, 0.4654, 0.5907, 0.3225).

Therefore, the optimal optimization order of four groups of bits is HJT617GH, HJT637GH, HJT737GH, HF647GHMY. It is consistent with field data statistical results. Compared with traditional scaling method, small perturbation of adjacent importance ratio does not lead to great changes in the bit optimization weight, and only a great disturbance can make the order change slightly, as shown in Figures 2 and 3.



Relationship Between the Disturbance of Adjacent Importance Ratio and Bit Optimization Weight



Figure 3 Relation

#### Relationship Between the Disturbance of Adjacent Importance Ratio and Bit Optimization Weight in Traditional Method

The method remarkably improves the stability and accuracy of the model decision results, retains the model adjustment flexibility and improves the accuracy of the optimization results.

### CONCLUSION

(a) The bit optimization selection hierarchy structure model is establishment by using Index scale AHP, which effectively guarantees the consistency of the matrix and simplifies the process of calculation of bit optimization.

(b) The fractal dimension is one of the key factors that decides whether the decision results of the model are accurate. Hierarchical thinking combining fractal dimension is proposed to select adjacent importance ratio, which effectively ensures the accuracy of decision results of bit selection hierarchy structure model and enhances the quantitative evaluation function and flexibility of the model.

(c) The optimization model is practical and can be applied to solve the optimization problem of different types of drill bits in different regions. The optimization criterion and bit program was cut after increased according to the actual need, and the standard material is easy to obtain.

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