CSCanada
Advances in Natural Science
Vol. 5, No. 2, 2012, pp. 100-114
DOI:10.3968/j.ans.1715787020120502.1835

ISSN 1715-7862 [PRINT]
ISSN 1715-7870 [ONLINE]
www.cscanada.net
www.cscanada.org

Prigogine's Dissipative Structures -- A Haimovician Analysis (Part I)

K.N. Prasanna Kumar[a],*; B.S. Kiranagi[b]; C.S. Bagewadi[c]

[a] Department of studies in Mathematics, Kuvempu University, Shimoga, Karnataka, India.
[b] Department of studies in Mathematics, Manasagangotri, University of Mysore, Karnataka, India.
[c] Chairman of Department of Mathematics and Department of Studies in Computer Science, Jnanasahyadri Kuvempu University, Shankarghatta, Shimoga district, Karnataka, India.
*Corresponding author.

Received 1 August 2011; accepted 14 September 2011

Abstract
A system of terrestrial organisms dissipating consumption of oxygen due to cellular respiration and parallel system of consumption of oxygen due to cellular respiration that contribute to the dissipation of the velocity of production of terrestrial organisms is investigated. It is shown that the time independence of the contributions portrays another system by itself and constitutes the equilibrium solution of the original time independent system. A system of dead organic matter that reduces the dissipation coefficient of the decomposer organism is annexed to the oxygen consumption-terrestrial organism system. With the methodology reinforced with the explanations, we write the governing equations with the nomenclature for the systems in the foregoing. Further papers extensively draw inferences upon such concatenation process, ipsofacto.

Key words: Dissipative structures; Prigogine; Plants-nutrients; Dead organic matter; Decomposer organisms

K.N. Prasanna Kumar, B.S. Kiranagi, C.S. Bagewadi (2012). Prigogine's Dissipative Structures -- A Haimovician Analysis (Part I). *Advances in Natural Science*, *5*(2), 100-114. Available from URL: http://www.cscanada.net/index.php/ans/article/view/j.ans.1715787020120502.1835
DOI: http://dx.doi.org/10.3968/j.ans.1715787020120502.1835

INTRODUCTION
In his celebrated paper Adolf Haimovici [1], studied the growth of a two species ecological system divided on age groups. In this paper, we establish that his processual regularities and procedural formalities can be applied for consummation of a system of oxygen consumption by terrestrial organisms. Notations are changed towards the end of obtaining higher number of equations in the holistic study of the global climate models. Quintessentially, Haimovician diurnal dynamics, are used to draw interesting inferences, from the simple fact that terrestrial organisms consume oxygen due to cellular respiration.

Fritjof Capra [2] in his scintillating and brilliant synthesis of such scientific breakthroughs as the "Theory of Dissipative structures", "Theory of Complexity", "GAIA theory", "Chaos Theory" in his much acclaimed "The Web of life" elucidates dissipative structures as the new paradigm in ecology.

Heylighen F.[3] also concretises the necessity of self-organization and adaptability. Matsuit, *et al.*[4] made a satellite based assessment of marine low cloud variability, atmospheric stability and diurnal cycle. Steven's B., Feingold G.[5] studied untangling aerosol effects on clouds and precipitation in a buffered system. Illan Koren and Graham Feingold [6] studied the cellular cloud precipitation system and corresponding oscillations generated thereof. One other study that eminently calls for such a study of application is by R. Wood[7] in which he studied the loss of cloud droplets by coalescence in warm clouds. On the same lines the investigation of Xue H., Fiengold G., where in indirect effects of aerosol on large eddy simulations of trade wind provides a rich repository and fertile ground for prosecution of investigation based on our theoretical analysis. Aerosol effects on clouds itself is a pointer to the food cycle --- dissipative structure discussed by Prigogine.

All the studies centre on the possibility of application of Haimovician analysis to "dissipative structures". In this paper we study the following systems:

(a) Oxygen consumption - Terrestrial organism.

(b) Dead organic matter - Decomposer organisms.

We elucidate the governing equations of (b). Methodology for obtaining of solution follows from the one herein given.

In the next part we analyze the following systems:

(c) Plant investment - Nutrients.

(d) Solar radiation - Chemical process.

(e) Systems structure - Change.

Green plants play a vital role in the flow of energy through all ecological cycles. Their roots take in water and mineral salts from the earth, and the resultant juices rise up to the leaves, where they combine with CO_2 from air leading to the formulation of sugar and other organic compounds. Here solar energy is converted into chemical energy and encapsulated in organic substances, while oxygen is released in air to be taken up again by other plants and by animals in the process of cellular respiration. By the blend of water and minerals with sunlight and CO_2, green plants form link between earth and sky. Bulk of cellulose and the other organic compounds produced through photosynthesis consists of heavy carbon and oxygen atoms, which plants take directly from the air in the form of CO_2 .Thus the weight of a wooden log comes almost entirely from air. A log burnt, combines oxygen and carbon combine once more in to CO_2, and in the light and heat of fire is recovered part of the solar energy that went into making the wood.

As terrestrial organisms dissipate oxygen in the atmosphere, due to cellular respiration the plants nutrients are passed through the food web, while energy is dissipated as heat through respiration and as waste through excretion. Dead animals and plants are disintegrated by decomposer organisms, which break them into basic nutrients to be taken up by plants. Nutrients and other basic elements continually cycle through the ecological system, while energy is dissipated at each stage in accord with Eugene Odum's dictum "matter circulates, energy dissipates". Waste generated by the ecological system as a whole is the heat energy of cellular respiration, which is radiated into the atmosphere and is reimbursed continually by photosynthesis.

Prigogine's theory interlinks and entangles the main characteristics of living forms in to a coherent, cogent conceptualization and mathematical framework. We give a model for his framework. Perhaps the most fundamental necessity of the systemic dynamics is the optimality considerations. Taking cognizance of the critical issues involved emphasizes need for setting out dynamic programming in order to capture systemic structural changes.

Axiomatic predications of systemic dynamics in question are essentially "laws of accentuation and dissipation". It includes once over change, continuing change, process of change, functional relationships, predictability, cyclical growth, cyclical fluctuations, speculation theory, cobweb analyses, stagnation thesis, perspective analysis etc. Upshot of the above statement is data produce consequences and consequences produce data.

OXYGEN CONSUMPTION DUE TO CELLULAR RESPIRATION

Assumptions

a) Oxygen Consumption due to cellular respiration are classified into three categories;
   1) Category 1 representative of the consumption due to cellular respiration in the first interval vis-à-vis category1 of terrestrial organisms.
   2) Category 2 (second interval) comprising of consumption due to cellular respiration corresponding to category 2 of terrestrial organisms.
   3) Category 3 constituting consumption due to cellular respiration which belong to higher age than that of category 1 and category 2. This is concomitant to category 3 of terrestrial organism.

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the terrestrial organisms made out of the total oxygen consumption due to cellular respiration would be in the fitness of things. For category 3. "Over and above" nomenclature could be used to encompass a wider range of consumption due to cellular respiration. Similarly, a "less than" scale for category 1 can be used.

b) The speed of growth of oxygen consumption due to cellular respiration under category 1 is proportional to the total amount of Oxygen consumption due to cellular respiration under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between consumption due to cellular respiration under category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable.

c) The dissipation in all the three categories is attributable to the following two phenomenon:
   1) Aging phenomenon: The aging process leads to transference of the balance of oxygen consumption due to cellular respiration to the next category, no sooner than the age of the terrestrial organism crosses the boundary of demarcation.
   2) Depletion phenomenon: Death of consumer viz-terrestrial organism dissipates the growth speed by an equivalent extent. The model is not concerned

with the end uses of consumption due to cellular respiration – dissipation other than for terrestrial organisms

Notations

G_{13}: Quantum of oxygen consumption due to cellular respiration in category
1 of terrestrial organism
G_{14}: Quantum of oxygen consumption due to cellular respiration in category
2 of terrestrial organism
G_{15}: Quantum of oxygen consumption due to cellular respiration in category
3 of terrestrial organism
$(a_{13})^{(1)}, (a_{14})^{(1)}, (a_{15})^{(1)}$ Accentuation coefficients
$(a'_{13})^{(1)}, (a'_{14})^{(1)}, (a'_{15})^{(1)}$ Dissipation coefficients

Formulation of the System

In the light of the assumptions stated in the foregoing, we infer the following:

(a) The growth speed in category 1 is the sum of a accentuation term $(a_{13})^{(1)}G_{14}$ and a dissipation term
$-(a'_{13})^{(1)}G_{13}$, the amount of dissipation taken to be proportional to the total quantum of oxygen consumption due to cellular respiration in the concomitant category of terrestrial organisms.

(b) The growth speed in category 2 is the sum of two parts $(a_{14})^{(1)}G_{13}$ and $-(a'_{14})^{(1)}G_{14}$ the inflow from the category 1 dependent on the total amount standing in that category.

(c) The growth speed in category 3 is equivalent to $(a_{15})^{(1)}G_{14}$ and $-(a'_{15})^{(1)}G_{15}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new quantum of oxygen consumption due to new entrants in terrestrial organisms and deceleration in the oxygen consumption attributable and ascribable to death of terrestrial organisms.

Governing Equations

The differential equations governing the above system can be written in the following form

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13} \quad (1)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14} \quad (2)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15} \quad (3)$$

$$(a_i)^{(1)} > 0 \quad , \qquad i = 13,14,15 \quad (4)$$

$$(a'_i)^{(1)} > 0 \quad , \qquad i = 13,14,15 \quad (5)$$

$$(a_{14})^{(1)} < (a'_{13})^{(1)} \quad (6)$$

$$(a_{15})^{(1)} < (a'_{14})^{(1)} \quad (7)$$

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = dt \quad (8)$$

$$\frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = dt \quad (9)$$

Or we write a single equation as

$$\frac{dG_{13}}{(a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}} = \frac{dG_{14}}{(a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}} = \frac{dG_{15}}{(a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15}} = dt \quad (10)$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α ,β ,γ all positive we can write equation (10) as

$$\frac{\alpha dG_{13}}{\alpha\left((a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13}\right)} = \frac{\beta dG_{14}}{\beta\left((a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14}\right)} = \frac{\gamma dG_{15}}{\gamma\left((a_{15})^{(1)}G_{14} - (a'_{15})^{(1)}G_{15}\right)} = dt \quad (11)$$

The general solution of the consumption of oxygen due to cellular respiration system can be written in the form $\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$ Where i=13,14,15 and C_{13},C_{14},C_{15} are arbitrary constant coefficients.

Stability Analysis

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} > 0$ all the components of the solution, ie all the three parts of the consumption of oxygen due to cellular respiration tend to zero, and the solution is stable with respect to the initial data.

2. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $\left(\lambda_{14} + (a'_{13})^{(1)}\right)G_{13}^0 - (a_{13})^{(1)}G_{14}^0 \neq 0, (\lambda_{14} < 0)$, the first two components of the solution tend to infinity as t→∞, and G_{15}→0, i. e. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 consumption of oxygen due to cellular respiration tends to zero.

3. If $(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$ and $\left(\lambda_{14} + (a'_{13})^{(1)}\right)G_{13}^0 - (a_{13})^{(1)}G_{14}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

Actual food cycles can be understood on a much broader canvass, in which nutrient elements appear in a variety of chemical compounds. Gaia theory, has refined indications of interweaving of living and non living systems throughout the biosphere. Key to comprehension of such dissipative structures is that these systems maintain themselves in a "stable state" far from equilibrium. For instance chemical and thermal equilibrium exists when all these processes come to a halt. Organism in equilibrium is a dead organism. Living organisms, like terrestrial organisms, continually maintain themselves in a state far from equilibrium. Notwithstanding the fact, that such a maintained state is stable over a period of time, the same overall holistic structure is maintained, despite continual ongoing flow and change of components.

Prigogine realized that classical thermodynamics is not the appropriate tool to explain systems far from equilibrium, owing to the fact mathematical structure is linear. Close on the heels to equilibrium, there will be "fluxes", "vortices", however weak nevertheless. System shall evolve towards a stationary state in which generation of "entropy" (disorder) is as small as possible. By implication, there shall be a minimization problem mathematically, around the equilibrium state. In and around this range, linear equation would explain the characteristics of the system.

On the other hand, away from "equilibrium", the "fluxes" are more emphasized. Result is increase in "entropy". When this occurs, the system no longer tends towards equilibrium. On the contrary, it may encounter instabilities that culminate into newer orders that move away from equilibrium. Thus, dissipative structures revitalize and resurrect complex forms away from equilibrium state.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system of oxygen consumption converges to equilibrium.

2. The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point.

3. Conditions 1 and 2 are independent of the size and direction of initial disturbance.

4. The actual shape of the time path of oxygen consumption in the atmosphere by the terrestrial organism is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance.

5. Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold.

6. Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question, in the present case terrestrial organisms –oxygen consumption-dead organic matter and decomposer organisms.

7. Some authors Nober F. J., Agee, Winfree were interested in such questions, whether growing system could produce full employment of all factors, whether or not there was a full employment natural rate growth path and perpetual oscillations around it. It is to be noted some systems pose extremely difficult stability problems. As an instance, one can quote example of pockets of open cells and drizzle in complex networks in marine stratocumulus. Other examples are clustering and synchronization of lightning flashes adjunct to thunderstorms, coupled studies of microphysics and aqueous chemistry.

TERRESTIAL ORGANISM PORTFOLIO

Assumptions

Terrestrial organisms are classified into three categories analogous to the stratification that was resorted to in consumption of oxygen due to cellular respiration sector. When consumption of oxygen due to cellular respiration in a particular category is transferred to the next sector, (such transference is attributed to the aging process of terrestrial organisms), terrestrial organisms from that category apparently would have become qualified for classification in the corresponding category, because we are in fact classifying terrestrial organisms based on stratification of consumption of oxygen due to cellular respiration

(1) Category 1 is representative of terrestrial organisms corresponding to oxygen consumptions due to cellular respiration under category 1

(2) Category 2 constitutes those terrestrial organisms whose age is higher than that specified under the head category 1 and is in correspondence with the similar classification of oxygen consumption due to cellular respiration.

(3) Category 3 of terrestrial organisms encompasses those terrestrial organisms with respect to category 3 of Oxygen Consumption due to cellular respiration

It is assumed for the sake of simplicity that amount of oxygen taken in water is slowly divided into that of utilization due to terrestrial organisms, Cellular respiration, clouds, etc..

a) The speed of growth of terrestrial organism sector in category 1 is a linear function of the amount of terrestrial organism sector in category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between balance in category 1 and category 2.

b) The dissipation coefficient in the growth model is attributable to two factors;

1. With the progress of time terrestrial organism sector gets aged and become eligible for transfer

to the next category. Notwithstanding Category 3 does not have such a provision for further transference

2. Terrestrial organism sector when become irretrievable(dead from which no cells can be obtained) are the other outlet that decelerates the speed of growth of terrestrial organism sector

c) Inflow into category 2 is only from category 1 in the form of transfer of balance of terrestrial organism sector from the category 1. This is evident from the age wise classification scheme. As a result, the speed of growth of category 2 is dependent upon the amount of inflow, which is a function of the quantum of balance of terrestrial organism sector under the category 1.

d) The balance of terrestrial organism sector in category 3 is because of transfer of balance from category 2. It is dependent on the amount of terrestrial organism sector under category 2.

Notations

T_{13}: Balance standing in the category 1 of terrestrial organism.

T_{14}: Balance standing in the category 2 of terrestrial organism.

T_{15}: Balance standing in the category 3 of terrestrial organism.

$(b_{13})^{(1)}, (b_{14})^{(1)}, (b_{15})^{(1)}$: Accentuation coefficients

$(b'_{13})^{(1)}, (b'_{14})^{(1)}, (b'_{15})^{(1)}$: Dissipation coefficients

Formulation of the System

a) The growth speed in category 1 is the sum of two parts:

1. A term $+(b_{13})^{(1)}T_{14}$ proportional to the amount of balance of terrestrial organisms in the category 2.
2. A term $-(b'_{13})^{(1)}T_{13}$ representing the quantum of balance dissipated from category 1 .This comprises of terrestrial organisms which have grown old, qualified to be classified under category 2 and loss of terrestrial organisms due to death of terrestrial organism (dead organic matter- for concatenated equations see end of the paper).

b) The growth speed in category 2 is the sum of two parts:

1. A term $+(b_{14})^{(1)}T_{13}$ constitutive of the amount of inflow from the category 1
2. A term $-(b'_{14})^{(1)}T_{14}$ the dissipation factor arising due to aging of terrestrial organism and the oxygen saved on account of death of terrestrial organisms.

c) The growth speed under category 3 is attributable to inflow from category 2 and oxygen consumption stalled irrevocably and irretrievably due to death of the terrestrial organisms, and hence cannot deplete oxygen quantum in the atmosphere due to cellular respiration any further.

Governing Equations

Following are the differential equations that govern the growth in the terrestrial organisms portfolio

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13} \tag{12}$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14} \tag{13}$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - (b'_{15})^{(1)}T_{15} \tag{14}$$

$$(b_i)^{(1)} > 0 \quad , \qquad i = 13,14,15 \tag{15}$$

$$(b'_i)^{(1)} > 0 \quad , \qquad i = 13,14,15 \tag{16}$$

$$(b_{14})^{(1)} < (b'_{13})^{(1)} \tag{17}$$

$$(b_{15})^{(1)} < (b'_{14})^{(1)} \tag{18}$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is

$$\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda' i t},\ i = 13,14,15$$

where $C'_{13}, C'_{14}, C'_{15}$ are arbitrary constant coefficients and $\alpha'_{13}, \alpha'_{14}, \alpha'_{15}, \gamma'_{13}, \gamma'_{14}, \gamma'_{15}$ corresponding multipliers to the characteristic roots of the terrestrial organism system.

OXYGEN CONSUMPTION DUE TO CELLULAR RESPIRATION – TERRESTRIAL ORGANISM – DUAL SYSTEM ANALYSIS

In the previous section, we studied the growth of oxygen consumption due to cellular respiration and terrestrial organisms separately. In this section, we study the two-portfolio model comprising six-storey oxygen consumption due to cellular respiration and terrestrial organisms. Scheme of age wise classification however remains the same. We make an explicit assumption that only category 2 of terrestrial organisms is responsible for the increase in the dissipation coefficient of the oxygen consumption due to cellular respiration. Terrestrial organisms of three categories dissipating three portfolios of oxygen consumption due to cellular respiration levels follows by mere substitution of corresponding variables. Dissipation coefficients of the terrestrial organisms portfolio are diminished by the contribution of all three categories of oxygen consumption due to cellular respiration portfolio of terrestrial organisms. This is to facilitate circumvention of the nonlinearity of the equations and consequent unsolvability thereof We will denote

1) $T_i(t), i = 13,14,15$, the three parts of the terrestrial organisms system analogously to the G_i of the consumption of oxygen due to cellular respiration.
2) By $(a_i'')^{(1)}(T_{14},t)$ $(T_{14} \geq 0, t \geq 0)$, the contribution of the terrestrial organisms to the dissipation coefficient of the oxygen consumption due to cellular respiration of terrestrial organisms
3) By$(-\ b_i'')^{(1)}(G_{13},\ G_{14},\ G_{15},\ t) = -\ (b_i'')^{(1)}(G,\ t)$, the contribution of the consumption of oxygen due to cellular respiration to the dissipation coefficient of the terrestrial organisms.

Terrestrial Organism – Oxygen Consumption System Governing Equations

The differential system of this model is now

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a_{13}')^{(1)} + (a_{13}'')^{(1)}(T_{14},t)\right]G_{13} \quad (19)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a_{14}')^{(1)} + (a_{14}'')^{(1)}(T_{14},t)\right]G_{14} \quad (20)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a_{15}')^{(1)} + (a_{15}'')^{(1)}(T_{14},t)\right]G_{15} \quad (21)$$

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b_{13}')^{(1)} - (b_{13}'')^{(1)}(G,t)\right]T_{13} \quad (22)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b_{14}')^{(1)} - (b_{14}'')^{(1)}(G,t)\right]T_{14} \quad (23)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b_{15}')^{(1)} - (b_{15}'')^{(1)}(G,t)\right]T_{15} \quad (24)$$

$+(a_{13}'')^{(1)}(T_{14},t) =$ First augmentation factor attributable to cellular respiration of terrestrial organism, to the dissipation of oxygen consumption.

$-(b_{13}'')^{(1)}(G,t) =$ First detrition factor contributed by oxygen consumption to the dissipation of terrestrial organisms Where we suppose.

(A) $(a_i)^{(1)}, (a_i')^{(1)}, (a_i'')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (b_i'')^{(1)} > 0,$
$i, j = 13,14,15$

(B) The functions $(a_i'')^{(1)}, (b_i'')^{(1)}$ are positive continuous increasing and bounded.
Definition of $(p_i)^{(1)}$, $(r_i)^{(1)}$:

$$(a_i'')^{(1)}(T_{14},t) \leq (p_i)^{(1)} \leq (\hat{A}_{13})^{(1)} \quad (25)$$

$$(b_i'')^{(1)}(G,t) \leq (r_i)^{(1)} \leq (b_i')^{(1)} \leq (\hat{B}_{13})^{(1)}$$

(C) $$\lim_{T_2 \to \infty} (a_i'')^{(1)}(T_{14},t) = (p_i)^{(1)} \quad (26)$$

$$\lim_{G \to \infty} (b_i'')^{(1)}(G,t) = (r_i)^{(1)} \quad (27)$$

Definition of $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}$ (28)

Where $(\hat{A}_{13})^{(1)}, (\hat{B}_{13})^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}$ are positive constants and i = 13, 14, 15
They satisfy Lipschitz condition:

$$|(a_i'')^{(1)}(T_{14}',t) - (a_i'')^{(1)}(T_{14},t)|$$

$$\leq (\hat{k}_{13})^{(1)}|T_{14} - T_{14}'|e^{-(\hat{M}_{13})^{(1)}t} \quad (29)$$

$$|(b_i'')^{(1)}(G',t) - (b_i'')^{(1)}(G,T)|$$

$$< (\hat{k}_{13})^{(1)}||G - G'||e^{-(\hat{M}_{13})^{(1)}t} \quad (30)$$

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(1)}(T_{14}',t)$ and $(a_i'')^{(1)}(T_{14},\ t)$. $(T'_{14},\ t)$ and $(T_{14},\ t)$ are points belonging to the interval $\left[(\hat{k}_{13})^{(1)}, (\hat{M}_{13})^{(1)}\right]$. It is to be noted that $(a_i'')^{(1)}(T_{14},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{13})^{(1)} = 1$ then the function $(a_i'')^{(1)}(T_{14},t)$, the first augmentation coefficient attributable to terrestrial organisms, would be absolutely continuous.

Definition of $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$

(D) $(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}$, are positive constants

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}, \frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1 \quad (31)$$

Definition of $(\hat{P}_{13})^{(1)}, (\hat{Q}_{13})^{(1)}$

(E) There exists two constants $(\hat{P}_{13})^{(1)}$ and $(\hat{Q}_{13})^{(1)}$ which together with

$(\hat{M}_{13})^{(1)}, (\hat{k}_{13})^{(1)}, (\hat{A}_{13})^{(1)} and\ (\hat{B}_{13})^{(1)}$ and the constants $(a_i)^{(1)}, (a_i')^{(1)}, (b_i)^{(1)}, (b_i')^{(1)}, (p_i)^{(1)}$, $(r_i)^{(1)}$, i = 13, 14, 15, satisfy the inequalities

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(a_i)^{(1)} + (a_i')^{(1)} + (\hat{A}_{13})^{(1)}$$

$$+ (\hat{P}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1 \quad (32)$$

$$\frac{1}{(\hat{M}_{13})^{(1)}}[(b_i)^{(1)} + (b_i')^{(1)} + (\hat{B}_{13})^{(1)} \cdot$$

$$+ (\hat{Q}_{13})^{(1)}(\hat{k}_{13})^{(1)}] < 1 \quad (33)$$

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions
Definition of $G_i(0), T_i(0)$

$$G_i(t) \leq \left(\hat{P}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq \left(\hat{Q}_{13}\right)^{(1)} e^{(\hat{M}_{13})^{(1)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof:

Consider operator $\mathcal{A}^{(1)}$ defined on the space of sextuples of continuous functions G_i, $T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy $G_i(0) = G_i^0$, $T_i(0) = T_i^0$, $G_i^0 \leq (\hat{P}_{13})^{(1)}$,

$$T_i^0 \le (\hat{Q}_{13})^{(1)}, \tag{34}$$

$$0 \le G_i(t) - G_i^0 \le (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \tag{35}$$

$$0 \le T_i(t) - T_i^0 \le (\hat{Q}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} t} \tag{36}$$

By

$$\bar{G}_{13}(t) = G_{13}^0 + \int_0^t \Big[(a_{13})^{(1)} G_{14}(s_{(13)}) - \Big((a'_{13})^{(1)} + (a''_{13})^{(1)}\big(T_{14}(s_{(13)}), s_{(13)}\big)\Big) G_{13}(s_{(13)})\Big] ds_{(13)} \tag{37}$$

$$\bar{G}_{14}(t) = G_{14}^0 + \int_0^t \Big[(a_{14})^{(1)} G_{13}(s_{(13)}) - \Big((a'_{14})^{(1)} + (a''_{14})^{(1)}\big(T_{14}(s_{(13)}), s_{(13)}\big)\Big) G_{14}(s_{(13)})\Big] ds_{(13)} \tag{38}$$

$$\bar{G}_{15}(t) = G_{15}^0 + \int_0^t \Big[(a_{15})^{(1)} G_{14}(s_{(13)}) - \Big((a'_{15})^{(1)} + (a''_{15})^{(1)}\big(T_{14}(s_{(13)}), s_{(13)}\big)\Big) G_{15}(s_{(13)})\Big] ds_{(13)} \tag{39}$$

$$\bar{T}_{13}(t) = T_{13}^0 + \int_0^t \Big[(b_{13})^{(1)} T_{14}(s_{(13)}) - \Big((b'_{13})^{(1)} - (b''_{13})^{(1)}\big(G(s_{(13)}), s_{(13)}\big)\Big) T_{13}(s_{(13)})\Big] ds_{(13)} \tag{40}$$

$$\bar{T}_{14}(t) = T_{14}^0 + \int_0^t \Big[(b_{14})^{(1)} T_{13}(s_{(13)}) - \Big((b'_{14})^{(1)} - (b''_{14})^{(1)}\big(G(s_{(13)}), s_{(13)}\big)\Big) T_{14}(s_{(13)})\Big] ds_{(13)} \tag{41}$$

$$\bar{T}_{15}(t) = T_{15}^0 + \int_0^t \Big[(b_{15})^{(1)} T_{14}(s_{(13)}) - \Big((b'_{15})^{(1)} - (b''_{15})^{(1)}\big(G(s_{(13)}), s_{(13)}\big)\Big) T_{15}(s_{(13)})\Big] ds_{(13)} \tag{42}$$

Where $s_{(13)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(1)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

$$G_{13}(t) \le G_{13}^0 + \int_0^t \Big[(a_{13})^{(1)} \Big(G_{14}^0 + (\hat{P}_{13})^{(1)} e^{(\hat{M}_{13})^{(1)} s_{(13)}}\Big)\Big] ds_{(13)}$$

$$= \big(1 + (a_{13})^{(1)} t\big) G_{14}^0 + \frac{(a_{13})^{(1)} (\hat{P}_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \Big(e^{(\hat{M}_{13})^{(1)} t} - 1\Big) \tag{43}$$

From which it follows that

$$(G_{13}(t) - G_{13}^0) e^{-(\hat{M}_{13})^{(1)} t} \le$$

$$\frac{(a_{13})^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\big((\hat{P}_{13})^{(1)} + G_{14}^0\big) e^{\left(-\frac{(\hat{P}_{13})^{(1)} + G_{14}^0}{G_{14}^0}\right)} + (\hat{P}_{13})^{(1)} \right] \tag{44}$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for G_{14} , G_{15}, T_{13}, T_{14}, T_{15}

It is now sufficient to take $\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}}$, $\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} < 1$ and to choose $(\hat{\mathrm{P}}_{13})^{(1)}$ and $(\hat{\mathrm{Q}}_{13})^{(1)}$ large to have

$$\frac{(a_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[(\hat{P}_{13})^{(1)} + \big((\hat{P}_{13})^{(1)} + G_j^0\big) e^{-\left(\frac{(\hat{P}_{13})^{(1)} + G_j^0}{G_j^0}\right)} \right] \le (\hat{P}_{13})^{(1)} \tag{45}$$

$$\frac{(b_i)^{(1)}}{(\hat{M}_{13})^{(1)}} \left[\big((\hat{Q}_{13})^{(1)} + T_j^0\big) e^{-\left(\frac{(\hat{Q}_{13})^{(1)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{13})^{(1)} \right] \le (\hat{Q}_{13})^{(1)} \tag{46}$$

In order that the operator $\mathcal{A}^{(1)}$ transforms the space of sextuples of functions G_i ,T_i satisfying 34,35,36 into itself The operator $\mathcal{A}^{(1)}$ is a contraction with respect to the metric

$$d\Big(\big(G^{(1)}, T^{(1)}\big), \big(G^{(2)}, T^{(2)}\big)\Big) =$$

$$\sup_i \{ \max_{t \in \mathbb{R}_+} \big|G_i^{(1)}(t) - G_i^{(2)}(t)\big| e^{-(\hat{M}_{13})^{(1)} t},$$

$$\max_{t \in \mathbb{R}_+} \big|T_i^{(1)}(t) - T_i^{(2)}(t)\big| e^{-(\hat{M}_{13})^{(1)} t} \} \tag{47}$$

Indeed if we denote

Definition of $\tilde{G}, \tilde{T}$:

$$\big(\tilde{G}, \tilde{T}\big) = \mathcal{A}^{(1)}(G, T) \tag{48}$$

It results

$$\big|\tilde{G}_{13}^{(1)} - \tilde{G}_i^{(2)}\big| \le$$

$$\int_0^t (a_{13})^{(1)} \big|G_{14}^{(1)} - G_{14}^{(2)}\big| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} ds_{(13)} +$$

$$\int_0^t \{(a'_{13})^{(1)} \big|G_{13}^{(1)} - G_{13}^{(2)}\big| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{-(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$(a''_{13})^{(1)}\big(T_{14}^{(1)}, s_{(13)}\big) \big|G_{13}^{(1)} - G_{13}^{(2)}\big| e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} +$$

$$G_{13}^{(2)} \big|(a''_{13})^{(1)}\big(T_{14}^{(1)}, s_{(13)}\big) -$$

$$(a''_{13})^{(1)}\big(T_{14}^{(2)}, s_{(13)}\big)\big| \; e^{-(\hat{M}_{13})^{(1)} s_{(13)}} e^{(\hat{M}_{13})^{(1)} s_{(13)}} \} ds_{(13)} \tag{49}$$

Where $s_{(13)}$ represents integrand that is integrated over the interval [0,t]

From the hypotheses on 25,26,27,28 and 29 it follows

$$\left|G^{(1)} - G^{(2)}\right| e^{-(\widehat{M}_{13})^{(1)}t} \leq$$

$$\frac{1}{(\widehat{M}_{13})^{(1)}}\left((a_{13})^{(1)} + (a'_{13})^{(1)} + (\widehat{A}_{13})^{(1)} + (\widehat{P}_{13})^{(1)}(\widehat{k}_{13})^{(1)}\right) d\left(\left(G^{(1)}, T^{(1)}; G^{(2)}, T^{(2)}\right)\right) \quad (50)$$

And analogous inequalities for G_i and T_i. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{13})^{(1)}$ and $(b''_{13})^{(1)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\widehat{P}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ *and* $(\widehat{Q}_{13})^{(1)} e^{(\widehat{M}_{13})^{(1)}t}$ respectively of $\mathbb{R}_+$.

(51)

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(1)}$ and $(b''_i)^{(1)}, i = 13,14,15$ depend only on T_{14} and respectively on G(*and not on t*) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any *t* where G_i (*t*)=0 and T_i (*t*)=0 (52)

From 19 to 24 it results

$$G_i(t) \geq G_i^0 e^{\left[-\int_0^t \{(a'_i)^{(1)} - (a''_i)^{(1)}(T_{14}(s_{(13)}), s_{(13)})\} ds_{(13)}\right]} \geq 0$$

$$T_i(t) \geq T_i^0 e^{(-(b'_i)^{(1)}t)} > 0 \quad \text{for } t > 0$$

Definition of

$$\left((\widehat{M}_{13})^{(1)}\right)_1,\ \left((\widehat{M}_{13})^{(1)}\right)_2 \text{ and } \left((\widehat{M}_{13})^{(1)}\right)_3 :$$

Remark 3: if G_{13} is bounded, the same property have also G_{14} and G_{15} . indeed if (53)

$G_{13} < (\widehat{M}_{13})^{(1)}$ it follows $\frac{dG_{14}}{dt} \leq \left((\widehat{M}_{13})^{(1)}\right)_1 - (a'_{14})^{(1)} G_{14}$

and by integrating

$$G_{14} \leq \left((\widehat{M}_{13})^{(1)}\right)_2 = G_{14}^0 + 2(a_{14})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_1 / (a'_{14})^{(1)}$$

In the same way , one can obtain

$$G_{15} \leq \left((\widehat{M}_{13})^{(1)}\right)_3 = G_{15}^0 + 2(a_{15})^{(1)}\left((\widehat{M}_{13})^{(1)}\right)_2 / (a'_{15})^{(1)}$$

If G_{14} *or* G_{15} is bounded, the same property follows for G_{13}, G_{15} and G_{13} ,G_{14} respectively.

Remark 4: If G_{13} is bounded, from below, the same property holds for G_{14} and G_{15} .The proof is analogous with the preceding one. An analogous property is true if G_{14} is bounded from below. (54)

Remark 5: If T_{13} is bounded from below and $\lim_{t\to\infty}((b''_i)^{(1)}(G(t), t)) = (b'_{14})^{(1)}$ then $T_{14} \to \infty$.

Definition of $(m)^{(1)}$ and ε_1:
Indeed let t_1 be so that for $t > t_1$

$$(b_{14})^{(1)} - (b''_i)^{(1)}(G(t), t) < \varepsilon_1, T_{13}(t) > (m)^{(1)}$$

Then $\frac{dT_{14}}{dt} \geq (a_{14})^{(1)}(m)^{(1)} - \varepsilon_1 T_{14}$ which leads to

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{\varepsilon_1}\right)(1 - e^{-\varepsilon_1 t}) + T_{14}^0 e^{-\varepsilon_1 t}$$

If we take t such that $e^{-\varepsilon_1 t} = \frac{1}{2}$ it results

$$T_{14} \geq \left(\frac{(a_{14})^{(1)}(m)^{(1)}}{2}\right), \quad t = log\frac{2}{\varepsilon_1}$$

By taking now ε_1 sufficiently small one sees that T_{14} is unbounded. The same property holds for T_{15} if $\lim_{t\to\infty}(b''_{15})^{(1)}(G(t), t) = (b'_{15})^{(1)}$ We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ (56)

(a) $(\sigma_1)^{(1)}, (\sigma_2)^{(1)}, (\tau_1)^{(1)}, (\tau_2)^{(1)}$ four constants satisfying

$$-(\sigma_2)^{(1)} \leq -(a'_{13})^{(1)} + (a'_{14})^{(1)} - (a''_{13})^{(1)}(T_{14}, t) + (a''_{14})^{(1)}(T_{14}, t) \leq -(\sigma_1)^{(1)} \quad (57)$$

$$-(\tau_2)^{(1)} \leq -(b'_{13})^{(1)} + (b'_{14})^{(1)} - (b''_{13})^{(1)}(G, t) - (b''_{14})^{(1)}(G, t) \leq -(\tau_1)^{(1)} \quad (58)$$

Definition of $(\nu_1)^{(1)}, (\nu_2)^{(1)}, (u_1)^{(1)}, (u_2)^{(1)}, \nu^{(1)}, u^{(1)}$

(b) By $(\nu_1)^{(1)} > 0$, $(\nu_2)^{(1)} < 0$ and respectively $(u_1)^{(1)} > 0$, $(u_2)^{(1)} < 0$ the roots of the equations

$$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0 \text{ and} \quad (59)$$

$$(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_1)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0 \text{ and} \quad (60)$$

Definition of $(\bar{\nu}_1)^{(1)}, , (\bar{\nu}_2)^{(1)}, (\bar{u}_1)^{(1)}, (\bar{u}_2)^{(1)}$ (61)

By $(\bar{\nu}_1)^{(1)} > 0$, $(\bar{\nu}_2)^{(1)} < 0$ and respectively $(\bar{u}_1)^{(1)} > 0$, $(\bar{u}_2)^{(1)} < 0$ the (62)

roots of the equations

$$(a_{14})^{(1)}\left(\nu^{(1)}\right)^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)} = 0 \quad (63)$$

and $(b_{14})^{(1)}\left(u^{(1)}\right)^2 + (\tau_2)^{(1)}u^{(1)} - (b_{13})^{(1)} = 0$ (64)

Definition of $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}, (\nu_0)^{(1)}$

(c) If we define $(m_1)^{(1)}, (m_2)^{(1)}, (\mu_1)^{(1)}, (\mu_2)^{(1)}$ by (65)

$$(m_2)^{(1)} = (\nu_0)^{(1)}, (m_1)^{(1)} = (\nu_1)^{(1)}, \ if\ (\nu_0)^{(1)} < (\nu_1)^{(1)} \quad (66)$$

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\bar{\nu}_1)^{(1)}$,

$if\ (\nu_1)^{(1)} < (\nu_0)^{(1)} < (\bar{\nu}_1)^{(1)}$,

and $(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}$ (67)

$(m_2)^{(1)} = (\nu_1)^{(1)}, (m_1)^{(1)} = (\nu_0)^{(1)},\ if\ (\bar{\nu}_1)^{(1)} < (\nu_0)^{(1)}$ (68)

and analogously

$(\mu_2)^{(1)} = (u_0)^{(1)}, (\mu_1)^{(1)} = (u_1)^{(1)},\ if\ (u_0)^{(1)} < (u_1)^{(1)}$ (69)

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (\bar{u}_1)^{(1)}$,

$if\ (u_1)^{(1)} < (u_0)^{(1)} < (\bar{u}_1)^{(1)}$,

and $(u_0)^{(1)} = \frac{T_{13}^0}{T_{14}^0}$ (70)

$(\mu_2)^{(1)} = (u_1)^{(1)}, (\mu_1)^{(1)} = (u_0)^{(1)}, if\ (\bar{u}_1)^{(1)} < (u_0)^{(1)}$ (71)

where $(u_1)^{(1)}$, $(\bar{u}_1)^{(1)}$ are defined by 59 and 61 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities

$G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{13}(t) \le G_{13}^0 e^{(S_1)^{(1)} t}$ (72)

where $(p_i)^{(1)}$ is defined by equation 25

$\frac{1}{(m_1)^{(1)}} G_{13}^0 e^{((S_1)^{(1)} - (p_{13})^{(1)})t} \le G_{14}(t) \le \frac{1}{(m_2)^{(1)}} G_{13}^0 e^{(S_1)^{(1)} t}$ (73)

$\left(\frac{(a_{15})^{(1)} G_{13}^0}{(m_1)^{(1)}((S_1)^{(1)} - (p_{13})^{(1)} - (S_2)^{(1)})}\left[e^{((S_1)^{(1)} - (p_{13})^{(1)})t} - e^{-(S_2)^{(1)} t}\right]\right.$

$+ G_{15}^0 e^{-(S_2)^{(1)} t} \le G_{15}(t) \le$

$\frac{(a_{15})^{(1)} G_{13}^0}{(m_2)^{(1)}((S_1)^{(1)} - (a'_{15})^{(1)})}\left[e^{(S_1)^{(1)} t} - e^{-(a'_{15})^{(1)} t}\right]$

$\left. + G_{15}^0 e^{-(a'_{15})^{(1)} t}\right)$ (74)

$T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ (75)

$\frac{1}{(\mu_1)^{(1)}} T_{13}^0 e^{(R_1)^{(1)} t} \le T_{13}(t) \le \frac{1}{(\mu_2)^{(1)}} T_{13}^0 e^{((R_1)^{(1)} + (r_{13})^{(1)})t}$ (76)

$\frac{(b_{15})^{(1)} T_{13}^0}{(\mu_1)^{(1)}((R_1)^{(1)} - (b'_{15})^{(1)})}\left[e^{(R_1)^{(1)} t} - e^{-(b'_{15})^{(1)} t}\right]$

$+ T_{15}^0 e^{-(b'_{15})^{(1)} t} \le T_{15}(t) \le$

$\frac{(a_{15})^{(1)} T_{13}^0}{(\mu_2)^{(1)}((R_1)^{(1)} + (r_{13})^{(1)} + (R_2)^{(1)})}\left[e^{((R_1)^{(1)} + (r_{13})^{(1)})t} - e^{-(R_2)^{(1)} t}\right]$

$+ T_{15}^0 e^{-(R_2)^{(1)} t}$ (77)

Definition of $(S_1)^{(1)}, (S_2)^{(1)}, (R_1)^{(1)}, (R_2)^{(1)}$:

Where $(S_1)^{(1)} = (a_{13})^{(1)}(m_2)^{(1)} - (a'_{13})^{(1)}$

$(S_2)^{(1)} = (a_{15})^{(1)} - (p_{15})^{(1)}$

$(R_1)^{(1)} = (b_{13})^{(1)}(\mu_2)^{(1)} - (b'_{13})^{(1)}$ (78)

$(R_2)^{(1)} = (b'_{15})^{(1)} - (r_{15})^{(1)}$ (79)

Proof : From 19,20,21,22,23,24, we obtain

$\frac{d\nu^{(1)}}{dt} = (a_{13})^{(1)} - \left((a'_{13})^{(1)} - (a'_{14})^{(1)} + (a''_{13})^{(1)}(T_{14}, t)\right)$

$- (a''_{14})^{(1)}(T_{14}, t)\nu^{(1)} - (a_{14})^{(1)}\nu^{(1)}$ (80)

Definition of $\nu^{(1)}$: $\boxed{\nu^{(1)} = \frac{G_{13}}{G_{14}}}$

It follows $-\left((a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_2)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$

$\le \frac{d\nu^{(1)}}{dt} \le -\left((a_{14})^{(1)}(\nu^{(1)})^2 + (\sigma_1)^{(1)}\nu^{(1)} - (a_{13})^{(1)}\right)$ (81)

From which one obtains

Definition of $(\bar{\nu}_1)^{(1)}, (\nu_0)^{(1)}$:

(a) For $0 < \boxed{(\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}} < (\nu_1)^{(1)} < (\bar{\nu}_1)^{(1)}$

$\nu^{(1)}(t) \ge \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{\left[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_0)^{(1)})\, t\right]}}{1 + (C)^{(1)} e^{\left[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_0)^{(1)})\, t\right]}}$,

$\boxed{(C)^{(1)} = \frac{(\nu_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\nu_2)^{(1)}}}$ (82)

it follows $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\nu_1)^{(1)}$

In the same manner , we get

$\nu^{(1)}(t) \le \frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})\, t\right]}}{1 + (\bar{C})^{(1)} e^{\left[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})\, t\right]}}$,

$\boxed{(\bar{C})^{(1)} = \frac{(\bar{\nu}_1)^{(1)} - (\nu_0)^{(1)}}{(\nu_0)^{(1)} - (\bar{\nu}_2)^{(1)}}}$ (83)

From which we deduce $(\nu_0)^{(1)} \le \nu^{(1)}(t) \le (\bar{\nu}_1)^{(1)}$

(b) If $0 < (\nu_1)^{(1)} < (\nu_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0} < (\bar{\nu}_1)^{(1)}$ we find like in the previous case,

$(\nu_1)^{(1)} \le \frac{(\nu_1)^{(1)} + (C)^{(1)}(\nu_2)^{(1)} e^{[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_2)^{(1)})\, t]}}{1 + (C)^{(1)} e^{[-(a_{14})^{(1)}((\nu_1)^{(1)} - (\nu_2)^{(1)})\, t]}} \le \nu^{(1)}(t) \le$

$\frac{(\bar{\nu}_1)^{(1)} + (\bar{C})^{(1)}(\bar{\nu}_2)^{(1)} e^{[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})\, t]}}{1 + (\bar{C})^{(1)} e^{[-(a_{14})^{(1)}((\bar{\nu}_1)^{(1)} - (\bar{\nu}_2)^{(1)})\, t]}} \le (\bar{\nu}_1)^{(1)}$ (84)

(c) if $0 < (v_1)^{(1)} \leq (\bar{v}_1)^{(1)} \leq \boxed{(v_0)^{(1)} = \frac{G_{13}^0}{G_{14}^0}}$, we obtain

$$(v_1)^{(1)} \leq v^{(1)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(1)}+(\bar{C})^{(1)}(\bar{v}_2)^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}}{1+(\bar{C})^{(1)}e^{\left[-(a_{14})^{(1)}\left((\bar{v}_1)^{(1)}-(\bar{v}_2)^{(1)}\right)t\right]}} \leq (v_0)^{(1)} \quad (85)$$

And so with the notation of the first part of condition (c) , we have

Definition of $v^{(1)}(t)$:-

$$(m_2)^{(1)} \leq v^{(1)}(t) \leq (m_1)^{(1)}, \quad \boxed{v^{(1)}(t) = \frac{G_{13}(t)}{G_{14}(t)}} \quad (86)$$

In a completely analogous way, we obtain

Definition of $u^{(1)}(t)$:-

$$(\mu_2)^{(1)} \leq u^{(1)}(t) \leq (\mu_1)^{(1)}, \quad \boxed{u^{(1)}(t) = \frac{T_{13}(t)}{T_{14}(t)}} \quad (87)$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case :

If $(a_{13}'')^{(1)} = (a_{14}'')^{(1)}, then\ (\sigma_1)^{(1)} = (\sigma_2)^{(1)}$ and in this case $(v_1)^{(1)} = (\bar{v}_1)^{(1)}$ if in addition $(v_0)^{(1)} = (v_1)^{(1)}$ then $v^{(1)}(t) = (v_0)^{(1)}$ and as a consequence $G_{13}(t) = (v_0)^{(1)}G_{14}(t)$ t this also defines $(v_0)^{(1)}$ for the special case.

Analogously if $(b_{13}'')^{(1)} = (b_{14}'')^{(1)}, then\ (\tau_1)^{(1)} = (\tau_2)^{(1)}$, then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ and then $(u_1)^{(1)} = (\bar{u}_1)^{(1)}$ if in addition $(u_0)^{(1)} = (u_1)^{(1)}$ then $T_{13}(t) = (u_0)^{(1)}T_{14}(t)$

This is an important consequence of the relation between $(v_1)^{(1)}$ and $(\bar{v}_1)^{(1)}$, and definition of $(u_0)^{(1)}$.

STATIONARY SOLUTIONS AND STABILITY

Stationary solutions and stability curve representative of the variation of oxygen consumption due to cellular respiration of terrestrial organisms vis-a-vis that of terrestrial organism variation curve lies below the tangent at $G=G_0$ for $G<G_0$ and above the tangent for $G>G_0$. Wherever such a situation occurs the point G_0 is called the "point of inflexion". In this case, the tangent has a positive slope that simply means the rate of change of oxygen consumption due to cellular respiration is greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

Stationary value:

In all the cases $G=G_0$, $G<G_0$, $G>G_0$ the condition that the rate of change of oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of (G) is that the rate of change of oxygen consumption function at $G_0$is zero.
2. A sufficient condition for the stationary value at G_0 , to be maximum is that the acceleration of the oxygen consumption is less than zero.
3. A sufficient condition for the stationary value at G_0, be minimum is that acceleration of oxygen consumption is greater than zero.
4. With the rate of change of G namely oxygen consumption defined as the accentuation term and the dissipation term, we are sure that the rate of change of oxygen consumption is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationariness.

We can prove the following

Theorem 3: If $(a_i'')^{(1)} and\ (b_i'')^{(1)}$ are independent ont, and the conditions (with the notations 25,26,27,28) (88)

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} < 0$$

$$(a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a_{13})^{(1)}(p_{13})^{(1)} + (a'_{14})^{(1)}(p_{14})^{(1)} + (p_{13})^{(1)}(p_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} > 0$$

$$(b'_{13})^{(1)}(b'_{14})^{(1)} - (b_{13})^{(1)}(b_{14})^{(1)} - (b_{13})^{(1)}(r_{14})^{(1)} - (b'_{14})^{(1)}(r_{14})^{(1)} + (r_{13})^{(1)}(r_{14})^{(1)} < 0$$

with $(p_{13})^{(1)}(r_{14})^{(1)}$ as defined by equation 25 are satisfied, then the system

$$(a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14})\right]G_{13} = 0 \quad (89)$$

$$(a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14})\right]G_{14} = 0 \quad (90)$$

$$(a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14})\right]G_{15} = 0 \quad (91)$$

$$(b_{13})^{(1)}T_{14} - [(b'_{13})^{(1)} - (b''_{13})^{(1)}(G)\ ]T_{13} = 0 \quad (92)$$

$$(b_{14})^{(1)}T_{13} - [(b'_{14})^{(1)} - (b''_{14})^{(1)}(G)\ ]T_{14} = 0 \quad (93)$$

$$(b_{15})^{(1)}T_{14} - [(b'_{15})^{(1)} - (b''_{15})^{(1)}(G)\ ]T_{15} = 0 \quad (94)$$

has a unique positive solution, which is an equilibrium solution for the system (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{13},G_{14} if

$$F(T) = (a'_{13})^{(1)}(a'_{14})^{(1)} - (a_{13})^{(1)}(a_{14})^{(1)} + (a'_{13})^{(1)}(a''_{14})^{(1)}(T_{14}) + (a'_{14})^{(1)}(a''_{13})^{(1)}(T_{14}) + (a''_{13})^{(1)}(T_{14})(a''_{14})^{(1)}(T_{14}) = 0 \quad (95)$$

Definition and uniqueness of T_{14}^*

After hypothesis $f(0)<0, f(\infty)>0$ and the functions $(a_i'')^{(1)}(T_{14})$ being increasing, it follows that there exists a unique T_{14}^* for which $f(T_{14}^*)=0$. With this value , we obtain from the three first equations

$$G_{13}=\frac{(a_{13})^{(1)}G_{14}}{[(a_{13}')^{(1)}+(a_{13}'')^{(1)}(T_{14}^*)]}\ ,$$

$$G_{15}=\frac{(a_{15})^{(1)}G_{14}}{[(a_{15}')^{(1)}+(a_{15}'')^{(1)}(T_{14}^*)]} \tag{96}$$

(b) By the same argument, the equations 92,93 admit solutions G_{13}, G_{14} if

$$\varphi(G)=(b_{13}')^{(1)}(b_{14}')^{(1)}-(b_{13})^{(1)}(b_{14})^{(1)}-$$

$$\left[(b_{13}')^{(1)}(b_{14}'')^{(1)}(G)+(b_{14}')^{(1)}(b_{13}'')^{(1)}(G)\right]+$$

$$(b_{13}'')^{(1)}(G)(b_{14}'')^{(1)}(G)=0 \tag{97}$$

Where in $G(G_{13},G_{14},G_{15})$,G_{13},G_{15}must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{14} taking into account the hypothesis $\varphi(0)>0$, $\varphi(\infty)<0$ it follows that there exists a unique G_{14}^* such that $\varphi(G^*)=0$

Finally we obtain the unique solution of 89 to 94

G_{14}^* given by $\varphi(G^*)=0$, T_{14}^* given by $f(T_{14}^*)=0$ and

$$G_{13}^*=\frac{(a_{13})^{(1)}G_{14}^*}{[(a_{13}')^{(1)}+(a_{13}'')^{(1)}(T_{14}^*)]}\ ,$$

$$G_{15}^*=\frac{(a_{15})^{(1)}G_{14}^*}{[(a_{15}')^{(1)}+(a_{15}'')^{(1)}(T_{14}^*)]} \tag{98}$$

$$T_{13}^*=\frac{(b_{13})^{(1)}T_{14}^*}{[(b_{13}')^{(1)}-(b_{13}'')^{(1)}(G^*)]}\ ,$$

$$T_{15}^*=\frac{(b_{15})^{(1)}T_{14}^*}{[(b_{15}')^{(1)}-(b_{15}'')^{(1)}(G^*)]} \tag{99}$$

Obviously, these values represent an equilibrium solution of 19,20,21,22,23,24.

ASYMPTOTIC STABILITY ANALYSIS

Theorem 4: If the conditions of the previous theorem are satisfied and if the functions $(a_i'')^{(1)}$and $(b_i'')^{(1)}$Belong to $C^{(1)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

Proof: Denote

Definition of $\mathbb{G}_i,\mathbb{T}_i$:-

$$G_i=G_i^*+\mathbb{G}_i\ ,\ T_i=T_i^*+\mathbb{T}_i \tag{100}$$

$$\frac{\partial(a_{14}'')^{(1)}}{\partial T_{14}}(T_{14}^*)=(q_{14})^{(1)}\ ,\ \frac{\partial(b_i'')^{(1)}}{\partial G_j}(G^*)=s_{ij} \tag{101}$$

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain from 19 to 24

$$\frac{d\mathbb{G}_{13}}{dt}=-\left((a_{13}')^{(1)}+(p_{13})^{(1)}\right)\mathbb{G}_{13}+$$

$$(a_{13})^{(1)}\mathbb{G}_{14}-(q_{13})^{(1)}G_{13}^*\mathbb{T}_{14} \tag{102}$$

$$\frac{d\mathbb{G}_{14}}{dt}=-\left((a_{14}')^{(1)}+(p_{14})^{(1)}\right)\mathbb{G}_{14}+$$

$$(a_{14})^{(1)}\mathbb{G}_{13}-(q_{14})^{(1)}G_{14}^*\mathbb{T}_{14} \tag{103}$$

$$\frac{d\mathbb{G}_{15}}{dt}=-\left((a_{15}')^{(1)}+(p_{15})^{(1)}\right)\mathbb{G}_{15}+$$

$$(a_{15})^{(1)}\mathbb{G}_{14}-(q_{15})^{(1)}G_{15}^*\mathbb{T}_{14} \tag{104}$$

$$\frac{d\mathbb{T}_{13}}{dt}=-\left((b_{13}')^{(1)}-(r_{13})^{(1)}\right)\mathbb{T}_{13}+$$

$$(b_{13})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(13)(j)}T_{13}^*\mathbb{G}_j\right) \tag{105}$$

$$\frac{d\mathbb{T}_{14}}{dt}=-\left((b_{14}')^{(1)}-(r_{14})^{(1)}\right)\mathbb{T}_{14}+$$

$$(b_{14})^{(1)}\mathbb{T}_{13}+\sum_{j=13}^{15}\left(s_{(14)(j)}T_{14}^*\mathbb{G}_j\right) \tag{106}$$

$$\frac{d\mathbb{T}_{15}}{dt}=-\left((b_{15}')^{(1)}-(r_{15})^{(1)}\right)\mathbb{T}_{15}+$$

$$(b_{15})^{(1)}\mathbb{T}_{14}+\sum_{j=13}^{15}\left(s_{(15)(j)}T_{15}^*\mathbb{G}_j\right) \tag{107}$$

The characteristic equation of this system is

$$\left((\lambda)^{(1)}+(b_{15}')^{(1)}-(r_{15})^{(1)}\right)\{\left((\lambda)^{(1)}+(a_{15}')^{(1)}+(p_{15})^{(1)}\right)$$

$$\left[\left((\lambda)^{(1)}+(a_{13}')^{(1)}+(p_{13})^{(1)}\right)(q_{14})^{(1)}G_{14}^*+(a_{14})^{(1)}(q_{13})^{(1)}G_{13}^*\right)\Big]$$

$$\left((\lambda)^{(1)}+(b_{13}')^{(1)}-(r_{13})^{(1)}\right)s_{(14),(14)}T_{14}^*+(b_{14})^{(1)}s_{(13),(14)}T_{14}^*\Big)$$

$$+\left((\lambda)^{(1)}+(a_{14}')^{(1)}+(p_{14})^{(1)}\right)(q_{13})^{(1)}G_{13}^*+(a_{13})^{(1)}(q_{14})^{(1)}G_{14}^*\Big)$$

$$\left((\lambda)^{(1)}+(b_{13}')^{(1)}-(r_{13})^{(1)}\right)s_{(14),(13)}T_{14}^*+(b_{14})^{(1)}s_{(13),(13)}T_{13}^*\Big)$$

$$\left((\lambda)^{(1)}\right)^2+\left((a_{13}')^{(1)}+(a_{14}')^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$\left((\lambda)^{(1)}\right)^2+\left((b_{13}')^{(1)}+(b_{14}')^{(1)}-(r_{13})^{(1)}+(r_{14})^{(1)}\right)(\lambda)^{(1)}\Big)$$

$$+\left((\lambda)^{(1)}\right)^2+\left((a_{13}')^{(1)}+(a_{14}')^{(1)}+(p_{13})^{(1)}+(p_{14})^{(1)}\right)(\lambda)^{(1)}\Big)(q_{15})^{(1)}G_{15}$$

$$+\left((\lambda)^{(1)}+(a_{13}')^{(1)}+(p_{13})^{(1)}\right)\left((a_{15})^{(1)}(q_{14})^{(1)}G_{14}^*+(a_{14})^{(1)}(a_{15})^{(1)}(q_{13})^{(1)}G_{13}^*\right)$$

$$\left((\lambda)^{(1)}+(b_{13}')^{(1)}-(r_{13})^{(1)}\right)s_{(14),(15)}T_{14}^*+(b_{14})^{(1)}s_{(13),(15)}T_{13}^*\Big)\}=0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

More often than not, models begin with the assumption of 'steady state' and then proceed to trace out the path, which will be followed when the steady state is subjected to some kind of exogenous disturbance. Breathing pattern of terrestrial organisms is another parametric

representation to be taken into consideration. It cannot be taken for granted that the sequence generated in this manner will tend to equilibrium i.e. a traverse from one steady state to another.

In our model, we have, used the tools and techniques by Haimovici, Levin, Volttera, Lotka have brought out implications of steady state, stability, asymptotic stability, behavioral aspects of the solution without any such assumptions, such as those mentioned in the foregoing.

In the following, we give equations for the 'dead organic matter-decomposer organism-terrestrial organism-oxygen consumption' system. Solutions and sine-qua-non theoretical aspects are dealt in the next paper (part II).

GOVERNING EQUATIONS

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - (a'_{13})^{(1)}G_{13} \quad (1a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - (a'_{14})^{(1)}G_{14} \quad (2a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left(a'_{15}\right)^{(1)}G_{15} \quad (3a)$$

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - (b'_{13})^{(1)}T_{13} \quad (4a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - (b'_{14})^{(1)}T_{14} \quad (5a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left(b'_{15}\right)^{(1)}T_{15} \quad (6a)$$

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - (a'_{16})^{(2)}G_{16} \quad (7a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - (a'_{17})^{(2)}G_{17} \quad (8a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - (a'_{18})^{(2)}G_{18} \quad (9a)$$

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - (b'_{16})^{(2)}T_{16} \quad (10a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - (b'_{17})^{(2)}T_{17} \quad (11a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - (b'_{18})^{(2)}T_{18} \quad (12a)$$

GOVERNING EQUATIONS OF DUAL CONCATENATED SYSTEMS TERRESTRIAL ORGANISMS- OXYGEN CONSUMPTION SYSTEM

$\left(-b''_i\right)^{(1)}(G_{13}, G_{14}, G_{15}, t) = -\left(b''_i\right)^{(1)}(G, t)$, i=13,14,15

the contribution of the consumption of oxygen due to cellular respiration to the dissipation coefficient of the terrestrial organisms

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\right]G_{13} \quad (13a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\right]G_{14} \quad (14a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[\left(a'_{15}\right)^{(1)}\boxed{+\left(a''_{15}\right)^{(1)}(T_{14},t)}\right]G_{15} \quad (15a)$$

Where

$\boxed{+(a''_{13})^{(1)}(T_{14},t)}$, $\boxed{+(a''_{14})^{(1)}(T_{14},t)}$, $\boxed{+\left(a''_{15}\right)^{(1)}(T_{14},t)}$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\right]T_{13} \quad (16a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\right]T_{14} \quad (17a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[\left(b'_{15}\right)^{(1)}\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}\right]T_{15} \quad (18a)$$

Where

$\boxed{-(b''_{13})^{(1)}(G,t)}$, $\boxed{-(b''_{14})^{(1)}(G,t)}$, $\boxed{-\left(b''_{15}\right)^{(1)}(G,t)}$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption

DEAD ORGANIC MATTER-DECOMPOSER ORGANISM SYSTEM

$\left(-b''_i\right)^{(2)}(G_{16}, G_{17}, G_{18}, t) = -\left(b''_i\right)^{(2)}(G_{19}, t)$

i=16,17,18 the factor arising out of the decomposer organism disintegrating dead organic matter

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\right]G_{16} \quad (19a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\right]G_{17} \quad (20a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\right]G_{18} \quad (21a)$$

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\right]T_{16} \quad (22a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\right]T_{17} \quad (23a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\right]T_{18} \quad (24a)$$

Where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 of decomposer organism due to disintegration of dead organic matter by decomposer organism.

GOVERNING EQUATIONS OF CONCATENATED SYSTEM OF TWO CONCATENATED DUAL SYSTEMS

Terrestrial Organisms - Dead Organic Matter System Dead Organic Matter Dissipates Terrestrial Organism

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\boxed{-(a''_{13})^{(1,1)}(T_{14},t)}\right]G_{16} \quad (25a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\boxed{-(a''_{14})^{(1,1)}(T_{14},t)}\right]G_{17} \quad (26a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\boxed{-(a''_{15})^{(1,1)}(T_{14},t)}\right]G_{18} \quad (27a)$$

Where $+(a''_{16})^{(2)}(T_{17}, t)$, $+(a''_{17})^{(2)}(T_{17}, t)$, $+(a''_{18})^{(2)}(T_{17}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism $-(a''_{13})^{(1,1)}(T_{14}, t)$, $-(a''_{13})^{(1,1)}(T_{14}, t)$, $-(a''_{15})^{(1,1)}(T_{14}, t)$ are second detrition coefficients for category 1, 2 and 3 due to terrestrial organisms

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\boxed{+(b''_{16})^{(2,2)}(G_{19},t)}\right]T_{13} \quad (28a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\boxed{+(b''_{17})^{(2,2)}(G_{19},t)}\right]T_{14} \quad (29a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\boxed{+(b''_{18})^{(2,2)}(G_{19},t)}\right]T_{15} \quad (30a)$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption and $+(b''_{16})^{(2,2)}(G_{19}, t)$, $+(b''_{17})^{(2,2)}(G_{19}, t)$, $+(b''_{18})^{(2,2)}(G_{19}, t)$ are second augmentation coefficients for category 1, 2 and 3 due to dead organic matter.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\right]G_{13} \quad (31a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\right]G_{14} \quad (32a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\right]G_{15} \quad (33a)$$

Where $+(a''_{13})^{(1)}(T_{14}, t)$, $+(a''_{14})^{(1)}(T_{14}, t)$, $+(a''_{15})^{(1)}(T_{14}, t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\right]T_{16} \quad (34a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\right]T_{17} \quad (35a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\right]T_{18} \quad (36a)$$

Where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 of decomposer organism due to disintegration of dead organic matter by decomposer organism.

GOVERNING EQUATIONS OF THE OXYGEN CONSUMPTION-DECOMPOSER ORGANISM SYSTEM DECOMPOSER ORGANISM DISSIPATES OXYGEN CONSUMPTION-OXIDATION CASE

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - [(b'_{16})^{(2)} - (b''_{16})^{(2)}(G_{19},t) - (b''_{13})^{(1,1)}(G,t)]T_{16} \quad (37a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - [(b'_{17})^{(2)} - (b''_{17})^{(2)}(G_{19},t) - (b''_{14})^{(1,1)}(G,t)]T_{17} \quad (38a)$$

$$\frac{dT_{6}}{dt} = (b_{18})^{(2)}T_{17} - [(b'_{18})^{(2)} - (b''_{18})^{(2)}(G_{19},t) - (b''_{15})^{(1,1)}(G,t)]T_{18} \quad (39a)$$

Where $-(b''_{16})^{(2)}(G_{19}, t)$, $-(b''_{17})^{(2)}(G_{19}, t)$, $-(b''_{18})^{(2)}(G_{19}, t)$ are first detrition coefficients for category 1, 2 and 3 due to dead organic matter $-(b''_{13})^{(1,1)}(G, t)$, $-(b''_{14})^{(1,1)}(G, t)$, $-(b''_{15})^{(1,1)}(G, t)$ are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - [(a'_{13})^{(1)} + (a''_{13})^{(1)}(T_{14},t) + (a''_{16})^{(2,2)}(T_{17}, t)]G_{13} \quad (40a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - [(a'_{14})^{(1)} + (a''_{14})^{(1)}(T_{14},t) + (a''_{17})^{(2,2)}(T_{17},t)]G_{14} \quad (41a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - [(a'_{15})^{(1)} + (a''_{15})^{(1)}(T_{14},t) + (a''_{18})^{(2,2)}(T_{17},t)]G_{15} \quad (42a)$$

Where $(a''_{13})^{(1)}(T_{14},t)$, $(a''_{14})^{(1)}(T_{14},t)$, $(a''_{15})^{(1)}(T_{14},t)$, are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism $+(a''_{16})^{(2.2)}(T_{17},t)$, $+(a''_{17})^{(2.2)}(T_{17},t)$, $+(a''_{18})^{(2.2)}(T_{17},t)$ are second augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\right]G_{16} \quad (43a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\right]G_{17} \quad (44a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\right]G_{18} \quad (45a)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$, are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\right]T_{13} \quad (46a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\right]T_{14} \quad (47a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\right]T_{15} \quad (48a)$$

Where $-(b''_{13})^{(1)}(G, t)$, $-(b''_{14})^{(1)}(G, t)$, $-(b''_{15})^{(1)}(G, t)$ are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption governing equations of the system.

DECOMPOSER ORGANISM DISSIPATES OXYGEN CONSUMPTION—OXIDATION CASE TERRESTRIAL ORGANISMS DISSIPATES DEAD ORGANIC MATTER—DOG EATS DOG CASE

Dead Organic Matter (DOM)

$$\frac{dG_{16}}{dt} = (a_{16})^{(2)}G_{17} - \left[(a'_{16})^{(2)}\boxed{+(a''_{16})^{(2)}(T_{17},t)}\ \boxed{+(a''_{13})^{(1,1,1)}(T_{14},t)}\right]G_{16} \quad (49a)$$

$$\frac{dG_{17}}{dt} = (a_{17})^{(2)}G_{16} - \left[(a'_{17})^{(2)}\boxed{+(a''_{17})^{(2)}(T_{17},t)}\ \boxed{+(a''_{14})^{(1,1,1)}(T_{14},t)}\right]G_{17} \quad (50a)$$

$$\frac{dG_{18}}{dt} = (a_{18})^{(2)}G_{17} - \left[(a'_{18})^{(2)}\boxed{+(a''_{18})^{(2)}(T_{17},t)}\ \boxed{+(a''_{15})^{(1,1,1)}(T_{14},t)}\right]G_{18} \quad (51a)$$

Where $+(a''_{16})^{(2)}(T_{17},t)$, $+(a''_{17})^{(2)}(T_{17},t)$, $+(a''_{18})^{(2)}(T_{17},t)$ are first augmentation coefficients for category 1, 2 and 3 due to decomposer organism. And $+(a''_{13})^{(1,1,1)}(T_{14},t)$, $+(a''_{14})^{(1,1,1)}(T_{14},t)$, $+(a''_{15})^{(1,1,1)}(T_{14},t)$ are second augmentation coefficient for category 1, 2 and 3 due to terrestrial organisms.

Terrestrial Organisms (TO)

$$\frac{dT_{13}}{dt} = (b_{13})^{(1)}T_{14} - \left[(b'_{13})^{(1)}\boxed{-(b''_{13})^{(1)}(G,t)}\ \boxed{-(b''_{16})^{(2,2,2)}(G_{19},t)}\right]T_{13} \quad (52a)$$

$$\frac{dT_{14}}{dt} = (b_{14})^{(1)}T_{13} - \left[(b'_{14})^{(1)}\boxed{-(b''_{14})^{(1)}(G,t)}\ \boxed{-(b''_{17})^{(2,2,2)}(G_{19},t)}\right]T_{14} \quad (53a)$$

$$\frac{dT_{15}}{dt} = (b_{15})^{(1)}T_{14} - \left[(b'_{15})^{(1)}\boxed{-(b''_{15})^{(1)}(G,t)}\right.$$

$$\left.\boxed{-(b''_{18})^{(2,2,2)}(G_{19},t)}\right]T_{15} \quad (54a)$$

Where $-(b''_{13})^{(1)}(G,t)$, $-(b''_{14})^{(1)}(G,t)$, $-(b''_{15})^{(1)}(G,t)$, are first detrition coefficients for category 1, 2 and 3 due to oxygen consumption $-(b''_{16})^{(2,2,2)}(G_{19},t)$,$-(b''_{17})^{(2,2,2)}(G_{19},t)$, $-(b''_{18})^{(2,2,2)}(G_{19},t)$, are second detrition coefficient for category 1, 2 and 3 due to dead organic matter.

Oxygen Consumption (OC)

$$\frac{dG_{13}}{dt} = (a_{13})^{(1)}G_{14} - \left[(a'_{13})^{(1)}\boxed{+(a''_{13})^{(1)}(T_{14},t)}\right.$$

$$\left.\boxed{+(a''_{16})^{(2,2,2)}(T_{17},t)}\right]G_{13} \quad (55a)$$

$$\frac{dG_{14}}{dt} = (a_{14})^{(1)}G_{13} - \left[(a'_{14})^{(1)}\boxed{+(a''_{14})^{(1)}(T_{14},t)}\right.$$

$$\left.\boxed{+(a''_{17})^{(2,2,2)}(T_{17},t)}\right]G_{14} \quad (56a)$$

$$\frac{dG_{15}}{dt} = (a_{15})^{(1)}G_{14} - \left[(a'_{15})^{(1)}\boxed{+(a''_{15})^{(1)}(T_{14},t)}\right.$$

$$\left.\boxed{+(a''_{18})^{(2,2,2)}(T_{17},t)}\right]G_{15} \quad (57a)$$

Where $+(a''_{13})^{(1)}(T_{14},t)$, $+(a''_{14})^{(1)}(T_{14},t)$,$+(a''_{15})^{(1)}(T_{14},t)$ are first augmentation coefficients for category 1, 2 and 3 due to terrestrial organism $+(a''_{16})^{(2,2,2)}(T_{17},t)$, $+(a''_{17})^{(2,2,2)}(T_{17},t)$, $+(a''_{18})^{(2,2,2)}(T_{17},t)$, are second augmentation coefficient for category 1, 2 and 3 due to decomposer organism.

Decomposer Organism (DO)

$$\frac{dT_{16}}{dt} = (b_{16})^{(2)}T_{17} - \left[(b'_{16})^{(2)}\boxed{-(b''_{16})^{(2)}(G_{19},t)}\right.$$

$$\left.\boxed{-(b''_{13})^{(1,1,1)}(G,t)}\right]T_{16} \quad (58a)$$

$$\frac{dT_{17}}{dt} = (b_{17})^{(2)}T_{16} - \left[(b'_{17})^{(2)}\boxed{-(b''_{17})^{(2)}(G_{19},t)}\right.$$

$$\left.\boxed{-(b''_{14})^{(1,1,1)}(G,t)}\right]T_{17} \quad (59a)$$

$$\frac{dT_{18}}{dt} = (b_{18})^{(2)}T_{17} - \left[(b'_{18})^{(2)}\boxed{-(b''_{18})^{(2)}(G_{19},t)}\right.$$

$$\left.\boxed{-(b''_{15})^{(1,1,1)}(G,t)}\right]T_{18} \quad (60a)$$

Where $-(b''_{16})^{(2)}(G_{19},t)$, $-(b''_{17})^{(2)}(G_{19},t)$, $-(b''_{18})^{(2)}(G_{19},t)$, are first detrition coefficients for category 1, 2 and 3 due to dead organic matter. $-(b''_{13})^{(1,1,1)}(G,t)$, $-(b''_{14})^{(1,1,1)}(G,t)$, $-(b''_{15})^{(1,1,1)}(G,t)$, are second detrition coefficients for category 1, 2 and 3 due to oxygen consumption.

ACKNOWLEDGMENTS

The introduction is a collection of information from, articles, abstracts of the articles, paper reports, home pages of the authors, textbooks, research papers, and various other sources including the internet including Wikipedia. We acknowledge all authors who have contributed to the same. Should there be any act of omission or commission on the part of the authors in not referring to the author, it is authors' sincere entreat, earnest beseech, and fervent appeal to pardon such lapses as has been done or purported to have been done in the foregoing. With great deal of compunction and contrition, the authors beg the pardon of the respective sources. References list is only illustrative and not exhaustive. We have put all concerted efforts and sustained endeavors to incorporate the names of all the sources from which information has been extracted. It is because of such eminent, erudite, and esteemed people allowing us to piggy ride on their backs, we have attempted to see little forward, or so we think.

REFERENCES

[1] Haimovici, A. (1982). On the Growth of a Two Species Ecological System Divided on Age Groups. *Tensor, 37*.

[2] Capra, Frtjof (--). The Web of Life. In Flamingo, Harper Collins (Ed.), *Dissipative Structures* (pp.172-188).

[3] Heylighen, F. (2001). The Science of Self-Organization and Adaptivity. In L. D. Kiel (Ed.), *Knowledge Management, Organizational Intelligence and Learning, and Complexity.* The Encyclopedia of Life Support Systems ((Eolss).Oxford: Eolss Publishers. http://www.eolss.net

[4] Matsui, T., Masunaga, H., Kreidenweis, S. M., Pielke Sr., R. A., Tao, W. K., Chin, M., & Kaufman, Y. J. (2006). Satellite-Based Assessment of Marine Low Cloud Variability Associated With Aerosol, Atmospheric Stability, and the Diurnal Cycle. *J. Geophys. Res., 111*, D17204,

[5] Stevens, B., Feingold, G., Cotton, W. R., & Walko, R. L. (--). Elements of the Microphysical Structure of Numerically Simulated Nonprecipitating Stratocumulus. *J. Atmos. Sci., 53*, 980-1006.

[6] Feingold, G., Koren, I.,Wang, H. L., Xue, H.W., & Brewer, W. A. (2010). Precipitation-Generated Oscillations in Open Cellular Cloud Fields. *Nature*, *466*(7308), 849-852.

[7] Wood, R. (2006). The Rate of Loss of Cloud Droplets By Coalescence in Warm Clouds. *J. Geophys. Res., 111*, (D21205), 6.

[8] Rund, H. (1959). The Differential Geometry of Finsler Spaces. *Grund. Math. Wiss.*. Berlin: Springer-Verlag.

[9] Dold, A. (1995). Lectures on Algebraic Topology. *Classics In Mathematics* (1972 Ed.). Berlin: Springer-Verlag.

[10] Levin, S. (1976). Some Mathematical Questions In Biology. *Lectures on MatheMatics in Life Sciences, 8,*1-186.The American Mathematical Society, Providence, Rhode Island.

