Effect of Eccentricity on Annulus Pressure of Aerated Drilling

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INTRODUCTION
In order to effectively develop complex medium and small oil and gas field, fault-block oil-gas field, oil sheet, oilfield with low pressure and low permeability, heavy oil or super heavy oil field and old oilfield, well drilling technologies which include horizontal well and underbalanced drilling and so on successively appear in twentieth century 20 and 50, then the horizontal drilling well and underbalanced drilling technology rapidly develop and are extensively applied in the world[1-3].

In 1996, assuming multiphase fluid can be seen as homogeneous mixture of three-phase of gas, liquid, solid, Guo et al established the calculation model for predicting the most optimal air injection rate and cuttings transport capacity.

In 1998, considering the existence of slippage, Bijleveld et al established steady flow model to calculate bottom-hole pressure and important parameters of two-phase flow during underbalanced drilling. The same year, Zhou Kaiji established steady state model of multiphase flow of underbalanced drilling through calculating continuity equation of gas, liquid phase, momentum equation and energy equation.

In 2000, combining experimental and theoretical study, Lage[6-7] et al established mechanics model of concentric annulus of two-phase flow, the model includes the prediction method of flow pattern and the calculation formula of gas phase volume fraction and pressure drop of bubbly flow, dispersed bubble flow, slug flow, churn flow and annular flow.

In 2002, Perez-Tellez[8-11] et al established pressure prediction model of comprehensive underbalanced drilling in the previous studies of underbalanced drilling model.
In summary, most of theoretical formula is based on pipe flow or concentric annulus flow, only a few formulas considered the effect of the eccentric drill pipe on annulus pressure.

1. AERATED DRILLING ECCENTRIC ANNULUS PRESSURE DROP CALCULATION DERIVATION

The basic hypothesis:

(a) External environment does not work on the multiphase flow control unit, multiphase flow control unit also does not work on external environment.

(b) Multiphase flow is under the state of steady flow.

(c) Ignore gas dissolved in the liquid phase and no chemical reaction and heat exchange between gas phase and liquid phase.

(d) Neglect the effect of thermal resistance on the annulus fluid temperature, and annulus multiphase fluid temperature is equal to the formation temperature.

(e) Without considering drill string rotation and fluid invasion, the mass force is only the gravity.

The angle between wellbore and horizontal direction is \( \theta \). According to the conservation of energy, for the unit mass of multiphase fluid, the differential expression of steady flow mechanical energy conservation equation can be expressed:

\[
\frac{dp}{\rho} + g \cdot \sin \theta \cdot dz + v \cdot dv + dE = 0. \tag{1}
\]

Under the condition of equal pressure drop, the pressure drop calculation formula of circular pipe flow and annulus flow, the concentric annulus diameter can be showed:

\[
D_v = \frac{\sqrt{3}}{\sqrt{3}}(D_2 - D_1) = 0.816497(D_2 - D_1). \tag{2}
\]

Equation (2) can be expressed:

\[
\frac{dp}{dz} = \frac{\rho E}{dz} + \rho g \sin \theta + \rho v \frac{dv}{dz}. \tag{3}
\]

By the circular pipe flow pressure drop and the definition of equivalent diameter, we can get the axial direction equivalent diameter of eccentric annulus:

\[
D_v = \frac{2}{\sqrt{3}}K_a(D_2 - D_1) = 0.816497\sqrt{K_a}(D_1 - D_2). \tag{4}
\]

Equation (4) can be expressed:

\[
\frac{dp}{dz} = \left( \frac{\partial p}{\partial z} \right)_h + \left( \frac{\partial p}{\partial z} \right)_p + \left( \frac{\partial p}{\partial z} \right)_g. \tag{5}
\]

According to Equation (5), gravity pressure drop gradient, frictional pressure drop gradient and acceleration pressure drop gradient together form the total pressure drop gradient of annular multiphase.

(a) Gravity Pressure Drop Gradient

The pressure gradient caused by height variation:

\[
\frac{\partial p}{\partial z} = \rho g \sin \theta. \tag{6}
\]

Where the multiphase fluid mixture density can be showed:

\[
\rho = \rho_n H + \rho_s (1 - H) . \tag{7}
\]

Therefore, Equation (7) can be rewritten:

\[
\frac{\partial p}{\partial z} = \rho g \sin \theta[\rho_n H + \rho_s (1 - H)] \sin \theta . \tag{8}
\]

(b) Frictional Pressure Drop Gradient

According to the definition, the friction pressure drop gradient can be written:

\[
\frac{\partial p}{\partial z} = \frac{\lambda v}{2D_e} \rho = \frac{\lambda (m/A) v}{2D_e}. \tag{9}
\]

(c) Acceleration Pressure Drop Gradient

Owing to

\[
v = v_g + v_m = \frac{m_g}{\rho_g A} + \frac{m_m}{\rho_m A}. \tag{10}
\]

So the equation for acceleration pressure drop gradient can be showed:

\[
\frac{\partial p}{\partial z} = \rho v g \frac{dv}{dz} = \rho v \left[ \frac{d}{dz} \left( \frac{m_g/A}{\rho_g} \right) + \frac{d}{dz} \left( \frac{m_m/A}{\rho_m} \right) \right]. \tag{11}
\]

Compared with gas phase, multiphase fluid compressibility of liquid phase and solid phase can be ignored, we can obtain

\[
\frac{d}{dz} \left( \frac{m_g/A}{\rho_g} \right) << \frac{d}{dz} \left( \frac{m_m/A}{\rho_m} \right), \text{ so }
\]

\[
\frac{\partial p}{\partial z} = \rho v \frac{d}{dz} \left( \frac{m_g/A}{\rho_g} \right) = \rho v \left[ \frac{d}{dz} \left( \frac{m_g/A}{\rho_g} \right) - \frac{d}{dz} \left( \frac{m_m/A}{\rho_m} \right) \right].
\]

In addition, due to gas phase density changes in annulus is generally much greater than gas phase mass flow rate changes, the flowing equation can be obtained:

\[
\frac{d}{dz} \left( \frac{m_g/A}{\rho_g} \right) << \frac{d}{dz} \left( \frac{m_m/A}{\rho_m} \right). \tag{12}
\]

Equation (12) can be simplified to
\[
\left( \frac{\partial p}{\partial z} \right)_a = -\rho_v \left( \frac{m_g/A}{\rho_g^2} \right) \frac{d\rho_g}{dz}.
\] (14)

According to the gas state equation, we can know
\[
\rho_g = \frac{pM_g}{ZRT}.
\]

Where \( M_g \) represents the molecular weight of injected gas, so
\[
\frac{d\rho_g}{dz} = \frac{d}{dz} \left( \frac{pM_g}{ZRT} \right) = \frac{\rho_g \frac{dp}{dz}}{ZRT} - \frac{\frac{dM_g}{dz}}{ZRT} + \frac{p \frac{dM_g}{dz}}{ZRT^2} + \frac{pM_g}{ZRT^2} \frac{dT}{dz}.
\]

Simplifying Equation (15) by:
\[
\frac{d\rho_g}{dz} = \rho_g \left( \frac{1}{\rho_g} \frac{dp}{dz} \right) - \frac{\frac{dM_g}{dz}}{ZRT} \frac{Z}{dz} - \frac{\frac{dM_g}{dz}}{ZRT^2} \frac{dT}{dz}.
\] (16)

We can assume \( \frac{1}{\rho_g} \frac{dp}{dz} = \frac{1}{\rho_g} \frac{d\rho_g}{dz} \), so simplifying Equation (16) we can obtain:
\[
\frac{d\rho_g}{dz} = \rho_g \frac{dp}{dz}.
\] (17)

Substituting Equation (17) into Equation (14), we can obtain
\[
\left( \frac{\partial p}{\partial z} \right)_a = -\rho_v \left( \frac{m_g/A}{\rho_g^2} \right) \frac{d\rho_g}{dz} = -\rho_v \left( \frac{m_g/A}{\rho_g^2} \right) \frac{dp}{\rho_g^2} \frac{dp}{dz}.
\] (18)

Finally, substituting Equation (8), Equation (9), Equation (18) into Equation (5), we can obtain
\[
\frac{dp}{dz} = \left[ \rho_m H_i + \rho_g \left( 1 - H_i \right) \right] g \sin \theta + \frac{\lambda mv/(2DA)}{1 - \left[ \rho_m H_i + \rho_g \left( 1 - H_i \right) \right] vv_{rg}} .
\] (19)

Where
\[
a = \left[ \rho_m H_i + \rho_g \left( 1 - H_i \right) \right] g \sin \theta \]
\[
b = \frac{\lambda mv/(2A)}{} ,
\]
\[
c = \left[ \rho_m H_i + \rho_g \left( 1 - H_i \right) \right] vv_{rg} .
\]

Substituting Equation (4) into Equation (19), multiphase flow pressure drop calculation formula of aerated drilling eccentric annular can be expressed:
\[
\frac{dp}{dz} = \frac{a + b/\left( 0.816497 \sqrt{K_s(D_2 - D_1)} \right)}{1 - \frac{c}{p}} .
\] (20)

Where

\[
K_s = \frac{P_n(z_0)}{z_0^2} + \frac{2R_1}{R_1 + R_2} \left( \frac{P_n(z_0)}{z_0^2} - \frac{P_{n+1}(z_0)}{z_0^2} \right), \quad z_0 = \frac{1}{\sqrt{1 - \epsilon^2}} .
\]

2. ANNULAR PRESSURE CALCULATION AND ANALYSIS OF AERATED DRILLING IN ECCENTRIC ANNULUS

Based on the eccentric annulus pressure drop formula derived from above equations, C# language is used to compile calculation program of aerated drilling annulus pressure and calculate aerated drilling annulus pressure. Combining with calculation data to prepare graphs and analyze variation regularity of annulus pressure, then the effect of different eccentricity on the annulus pressure can be obtained.

The basic parameters: The well type is vertical well; The well depth is 3,350 m; The calculation step is 60 m.

<table>
<thead>
<tr>
<th>Name</th>
<th>Data</th>
<th>Unit</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground temperature</td>
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<td></td>
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<tr>
<td>Geothermal gradient</td>
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<td>°C/100 m</td>
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<tr>
<td>Casing</td>
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<td>mm</td>
<td>Inner diameter</td>
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<tr>
<td>Drill pipe</td>
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<td>mm</td>
<td>Outer diameter</td>
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<td>Drill collar</td>
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<td>mm</td>
<td>Outer diameter</td>
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<tr>
<td>Liquid injection</td>
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<td>m³/min</td>
<td></td>
</tr>
<tr>
<td>Liquid density</td>
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<td>g/cm³</td>
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<tr>
<td>Gas relative density</td>
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<td>Air is 1.0</td>
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<tr>
<td>Cutting density</td>
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<td>g/cm³</td>
<td></td>
</tr>
<tr>
<td>Cuttings sphericity</td>
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<td>Sphericity is 1.0</td>
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</tr>
<tr>
<td>Cutting size</td>
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<td>mm</td>
<td></td>
</tr>
<tr>
<td>Wellhead back pressure</td>
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<td>MPa</td>
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</tbody>
</table>

For the sake of simplifying calculation process, we ignore the joint and do not consider the presence of downhole motor. The following calculation analysis is taking the constant eccentricity in the whole well section as example.

2.1 Pressure Calculation Results

Table 2

| Annulus Pressure Calculation Data (Eccentricity of 0.5,1.0 for Example) |
|-----------------------------|--------------|-----------------|----------------|
| Well depth (m) | Centricity | Eccentricity is 0.5 | Eccentricity is 1.0 |
| 0 | 0.20 | 0.20 | 0.20 |
| 240 | 1.01 | 0.96 | 0.86 |
| 480 | 2.02 | 1.93 | 1.74 |
| 720 | 3.24 | 3.10 | 2.83 |
| 960 | 4.63 | 4.44 | 4.09 |

To be continued
Well depth (m) | Centricity | Eccentricity is 0.5 | Eccentricity is 1.0
---|---|---|---
1,200 | 6.14 | 5.91 | 5.48
1,440 | 7.75 | 7.48 | 6.97
1,680 | 9.45 | 9.13 | 8.54
1,860 | 10.77 | 10.41 | 9.76
2,220 | 13.49 | 13.06 | 12.30
2,370 | 14.65 | 14.20 | 13.39
2,610 | 16.54 | 16.04 | 15.16
2,850 | 18.47 | 17.92 | 16.95
3,090 | 20.42 | 19.83 | 18.78
3,115 | 20.63 | 20.03 | 18.98
3,175 | 21.26 | 20.63 | 19.51
3,235 | 21.89 | 21.23 | 20.04
3,295 | 22.53 | 21.83 | 20.58
3,350 | 23.11 | 22.38 | 21.07

**2.2 Regular Analysis**

Table 3

<table>
<thead>
<tr>
<th>Eccentricity</th>
<th>Annulus bottomhole pressure MPa</th>
<th>Differential pressure compared with non eccentricity MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23.11</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>22.98</td>
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</tr>
<tr>
<td>0.2</td>
<td>22.80</td>
<td>-0.31</td>
</tr>
<tr>
<td>0.3</td>
<td>22.62</td>
<td>-0.49</td>
</tr>
<tr>
<td>0.4</td>
<td>22.38</td>
<td>-0.73</td>
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<tr>
<td>0.5</td>
<td>22.19</td>
<td>-0.41</td>
</tr>
<tr>
<td>0.6</td>
<td>21.92</td>
<td>-0.92</td>
</tr>
<tr>
<td>0.7</td>
<td>21.73</td>
<td>-1.38</td>
</tr>
<tr>
<td>0.8</td>
<td>21.48</td>
<td>-1.63</td>
</tr>
<tr>
<td>0.9</td>
<td>21.30</td>
<td>-1.81</td>
</tr>
<tr>
<td>1.0</td>
<td>21.07</td>
<td>-2.04</td>
</tr>
</tbody>
</table>

The eccentricity of 0, 0.5, 1.0 is taken as example, annulus pressure data under different eccentricity is drawn in the same figure.

**CONCLUSION**

Aerated drilling annulus pressure follows curve distribution, pressure gradually decreases from the bottom to wellhead, the cross-sectional area changing position which includes casing shoe and junction of drill pipe does not show pressure mutation, this is because the gravitational pressure drop is dominant in the annulus drop, liquid is incompressible fluid. Although the variation of gas volume, gas density is very small in contrast with liquid density, aerated liquid density has little change in the position of area variation cross-section. In the same conditions, the eccentric annulus flow pressure drop is less than the concentric annulus, and the greater eccentricity is, the smaller flow pressure drop will be.
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NOMENCLATURE

\( \rho_m \) Solid-liquid mixed density, \( \rho_m = \rho_s \phi + \rho_l (1-\phi) \), kg/m³

\( \phi \) Volume ratio of solid phase particles in solid-liquid mixed phase

\( \rho_g \) Density of gas phase

\( \theta \) The angle between borehole axis and horizontal direction

\( \lambda \) Multiphase flow frictional resistance coefficient

\( p \) Well section average pressure, MPa

\( z \) Well depth, m

\( A \) Annulus cross-section area, m²

\( D_e \) Annular equivalent diameter, m

\( G \) Gravitational acceleration, m/s²

\( H_l \) Liquid holdup

\( E_g \) Proosity

\( m \) Multiphase flow mass flow rate, m = m_s + m_l + m_g, kg/s

\( v \) Average velocity of multiphase fluid, m/s

\( v_{sg} \) Superficial gas velocity, m/s

\( D_1 \) The diameter of drill string, m

\( D_2 \) The borehole diameter, m

\( R_1 \) Radius of inner pipe (radius of drill sting), m

\( R_2 \) Radius of outer pipe (radius of borehole), m

\( K_f \) Flow ratio

REFERENCES


