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Stress Analysis of the Integrity of Casing-Cement Ring-Structural Consolidation Body by Volume Fracturing

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Abstract

For unconventional oil and gas reservoirs, the main body at home and abroad adopts segmented fracturing technology for cemented well completion bridge plugs, and the application ratio accounts for more than 85%. In order to improve the development effect of the tight oil and gas reservoirs in the Ordos Basin and increase the output of single wells, the horizontal wells in the Sulige tight gas reservoir adopt casing cementing well completion + bridge plug fracturing. The integrity between the casing-cement ring and the formation consolidation body is the precondition for ensuring the effectiveness of the segmented fracturing closure and the transformation effect. In this paper, the wellbore is subjected to alternating pressure when the volume of horizontal wells is compressed. By analyzing the force of the casing-cement-soil consolidated body and the temperature and casing internal pressure, the calculated relationship between the force of the consolidated body and the displacement of the first and second cemented surfaces was obtained. It provides an important reference for guiding the on-site construction design and follow-up research.

Key words: Consolidated body; Cementitious surfaces; Temperature; Casing pressure

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INTRODUCTION

The casing well is widely used in the petroleum industry, and the casing is frequently damaged during use. This is due to the rheological properties of the formation, under the action of ground stress, the casing is subjected to varying pressures and this pressure tends to be stable over a sufficiently long period of time, which is the casing load determined by the elastic mechanics method. It is important to study the stress distribution law of the cement ring, and to improve the integrity of the cement ring in high temperature and high pressure well, reduce the pressure risk of the ring air belt, and ensure the long-term safe production of gas well is of great significance[1-2]. In order to prevent casing damage, the stress distribution of casing-cement ring-formation in the field of study is very important in the petroleum industry. Yin Youquan et al. studied the casing load in creep formation in ground stress field by finite element method but did not consider the influence of cement ring. Fang Jun et al., using finite element numerical method, analyses the stress and deformation of casing-cement-ring structure underground stress. The numerical method can only be used to calculate the specific problems, and it is difficult to make an in-depth analysis on the results obtained, and give the conclusion of the regularity. Based on the derivation of yield criterion and the analysis of the mechanical state of the elastic region and plastic zone of the consolidation body, a method for calculating the stress of the first and two cementation surfaces is obtained. Based on the analysis method of temperature and casing pressure, the method for calculating the influence of bottom hole temperature on the state of consolidation strength is obtained, which provides a reference for future academic research.

1. STRESS ANALYSIS OF CONSOLIDATION BODY

1.1 Yield Criterion of Materials

Generally, the yield criterion of a material is determined by the test and different engineering materials have different yield criteria. The two yield criteria that we need to use are discussed below, namely, the Tresca criterion and the mole criteria. Generally, metal materials are listed in the former one, and geological materials such as soil, rock, concrete, etc. are listed in the latter one.

(a) Tresca criterion(Chen,2011)

The first yield criterion for the combined stress state of metallic materials is presented in the year. This yield criterion assumes that yielding occurs when the maximum shear stress at one point reaches the limit value. If the criterion is expressed in principal stress expression, the maximum value in half of the absolute absolute value of the three principal stresses at yield is reached, the mathematical expression of the criterion:

$$\max\left(\frac{1}{2}\left|\sigma_1-\sigma_2\right|,\frac{1}{2}\left|\sigma_2-\sigma_3\right|,\frac{1}{2}\left|\sigma_3-\sigma_1\right|\right)=k \quad (2\text{-}1)$$

If the material constant is determined by a single-axis test, the following relationship can be obtained:

$$k=\frac{\sigma_0}{2} \quad (2\text{-}2)$$

Where: σ_0— Single axle load yield stress, *MPa*。

(b) Molarity criterion (Chen, 2016)

The Moore criterion derived from 1990 is based on the assumption that the maximum shear stress is the yield-critical factor. The critical value of the shear force τ is not a constant, but it is a function of the stress σ in the same plane at that point.

$$\left|\tau\right|=h(\sigma) \quad (2\text{-}3)$$

Where: $h(\sigma)$— a function determined by the experiment.

The mathematical expression of this equation is:

$$\left|\tau\right|=c-\sigma\tan\varphi \quad (2\text{-}4)$$

Where: c-cohesion, *MPa*;

φ— internal friction angle, o;

Both of the parameters in the preceding equation are determined experimentally and the yield criterion in relation to Eq. 2-4 becomes the molar-Coulomb criterion, and its viscosity is equal to the yield stress in pure shear. Therefore, the Mohr-Coulomb criterion can be seen as a generalization of the Tresca rule.

Equation 2-4, at that time $\sigma_1\geq\sigma_2\geq\sigma_3$, the mole criteria can be written as:

$$\frac{1}{2}(\sigma_1-\sigma_3)\cos\varphi=c-\left[\frac{1}{2}(\sigma_1+\sigma_3)+\frac{\sigma_1-\sigma_3}{2}\sin\varphi\right]\tan\varphi \quad (2\text{-}5)$$

This equation can be converted to:

$$\frac{\sigma_1}{f_t'}-\frac{\sigma_3}{f_c'}=1,\sigma_1\geq\sigma_2\geq\sigma_3 \quad (2\text{-}6)$$

Where:

$$f_t'=\frac{2c\cos\varphi}{1+\sin\varphi},f_c'=\frac{2c\cos\varphi}{1-\sin\varphi} \quad (2\text{-}7)$$

The sum f_t' and f_c' in Eq. 2-7 is simple tensile and compression strength, and sometimes the following expression is convenient for many:

$$m\sigma_1-\sigma_3=f_c',\sigma_1\geq\sigma_2\geq\sigma_3 \quad (2\text{-}8)$$

Where:

$$m=\frac{f_c'}{f_t'}=\frac{1+\sin\varphi}{1-\sin\varphi} \quad (2\text{-}9)$$

1.2 Casing-Cement Ring-Formation Consolidation Stress Analysis

According to its own characteristics, it can be known that since the elastic modulus of casing is much larger than that of the cement ring and the elastic modulus of the formation, the casing shall first bear most of the internal pressure during the casing pressure test, and the cement sheath and the formation shall bear far less than the internal pressure of the casing. According to this process, we need to make the following provisions for the system combination[7]:

(a) The casing, cement ring and formation are elasto-plastic isotropic materials;

(b) The characteristics of the cement and formation rocks can be obtained through experiments, including cohesion and internal friction angle;

(c) The original site stress is the horizontal stress;

(d) The casing, cement ring and formation shall be closely connected and free from sliding to meet the continuous conditions of stress and displacement;

(e) The borehole is vertical and the casing is in good condition.

Based on the analysis of drilling and cementing operation process, considering different hole size and well body structure(8-1/ 2, 5-1/ 2),(6, 4-1/ 2),(118, 3-1/ 2)), stress distribution model of casing-cement-ring-stratum consolidation is carried out according to rock mechanics and elastic-plastic mechanics theory, as shown in Figure. 1-1, 1-2.

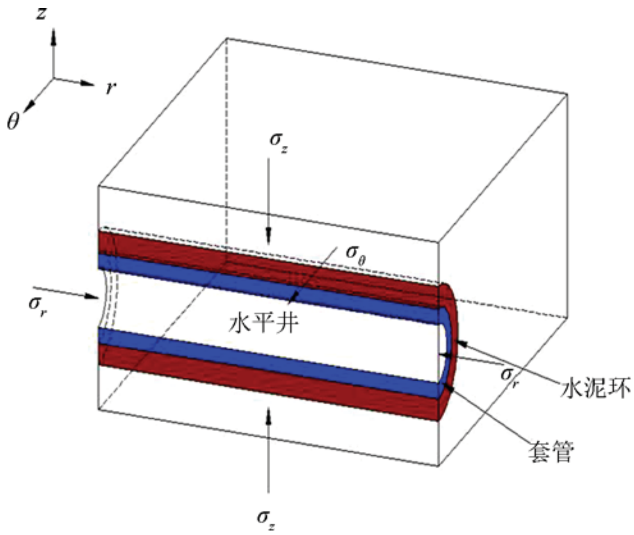

Figure 1-1
The Actual Three-Dimensional Mechanical Model

In the figure: the inside radius of the casing is r_1, m;The outer radius of the casing(i.e. the inside radius of the cement ring) is r_2,m;The radius of the hole(i.e. outside radius of the cement ring) is r_3,m;The formation radius(actual value) is r_4,m;The radius of plastic deformation of the sleeve is $r_{\rho 1}$,m.The radius of plastic deformation produced by the cement ring is $r_{\rho 2}$,mThe elastic modulus of the casing is P_1, MPa;Casing pressure is E_1,GPa;Sleeve Poisson's ratio is μ_1;Modulus of elasticity of cement ring is E_2,GPa Poisson's ratio of cement ring is μ_2;The elastic modulus of the formation is E_3,GPa;Poisson's Poisson's ratio is μ_3;The pressure q_1 MPa in the figure shows the inter-layer pressure between the outer wall of the casing and the inside diameter of the cement ring,q_4, MPa is the inter-layer pressure between the outer wall of the cement ring and the inside diameter of the hole, q_2,MPa is the stress on the plastic radius of the casing, q_3,MPa is the stress on the plastic radius of the cement ring,q_5,MPaIs the stress on the plastic radius of the formation.

1.2.1 Stress Analysis Of Casing Area

In order to make a more convenient representation, we break down the casing part of the combination system separately and make proper amplification as shown in the Figure 1-3

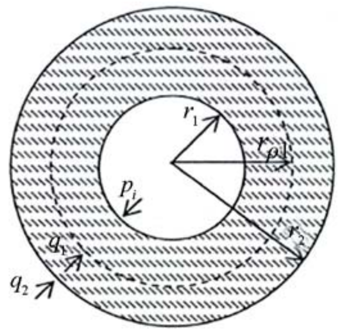

Figure 1-3
Schematic Diagram of Elastic-plastic Area of Casing

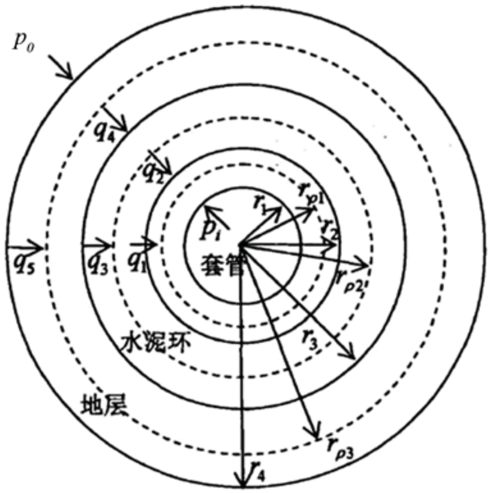

Figure 1-2
Combined System Elastoplastic Analysis Chart

Because this is an axisymmetric problem, all shear stress and shear strain are zero and σ_z=0can be known. The equation of balance equation and strain displacement is represented by:

$$\begin{cases} \dfrac{d\sigma_r}{dr} + \dfrac{\sigma_r - \sigma_\theta}{r} = 0 \\ \varepsilon_r = \dfrac{du_r}{dr}, \varepsilon_\theta = \dfrac{u_r}{r} \end{cases} \quad (2\text{-}10)$$

These control equations shall be solved using the following boundary conditions:

$$\begin{cases} \sigma_{r(r=r_1)} = -p_i \\ \sigma_{r(r=r_2)} = -q_2 \end{cases} \quad (2\text{-}11)$$

As the pressure in the casing is considered to be increasing, the results $\sigma_r \leq 0$且$\sigma_\theta > 0$,$\sigma_\theta > \sigma_z = 0 \geq \sigma_r$ can be observed at this time. Thus, in this case, the Tresca criterion is expressed as follows:

$$\sigma_\theta - \sigma_r = \sigma_0 \quad (2\text{-}12)$$

When the pressure p_i in the casing is greater than the maximum stress caused by plastic deformation of the casing, plastic deformation begins to occur at the inner wall of the casing. As the pressure inside the casing increases, the elasto-plastic boundary will move slowly toward the outer wall of casing. $r_{\rho 1}$ is used to represent the elasto-plastic boundary of the casing.

(a) Stress analysis on plastic area of casing

At that time $r_1 \leq r \leq r_{\rho 1}$, the casing was in plastic state. The balance equation becomes:

$$\frac{d\sigma_r^{套管(1)}}{dr} + \frac{\sigma_0}{r} = 0 \quad (2\text{-}13)$$

The differential equation of solution 2-13 is obtained by using the boundary conditions shown in Eq. 2-11:

$$\begin{cases} \sigma_r^{套管（1）} = \sigma_0 \ln \dfrac{r}{r_1} - p_i \\ \sigma_\theta^{套管（1）} = \sigma_0 \left(1 + \ln \dfrac{r}{r_1}\right) - p_i \end{cases} \tag{2-14}$$

Equation 2-14 is the stress component in the plastic zone of the sleeve.

(b) Stress analysis on elastic region of casing

At that time, when the sleeve is in the elastic region, the relation between elastic stress and strain can be applied:

$$\begin{cases} \sigma_r = \dfrac{A}{r^2} + B \\ \sigma_\theta = -\dfrac{A}{r^2} + B \end{cases} \tag{2-15}$$

Where: A and B are arbitrary constants.

The boundary conditions where the stress state $r=r_2$ of the elastic region must be satisfied, Eq. 2-14 is $r = r_{\rho 1}$ continuous at:

$$\sigma_{r(r=r_{\rho 1})}^{套管（1）} = \sigma_{r(r=r_{\rho 1})}^{套管（2）} \tag{2-16}$$

Where: A superscript(a) is the stress component in the plastic zone, and the superscript(b) is the stress component in the elastic region.

Using the boundary conditions in the elastic region, the two constants in Equation 2-15 can be determined as follows:

$$\begin{cases} \sigma_r^{套管（2）} = \dfrac{(q_2 - q_1) r_2^2}{r_2^2 - r_{\rho 1}^2} \left(\dfrac{r_{\rho 1}^2}{r^2} - 1 \right) - q_1 \\ \sigma_\theta^{套管（2）} = -\dfrac{(q_2 - q_1) r_2^2}{r_2^2 - r_{\rho 1}^2} \left(\dfrac{r_{\rho 1}^2}{r^2} + 1 \right) - q_1 \end{cases} \tag{2-17}$$

The elastic region, under the action of pressure, satisfies the Tresca criterion at the boundary of elasto-plastic, which can be simplified as follows:

$$\begin{cases} \sigma_r^{套管（2）} = \dfrac{\sigma_0}{2} \left(\dfrac{r_{\rho 1}^2}{r^2} - 1 \right) - q_1 \\ \sigma_\theta^{套管（2）} = \dfrac{\sigma_0}{2} \left(\dfrac{r_{\rho 1}^2}{r^2} + 1 \right) - q_1 \end{cases} \tag{2-18}$$

The stress solution of elastic region is given in Equation 2-18, when it is obtained:

$$q_2 = \frac{\sigma_0}{2} \left(\frac{r_{\rho 1}^2}{r_2^2} - 1 \right) - q_1 \tag{2-19}$$

Due to the known stress in the elastic region, the radial displacement of the outer diameter of the elastic region of the casing can be obtained:

$$u_{r(r=r_2)}^{套管} = \frac{1 + \mu_1}{E_1 \left(r_2^2 - r_{\rho 1}^2 \right)} \left\{ \left[(1 - 2\mu_1) r_{\rho 1}^2 r_2 + \frac{r_{\rho 1}^2 r_2^2}{r_2^2} \right] q_1 - \left[(1 - 2\mu_1) r_2^2 r_{\rho 1} + \frac{r_{\rho 1}^2 r_2^2}{r_2^2} \right] q_2 \right\} \tag{2-20}$$

1.2.2 Stress Analysis of Cement Ring Area

The cement ring is subjected to various complex conditions downhole, especially when the confining pressure is high, the cement stone will present elasto-plastic state, and the stress analysis chart of the cement ring is enlarged as shown in Figure1-4:

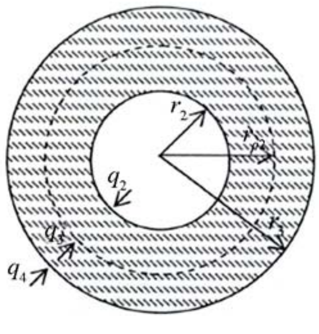

Figure 1-4
Schematic Diagram of Elastic Zone of Cement Ring

In the same way as analysis of the elastic-plastic method of casing, different cement stone yield criteria adopt the molar hydrogen yield criterion, that is:

$$f(\sigma) = \frac{1}{2}(\sigma_1 - \sigma_3)\cos\varphi - c - \left[\frac{1}{2}(\sigma_1 + \sigma_3) + \frac{\sigma_1 - \sigma_3}{2} \sin\varphi \right] \tan\varphi = 0 \tag{2-21}$$

Where: σ_1— maximum principal stress,*MPa*;

σ_3— minimum principal stress, *MPa*;

c-Cohesive force obtained from the cement stone experiment,MPa

φ— Internal friction angle, o.

When taken $q_2 \geq q_4$, Formula 2-21 can be simplified as:

$$f(\sigma)=\frac{1}{2}(\sigma_\theta-\sigma_r)+\frac{1}{2}(\sigma_\theta+\sigma_r)\sin\varphi-c\cos\varphi=0 \tag{2-22}$$

In the case $q_2 < q_4$ of Eq. 2-22 also satisfied.

The stress-strain relationship 2-15 in the elastic region of the casing just studied can also be applied to the cement-ring area:

$$f(\sigma)=-\frac{A}{r^2}+B\sin\varphi-c\cos\varphi=0$$

At that time $\frac{\partial f}{\partial r}<0$, the yield condition started from the inner wall was started, $r = r_2$,and the elastic limit of the cement ring was:

$$f(\sigma)=-\frac{A}{r_2^2}+B\sin\varphi-c\cos\varphi=0 \tag{2-23}$$

On-top instruction: when the stress of the inner wall of the cement ring exceeds the elastic limit, the plasticity shall be started from the inner wall of the cement ring, as shown in the previous section, since the cement ring is an axisymmetric drawing, the elastic-plastic interface is Q345B, and the plastic radius is $r_{\rho 2}$'at that time $r_2 \leq r \leqslant r_{\rho 2}$, and the plastic state is at that time $r_{\rho 2} \leq r \leq r_3$. They are in an elastic state.

(a) Stress analysis of plastic area of cement ring

In the plastic zone of the cement annulus, the stress needs to meet the equilibrium equation and the yield criterion. It can be seen from Eq. 2-8:

$$\sigma_r^{水泥环(1)}=m\sigma_\theta^{水泥环(1)}-f_c' \tag{2-24}$$

Substituting Equation 2-24 into the equilibrium equation:

$$\frac{d\sigma_r^{水泥环(1)}}{dr}+\frac{\sigma_\theta^{水泥环(1)}-\sigma_r^{水泥环(1)}}{r}=0 \tag{2-25}$$

To:

$$\frac{d\sigma_r^{水泥环(1)}}{r}+\frac{1-m}{r}\sigma_r^{水泥环(1)}+\frac{f_c'}{r}=0 \tag{2-26}$$

Solving:

$$\sigma_r^{水泥环(1)}=cr^{m-1}-\frac{f_c'}{1-m} \tag{2-27}$$

It can be known by boundary conditions that:

$$c=\left(\frac{f_c'}{1-m}-q_2\right)r_2^{1-m} \tag{2-28}$$

Substituting Equation 2-28 into Equation 2-27:

$$\sigma_r^{水泥环(1)}=\left(\frac{f_c'}{1-m}-q_2\right)\left(\frac{r}{r_2}\right)^{m-1}-\frac{f_c'}{1-m} \tag{2-29}$$

Substituting Equation 2-29 into Equation 2-24, that is:

$$\sigma_\theta^{水泥环(1)}=\frac{\left(\frac{f_c'}{1-m}-q_2\right)\left(\frac{r}{r_2}\right)^{m-1}-\left(\frac{-m}{1-m}\right)f_c'}{m} \tag{2-30}$$

The values m and $f'_c$are sorted into 2-29 and 2-30 respectively:

$$\begin{cases}\sigma_r^{水泥环(1)}=c\cot\varphi\left[1-\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{\frac{-2\sin\varphi}{1+\sin\varphi}}\right]\\ \sigma_\theta^{水泥环(1)}=c\cot\varphi\left[1-\frac{1-\sin\varphi}{1+\sin\varphi}\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{\frac{-2\sin\varphi}{1+\sin\varphi}}\right]\end{cases} \tag{2-31}$$

at that time $q_2 < q_4$ The stress distribution in the plastic zone of the cement annulus can be calculated using the same method:

$$\begin{cases}\sigma_r^{水泥环(1)}=c\cot\varphi\left[1-\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{\frac{2\sin\varphi}{1-\sin\varphi}}\right]\\ \sigma_\theta^{水泥环(1)}=c\cot\varphi\left[1-\frac{1+\sin\varphi}{1-\sin\varphi}\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{\frac{2\sin\varphi}{1-\sin\varphi}}\right]\end{cases} \tag{2-32}$$

Combine Formula 2-31 and Equation 2-32 into a formula:

$$\begin{cases}\sigma_r^{水泥环(1)}=c\cot\varphi\left[1-\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{M-1}\right]\\ \sigma_\theta^{水泥环(1)}=c\cot\varphi\left[1-M\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{M-1}\right]\end{cases} \tag{2-33}$$

Of which, at that time $q_2 \geq q_4$, $M=\frac{1}{m}$;At that time $q_2 < q_4$.M=m The stress on the elastic buckling of the cement ring is as follows:

$$q_3=c\cot\varphi\left[\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r_{\rho 2}}{r_2}\right)^{M-1}-1\right] \tag{2-34}$$

(b) Stress analysis on elastic region of cement ring

In the same way as the elastic region analysis method of the casing, we can see that:

$$\begin{cases} \sigma_r^{水泥环(2)} = \dfrac{\left(q_4 - q_3\right) r_3^2}{r_3^2 - r_{\rho 2}^2}\left(\dfrac{r_{\rho 2}^2}{r^2} - 1\right) - q_3 \\ \sigma_\theta^{水泥环(2)} = -\dfrac{\left(q_4 - q_3\right) r_3^2}{r_3^2 - r_{\rho 2}^2}\left(\dfrac{r_{\rho 2}^2}{r^2} + 1\right) - q_3 \end{cases} \quad (2\text{-}35)$$

At the point where the inner boundary of the elastic region of the cement annulus is the same as the outside surface of the plastic area, $r_{\rho 2}$ we can see that:

$$\begin{cases} \sigma_{r(r=r_{\rho 2})}^{水泥环(2)} = -q_3 \\ \sigma_{\theta(r=r_{\rho 2})}^{水泥环(2)} = -\dfrac{2\left(q_4 - q_3\right) r_3^2}{r_3^2 - r_{\rho 2}^2} - q_3 \end{cases} \quad (2\text{-}36)$$

Substituting Equation 2-36 into the yield criterion, Eq. 2-22:

$$q_4 = \frac{q_3\left(r_{\rho 2}^2 \sin\varphi + r_3^2\right) + c\cos\varphi\left(r_{\rho 2}^2 - r_3^2\right)}{r_3^2\left(1 + \sin\varphi\right)} \quad (2\text{-}37)$$

(c) Displacement in plastic zone of cement ring

According to formula 2-24, the plastic potential function of molar hydrogen conductivity can be written as follows:

$$g = \beta^* \sigma_1 - \sigma_3 - f^*$$

Where: f^*— constant;

β^*— Expansion factor, $\beta^* = \left(1 + \sin\varphi^*\right) / \left(1 - \sin\varphi^*\right)$。

φ^*— shear expansion angle, o;Internal friction angle is generally less than or equal to internal friction angle. At that time φ^*=0, β^*=1 the plastic flow rule was used:

$$\varepsilon^P = d\lambda \frac{\partial g}{\partial \sigma_1} + d\lambda \frac{\partial g}{\partial \sigma_2} + d\lambda \frac{\partial g}{\partial \sigma_3} = 0$$

That means:

$$\varepsilon^P = \varepsilon_r^P + \varepsilon_z^P + \varepsilon_\theta^P = 0$$

Therefore:

$$\varepsilon = \left(\varepsilon_r^P + \varepsilon_r^e\right) + \left(\varepsilon_z^P + \varepsilon_z^e\right) + \left(\varepsilon_\theta^P + \varepsilon_\theta^e\right) = \varepsilon_r^e + \varepsilon_z^e + \varepsilon_\theta^e$$

The strain in the plastic zone according to the law of volume elasticity is:

$$\varepsilon_r^e + \varepsilon_z^e + \varepsilon_\theta^e = \left(\frac{1 - 2\mu_2}{E_2}\right)\left(\sigma_r + \sigma_\theta + \sigma_z\right) \quad (2\text{-}38)$$

As this model ignores axial direction factors, it is:ε_z=0 According to the above formula, the plastic flow rule in the direction is as follows:

$$d\varepsilon_z^P = d\lambda \frac{\partial g}{\partial \sigma_z} = 0 \quad (2\text{-}39)$$

Where:$d\lambda$ — the size of the plastic strain increment, which is a non-negative scalar factor.

Therefore, according to the generalized Hug law in the plastic zone:

$$\sigma_z = \mu_2\left(\sigma_r + \sigma_\theta\right) \quad (2\text{-}40)$$

Substituting Equation 2-40 into Equation 2-38:

$$\varepsilon_r + \varepsilon_\theta = \frac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right)}{E_2}\left(\sigma_r + \sigma_\theta\right) \quad (2\text{-}41)$$

Substituting Equation 2-33 into Equation 2-41:

$$\frac{du^P}{dr} + \frac{u^P}{r} = \frac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right) c\cot\varphi}{E_2}\left[2 - \left(1 + \frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{M-1}\left(M + 1\right)\right] \quad (2\text{-}42)$$

Points are to be integrated on:

$$u_r^P = \frac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right) c\cot\varphi}{E_2}\left[1 - \left(1 + \frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{M-1}\right] r + \frac{C}{r} \quad (2\text{-}43)$$

The displacement of the plastic zone can be obtained by Eq. 2-43, i.e.:

$$\begin{cases} u_{r(r=r_2)}^P = \dfrac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right) c\cot\varphi}{E_2}\left[1 - \left(1 + \dfrac{q_2}{c\cot\varphi}\right)\right] r_2 + \dfrac{C}{r_2} \\ u_{r(r=r_{\rho 2})}^P = \dfrac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right) c\cot\varphi}{E_2}\left[1 - \left(1 + \dfrac{q_2}{c\cot\varphi}\right)\left(\dfrac{r_{\rho 2}}{r_2}\right)^{M-1}\right] r_{\rho 2} + \dfrac{C}{r_{\rho 2}} \end{cases} \quad (2\text{-}44)$$

Substituting Equation 2-34 into Equation 2-44:

$$u_{r(r=r_{\rho 2})}^P = -\frac{\left(1 - 2\mu_2\right)\left(1 + \mu_2\right) q_3 r_{\rho 2}}{E_2} + \frac{C}{r_{\rho 2}} \quad (2\text{-}45)$$

Where 2-44 and 2-45 are the displacement of the boundary in the plastic zone of the cement annulus and the displacement of the plastic outside boundary of the cement annulus, respectively.

(d) Displacement in elastic region of cement ring

According to the expression of plane radial displacement, the displacement in the elastic region of the cement ring can be obtained:

$$u_r^{水泥环(2)} = \frac{1+u_2}{E_2\left(r_3^2 - r_{\rho 2}^2\right)}\left\{\left[\left(1-2\mu_2\right)r_{\rho 2}^2 r + \frac{r_{\rho 2}^2 r_3^2}{r}\right]q_3 - \left[\left(1-2\mu_2\right)r_3^2 r + \frac{r_{\rho 2}^2 r_3^2}{r}\right]q_4\right\} \tag{2-46}$$

It can be seen from Eq. 2-46 that the radial displacement at the position $r_{\rho 2}$ is:

$$u_{r(r=r_{\rho 2})}^{水泥环(2)} = \frac{1+u_2}{E_2\left(r_3^2 - r_{\rho 2}^2\right)}\left\{\left[\left(1-2\mu_2\right)r_{\rho 2}^3 + r_{\rho 2} r_3^2\right]q_3 - \left[\left(1-2\mu_2\right)r_3^2 r_{\rho 2} + r_{\rho 2} r_3^2\right]q_4\right\}$$

Substituting Equation 2-37 into the previous equation:

$$u_{r(r=r_{\rho 2})}^{水泥环(2)} = \frac{\left(1+u_2\right)r_{\rho 2}}{E_2\left(1+\sin\varphi\right)}\left[\left(2\mu_2 - 1 + \sin\varphi\right)q_3 + c\cos\varphi\left(2-2\mu_2\right)\right] \tag{2-47}$$

In the same way, the radial displacement at the positioncan r_3 be obtained,

$$u_{r(r=r_3)}^{水泥环(2)} = \frac{1+u_2}{E_2\left(r_3^2 - r_{\rho 2}^2\right)}\left[\left(2-2\mu_2\right)r_{\rho 2}^2 r_3\right] + \frac{\left(1+\mu_2\right)\left(q_3\sin\varphi + c\cot\varphi\right)}{E_2\left(1+\sin\varphi\right)} \times \left[\left(1-2\mu_2\right)r_3 + \frac{r_{\rho 2}^2}{r_3}\right] \tag{2-48}$$

At this time, according to formula 2-45 and formula 2-47, find out:

$$C = \frac{\left(1+\mu_2\right)\left(2-2\mu_2\right)r_{\rho 2}^2}{E_2\left(1+\sin\varphi\right)}\left[q_3 + c\cos\varphi\right] \tag{2-49}$$

Substituting Eq. 2-49 in Eq. 2-43 gives plastic displacements of the plastic area of the cement annulus:

$$u_{(r)}^{水泥环(1)} = \frac{\left(1-2\mu_2\right)\left(1+u_2\right)c\cot\varphi}{E_2}\left[1-\left(1+\frac{q_2}{c\cot\varphi}\right)\left(\frac{r}{r_2}\right)^{M-1}\right]r + \frac{\left(1+\mu_2\right)\left(2-2\mu_2\right)r_{\rho 2}^2}{E_2\left(1+\sin\varphi\right)r} \times \left[q_3 + c\cos\varphi\right] \tag{2-50}$$

1.2.3 Force Analysis of Stratum Area

Since the stratum area is relatively wide, it is difficult to conduct modeling calculation in practice, so the radius of formation radius is limited to finite value. The stress analysis chart is shown in Figure 1-5:

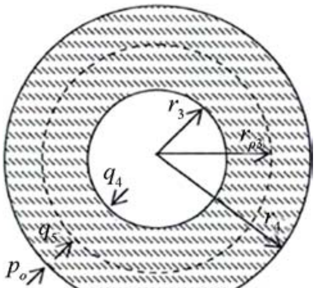

Figure 1-5
Schematic Diagram of Stratum Area

(a) Stress analysis of plastic area

at that time $r_3 \le r \le r_{\rho 3}$The cement annulus was then in the plastic area. The calculation method in the plastic zone is similar to that of the cement ring, so the stress in the formation in the plastic area is obtained. They are as follows:

$$\begin{cases} \sigma_r^{地层(1)} = c\cot\varphi\left[1-\left(1+\frac{q_4}{c\cot\varphi}\right)\left(\frac{r}{r_3}\right)^{M-1}\right] \\ \sigma_\theta^{地层(1)} = c\cot\left[1-M\left(1+\frac{q_4}{c\cot\varphi}\right)\left(\frac{r}{r_3}\right)^{M-1}\right] \end{cases} \tag{2-51}$$

Of which, at that time $q_4 \ge p_0$ $M = \frac{1}{m}$;At that time $q_4 <$ p_0 $M=m$. So we can know that the stress on the elastic-plastic elastic modulus of the formation is:

$$q_5 = c\cot\varphi\left[\left(1+\frac{q_4}{c\cot\varphi}\right)\left(\frac{r}{r_3}\right)^{M-1} - 1\right] \tag{2-52}$$

Similarly, the displacement of the plastic zone of the formation is:

$$u_{(r)}^{地层(1)} = \frac{\left(1-2\mu_3\right)\left(1+u_3\right)c\cot\varphi}{E_2}\left[1-\left(1+\frac{q_4}{c\cot\varphi}\right)\left(\frac{r}{r_3}\right)^{M-1}\right]r + \frac{\left(1+\mu_3\right)\left(2-2\mu_3\right)r_{\rho 3}^2}{E_3\left(1+\sin\varphi\right)r} \times \left[q_5 + c\cos\varphi\right] \tag{2-53}$$

(b) Stress analysis of elastic region

Based on the boundary conditions of the elastic region, the internal stress of the elastic region can be obtained, that is,

$$\begin{cases} \sigma_{r(r=r_{\rho 3})}^{水泥环(2)} = -q_5 \\ \sigma_{\theta(r=r_{\rho 3})}^{水泥环(2)} = -\dfrac{2(p_o - q_5) r_4^2}{r_4^2 - r_{\rho 3}^2} - q_5 \end{cases} \tag{2-54}$$

Substituting Equation 2-54 into Equation 2-22:

$$p_o = \frac{q_4 \left(r_{\rho 3}^2 \sin\varphi + r_4^2 \right) + c\cos\varphi \left(r_{\rho 3}^2 - r_4^2 \right)}{r_4^2 \left(1 + \sin\varphi \right)} \tag{2-55}$$

So far, it is possible to calculate the stress values of the elastic zones and plastic zones of the casing, the cement ring and the formation, and the displacement of the first cement surface and the second cementing surface.

CONCLUSION

(a) Through the yield criterion of the material, the relation between the force and the displacement of the first two interfaces is derived.

(b) The theoretical formula shows that as the pressure at the wellhead increases, the radial stress and circumferential stress of the cement stone increase, and the micro-annulus of the first cementation surface also increases.

(c) The theoretical formula shows that with the increase of the elastic modulus of the cement stone, the radial stress of the cement stone is decreasing, but the decreasing trend is gradually slow. Its circumferential stress also decreases, and the decreasing trend is obvious.

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