

Stabilization of a Kind of Nonlinear Discrete Singular Large-Scale Control Systems

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Abstract

This paper studies the state feedback stabilization of a kind of nonlinear discrete singular large-scale control systems by using Lyapunov matrix equation, generalized Lyapunov function method and matrix theory. There gives some sufficient conditions for determining the asymptotical stability and instability of the corresponding singular closed-loop large-scale systems while the subsystems are regular, causal and R-controllable. At last, an example is given to show the application of main result.

Key words: Nonlinear discrete singular large-scale system; Control system; Asymptotical stability; Stabilization; State feedback

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INTRODUCTION

With the development of modern control theory and the permeation into other application area, one kind of systems with extensive form has appeared which form follows as:

$$EX(t) = f(X(t), t, u(t))$$

Where $X(t) \in R^n$ is a n - state vector, $u(t) \in R^m$ is a m - control input vector, E is a $n \times n$ matrix, it is usually singular. This kind of systems generally is called as the singular control systems. It appeared large in many areas such as the economy management, the electronic network, robot, bioengineering, aerospace industry and navigation

and so forth. Singular large-scale control systems have a more practical background. The actual production process can be described preferably by singular large-scale control systems, particularly by discrete singular large-scale control systems. The causality of discrete singular systems makes related results complicated and challenging for us. At present, the research results of the problem above are seldom. The asymptotical stability (Sun & Peng, 2009; Sun & Chen, 2004) and stabilization (Yang & Zhang, 2004; Sun & Chen, 2011) of discrete linear singular large-scale systems has been considered by Lyapunov function method. This paper consider the state feedback stabilization of a kind of nonlinear discrete singular large-scale control systems by introduce weighted sum Lyapunov function method, and give its interconnecting parameters regions of stability.

DEFINITIONS AND PROBLEM FORMULATION

Consider the nonlinear discrete singular large-scale control systems with m subsystems:

$$E_i x_i(k+1) = A_{ii} x_i(k) + \sum_{j=1, j \neq k}^m A_{ij} x_j(k) + f_i(x(k), k) + B_i U_i(k) \quad (i=1, \dots, m)$$

(1)

where $x_i(k) \in R^{n_i}$ and $f_i(x(k), k) \in R^{n_i}$ are semi-state vector and vector function, respectively. $U_i \in R^m$ is a control input vector, $A_{ii}, E_i \in R^{n_i \times n_i}$, $B_i \in R^{n_i \times m_i}$, they are constant matrices;

$$\sum_{i=1}^m n_i = n, \quad \sum_{i=1}^m r_i = r, \quad \sum_{i=1}^m m_i = l,$$

denote: $x(k) = [x_1^T(k), x_2^T(k), \dots, x_m^T(k)] \in R^n$

$rank(E_i) = r_i \leq n_i$, $E = Block - diag(E_1, E_2, \dots, E_m)$,
 $rank(E) = r < n$, $B = Block - diag(B_1, B_2, \dots, B_m)$

Now we give some concepts about discrete singular system:

$$Ex(k+1) = Ax(k) \quad (2)$$

and discrete singular control system:

$$Ex(k+1) = Ax(k) + Bu(k) \quad (k = 1, \dots, N) \quad (3)$$

where E and A are $n \times n$ constant matrices, B is a

$n \times m$ constant matrix, $rank(E) = r < n$, $x(k) \in R^n$ is a semi-state vector, $u(k) \in R^m$ is a control input vector.

Definition 1 (Yang & Zhang, 2004): Discrete singular system (2) is said to be regular if

$$\det(zE - A) \neq 0, \text{ for some } z \in C.$$

Definition 2 (Yang & Zhang, 2004): The zero solution of discrete singular system (2) is said to be stable if for every $\varepsilon > 0$, there exists a $\delta > 0$, such that $\|x(k; k_0, x_0)\| < \varepsilon$, for all $k \geq k_0$, whenever the arbitrary initial consistency value $x(k_0) = x_0$ which satisfies $\|x_0\| < \delta$.

Definition 3 (Yang & Zhang, 2004): Discrete singular control system (3) is said to be causal if $x(k)$ can be uniquely determined by $x(0)$ and control input vectors $u(0), u(1), \dots, u(k)$ for any k ($0 \leq k \leq N$). Otherwise, it is said to be non-causal.

Now consider the isolated subsystems of systems:

$$E_i x_i(k+1) = A_i x_i(k) + B_i U_i(k) \quad (i=1,2,\dots,m) \quad (4)$$

Assume that all systems of systems (4) are R-controllable, we choose the linear control law

$$U_i(k) = -K_i x_i(k) \quad (i=1, \dots, m) \quad (5)$$

Then singular closed-loop large-scale systems of systems (1) are given by

$$E_i x_i(k+1) = (A_i - B_i K_i) x_i(k) + \sum_{j=1, j \neq i}^m A_j x_j(k) + f_i(x(k), k) \quad (i=1, \dots, m) \quad (6)$$

The corresponding closed-loop isolated subsystems are

$$E_i x_i(k+1) = (A_i - B_i K_i) x_i(k) \quad (i=1, \dots, m) \quad (7)$$

In order to investigate the stabilization of discrete singular large-scale control systems (1), we give the following lemmas:

Lemma 1 (Yang & Zhang, 2004): The system (3) is said to be R-controllable if

$$rank[zE - A \ B] = n$$

for some $z \in C$.

Lemma 2 (Yang & Zhang, 2004): Discrete singular control system (3) is said to be causal if and only if

$$\deg\{\det(zE - A)\} = rank(E)$$

Lemma 3 (Yang & Zhang, 2004): Assume that $u, v \in R^n$, $V \in R^{n \times n}$ is a positive semi-definite matrix, then $2u^T V v \leq e u^T V u + e^{-1} v^T V v$ holds for all $e > 0$.

Lemma 4 (Wo, 2004): Assume that the system (2) is regular and causal, then it is asymptotically stable if and only if given positive definite matrix W , there exists a positive semi-definite matrix V which satisfies

$$A^T V A - E^T V E = -E^T W E$$

Lemma 5 (Wo, 2004): Assume that the system (2) is regular, causal, and there exists a function $v(Ex)$ which satisfies the following conditions, then the sub-equilibrium state of systems (2) $Ex = 0$ is asymptotically stable.

(a) $v(Ex) = (Ex(k))^T V(Ex(k))$, where V is a positive semi-definite matrix, and $rank(E^T V E) = rank E = r$;

(b) $\Delta v(Ex(k)) \leq -(Ex(k))^T W(Ex(k))$, here W is a positive definite matrix.

MAIN RESULTS

Theorem 1: Assume that all isolated subsystems (4) of systems (1) are R-controllable, all closed-loop isolated subsystems (7) are regular, causal and asymptotically stable, and there exist real numbers $\mu > 0, \delta > 0$ which satisfies that

$$\begin{aligned} [A_j x_j(k)]^T [A_j x_j(k)] &\leq \mu [E_j x_j(k)]^T [E_j x_j(k)] \\ (i, j = 1, \dots, m, j \neq i) \end{aligned} \quad (8)$$

$$[f_i(x(k), k)]^T [f_i(x(k), k)] \leq \delta \sum_{i=1}^m [E_i x_i(k)]^T [E_i x_i(k)] \quad (9)$$

then when

$$3W_i - 2V_i - 3[\delta \lambda_M(V_i) + (m-1)\delta \lambda_M + (m-1)^2 \delta \lambda_M] I_i > 0 \quad (i=1, 2, \dots, m) \quad (10)$$

the zero solution of the singular closed-loop large-scale systems (6) are asymptotically stable, the discrete singular large-scale control systems (1) are stabilizable. The interconnecting parameter region of stability is given by (10). Here W_i is a positive definite and V_i is a positive semi-definite matrix from Lemma 4, and $\lambda_M(V_i)$ denotes the maximum eigenvalue of matrix V_i , $\lambda_M = \max_{1 \leq i \leq m} \{\lambda_M(V_i)\}$, and I_i is a $n_i \times n_i$ identity matrix.

Proof: systems (7) are regular and causal, as they are asymptotically stable, then given positive definite matrix W_i , Lyapunov matrix equation

$$(A_i - B_i K_i)^T V_i (A_i - B_i K_i) - E_i^T V_i E_i = -E_i^T W_i E_i$$

have positive semi-definite solution V_i .

Construct quadratic form

$$v_i [E_i x_i(k)] = [E_i x_i(k)]^T V_i [E_i x_i(k)]$$

as the scalar Lyapunov function of systems (7).

$$\text{Let } v[Ex(k)] = \sum_{i=1}^m v_i [E_i x_i(k)]$$

as the Lyapunov function of systems (1). We have

$$\begin{aligned} &\Delta v_i [E_i x_i(k)] \Big|_{(6)} \\ &= \left\{ [E_i x_i(k+1)]^T V_i [E_i x_i(k+1)] - [E_i x_i(k)]^T V_i [E_i x_i(k)] \right\} \Big|_{(6)} \\ &= [(A_i - B_i K_i) x_i(k) + \sum_{j=1, j \neq i}^m A_j x_j(k) + f_i(x(k), k)]^T \\ &\cdot V_i [(A_i - B_i K_i) x_i(k) + \sum_{j=1, j \neq i}^m A_j x_j(k) + f_i(x(k), k)] \\ &- [E_i x_i(k)]^T V_i [E_i x_i(k)] \\ &= x_i^T(k) \left[(A_i - B_i K_i)^T V_i (A_i - B_i K_i) - E_i^T V_i E_i \right] x_i(k) \\ &\quad + 2 \left[\sum_{j=1, j \neq i}^m A_j x_j(k) \right]^T V_i [(A_i - B_i K_i) x_i(k)] \\ &\quad + \left[\sum_{j=1, j \neq i}^m A_j x_j(k) \right]^T V_i \left[\sum_{j=1, j \neq i}^m A_j x_j(k) \right] \\ &\quad + 2[(A_i - B_i K_i) x_i(k)]^T V_i [f_i(x(k), k)] \\ &\quad + 2 \left[\sum_{j=1, j \neq i}^m A_j x_j(k) \right]^T V_i [f_i(x(k), k)] \\ &\quad + [f_i(x(k), k)]^T V_i [f_i(x(k), k)] \end{aligned}$$

By using Lemma 3, choose $e = 1$, we have

$$\begin{aligned} & \Delta v_i [E_i x_i(k)] \Big|_{(6)} \\ & \leq x_i^T(k) (-E_i^T W_i E_i) x_i(k) \\ & + 2[(A_{ii} - B_i K_i) x_i(k)]^T V_i [(A_{ii} - B_i K_i) x_i(k)] \\ & + 3 \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right]^T V_i \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right] \\ & + 3[f_i(x(k), k)]^T V_i [f_i(x(k), k)] \\ & \leq x_i^T(k) (-3E_i^T W_i E_i + 2E_i^T V_i E_i) x_i(k) \\ & + 3 \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right]^T V_i \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right] \\ & + 3\lambda_M(V_i) [f_i(x(k), k)]^T [f_i(x(k), k)] \end{aligned}$$

Noticing that

$$\begin{aligned} & \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right]^T V_i \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right] \\ & = \sum_{j=1, j \neq i}^m [A_{ij} x_j(k)]^T V_i \left[\sum_{j=1, j \neq i}^m A_{ij} x_j(k) \right] \\ & = \sum_{j=1, j \neq i}^m \sum_{s=1, s \neq i}^m (A_{ij} x_j(k))^T V_i A_{is} x_s(k) \\ & = \frac{1}{2} \sum_{j=1, j \neq i}^m \sum_{s=1, s \neq i}^m 2(A_{ij} x_j(k))^T V_i A_{is} x_s(k) \\ & \leq \frac{1}{2} \sum_{j=1, j \neq i}^m \sum_{s=1, s \neq i}^m [(A_{ij} x_j(k))^T V_i A_{ij} x_j(k) + (A_{is} x_s(k))^T V_i A_{is} x_s(k)] \\ & \leq \frac{1}{2} \lambda_M(V_i) \sum_{j=1, j \neq i}^m \sum_{s=1, s \neq i}^m [(A_{ij} x_j(k))^T A_{ij} x_j(k) \\ & + (A_{is} x_s(k))^T A_{is} x_s(k)] \\ & \leq \frac{\mu}{2} \lambda_M(V_i) \sum_{j=1, j \neq i}^m \sum_{s=1, s \neq i}^m [(E_j x_j(k))^T E_j x_j(k) \\ & + (E_s x_s(k))^T E_s x_s(k)] \\ & = \frac{\mu}{2} \lambda_M(V_i) \sum_{j=1, j \neq i}^m [(m-1)(E_j x_j(k))^T (E_j x_j(k)) \\ & + \sum_{s=1, s \neq i}^m (E_s x_s(k))^T E_s x_s(k)] \\ & = \frac{\mu}{2} \lambda_M(V_i) \left[(m-1) \sum_{j=1, j \neq i}^m (E_j x_j(k))^T (E_j x_j(k)) \right. \\ & \left. + (m-1) \sum_{s=1, s \neq i}^m (E_s x_s(k))^T E_s x_s(k) \right] \\ & = \frac{\mu}{2} \lambda_M(V_i) \left[2(m-1) \sum_{j=1, j \neq i}^m (E_j x_j(k))^T (E_j x_j(k)) \right] \\ & = (m-1) \mu \lambda_M(V_i) \sum_{j=1, j \neq i}^m (E_j x_j(k))^T (E_j x_j(k)) \end{aligned}$$

Thus

$$\begin{aligned} & \Delta v_i [E_i x_i(k)] \Big|_{(6)} \\ & \leq [E_i x_i(k)]^T (-3W_i + 2V_i) [E_i x_i(k)] \\ & + 3(m-1) \mu \lambda_M(V_i) \sum_{j=1, j \neq i}^m [E_j x_j(k)]^T [E_j x_j(k)] \\ & + 3\delta \lambda_M(V_i) \sum_{j=1}^m [E_j x_j(k)]^T [E_j x_j(k)] \\ & \Delta v[Ex(k)] \Big|_{(6)} = \sum_{i=1}^m \Delta v_i [E_i x_i(k)] \Big|_{(6)} \\ & \leq -\sum_{i=1}^m [E_i x_i(k)]^T \{3W_i - 2V_i - 3\delta \lambda_M(V_i) I_i \\ & - 3[(m-1)\delta \lambda_M + (m-1)^2 \mu \lambda_M] I_i\} [E_i x_i(k)] \end{aligned}$$

Noticing that

$$3W_i - 2V_i - 3[\delta \lambda_M(V_i) + (m-1)\delta \lambda_M + (m-1)^2 \mu \lambda_M] I_i > 0$$

by using Lemma 5, we know, $\lim_{k \rightarrow \infty} (Ex(k)) = 0$, therefore

$$\lim_{k \rightarrow \infty} E_i x_i(k) = 0 \quad (i = 1, \dots, m).$$

To prove

$$z_i(k) = Q_i^{-1} x_i(k) = \begin{bmatrix} z_i^{(1)}(k) \\ z_i^{(2)}(k) \end{bmatrix} \rightarrow 0 \quad (k \rightarrow \infty).$$

By noticing that systems (7) are regular and causal, there exists reversible, matrices P_i, Q_i ($i = 1, \dots, m$) which satisfy

$$P_i E_i Q_i = \begin{bmatrix} I_i^{(1)} & 0 \\ 0 & 0 \end{bmatrix}, \quad P_i (A_{ii} - B_i K_i) Q_i = \begin{bmatrix} M_i & 0 \\ 0 & I_i^{(2)} \end{bmatrix}$$

Let

$$P_i A_{ij} Q_j = \begin{bmatrix} A_{ij}^{(1)} & A_{ij}^{(12)} \\ A_{ij}^{(21)} & A_{ij}^{(2)} \end{bmatrix} \quad (i, j = 1, \dots, m, i \neq j)$$

$$z_i(k) = Q_i^{-1} x_i(k) = \begin{bmatrix} z_i^{(1)}(k) \\ z_i^{(2)}(k) \end{bmatrix},$$

Where $I_i^{(1)}, I_i^{(2)}$ is $r_i \times r_i$ and $(n_i - r_i) \times (n_i - r_i)$ identity matrix, respectively. $P_i, Q_i, M_i, A_{ij}^{(1)}, A_{ij}^{(12)}, A_{ij}^{(21)}, A_{ij}^{(2)}$ are corresponding dimension constant matrices. Thus the singular closed-loop large-scale systems (6) are equivalent to

$$\begin{cases} z_i^{(1)}(k+1) = M_i z_i^{(1)}(k) + \sum_{j=1, j \neq i}^m (A_{ij}^{(1)} z_j^{(1)}(k) + A_{ij}^{(12)} z_j^{(2)}(k)) \\ 0 = z_i^{(2)}(k) + \sum_{j=1, j \neq i}^m (A_{ij}^{(21)} z_j^{(1)}(k) + A_{ij}^{(2)} z_j^{(2)}(k)) \end{cases}$$

Noticing that

$$P_i E_i x_i = P_i E_i Q_i Q_i^{-1} x_i = P_i E_i Q_i z_i(k) = \begin{bmatrix} z_i^{(1)}(k) \\ 0 \end{bmatrix}$$

we have $z_i^{(1)}(k) = (I_i^{(1)} \ 0) P_i E_i x_i$. $\lim_{k \rightarrow \infty} z_i^{(1)}(k) = 0$ holds

from $\lim_{k \rightarrow \infty} E_i x_i(k) = 0$.

Noticing that

$$P_i A_{ij} x_j(k) = \begin{pmatrix} A_{ij}^{(0)} & A_{ij}^{(12)} \\ A_{ij}^{(21)} & A_{ij}^{(2)} \end{pmatrix} \begin{pmatrix} z_j^{(0)}(k) \\ z_j^{(2)}(k) \end{pmatrix}$$

we have

$$(A_{ij}^{(21)} z_j^{(0)}(k) + A_{ij}^{(2)} z_j^{(2)}(k)) = (0 \quad I_i^{(2)}) P_i A_{ij} x_j(k).$$

Noticing that $\lim_{k \rightarrow \infty} A_{ij} x_j(k) = 0$ holds from (8), that is

$$\lim_{k \rightarrow \infty} (A_{ij}^{(21)} z_j^{(0)}(k) + A_{ij}^{(2)} z_j^{(2)}(k)) = 0, \text{ so } \lim_{k \rightarrow \infty} z_i^{(2)}(k) = 0.$$

Hence, $\lim_{k \rightarrow \infty} z_i(k) = 0$, that is $\lim_{k \rightarrow \infty} x(k) = 0$. The

Theorem 1 is proved.

Theorem 2: Assume that all subsystems (4) of system (1) are R-controllable, all closed-loop isolated systems (7) are regular, causal, and given positive definite matrix W_i , there exists a positive semi-definite matrix V_i which satisfies

$$(A_{ii} - B_i K_i)^T V (A_{ii} - B_i K_i) - E_i^T V_i E_i = E_i^T W_i E_i \quad (i=1, \dots, m), \quad (11)$$

if there exists a real number $m > 0$ which satisfies

$$[A_{ij} x_j(k)]^T [A_{ij} x_j(k)] \leq \mu [E_j x_j(k)]^T [E_j x_j(k)] \quad (i, j = 1, \dots, m, i \neq j), \quad (12)$$

when

$$W_i - 2V_i - [7\delta_M(V_i) + 7(m-1)\delta_M + 15(m-1)^2 \mu_M] I_i > 0 \quad (i=1, \dots, m), \quad (13)$$

the zero solution of the discrete singular closed-loop large-scale systems (6) are unstable, the discrete singular large-scale control systems(1) are not stabilizable.

Proving is similar with Theorem 1, here it can be omitted.

EXAMPLE

Consider the following 5-order discrete singular large-scale control system which consists of two sub-systems

$$E_i x_i(k+1) = A_{ii} x_i(k) + \sum_{j=1, j \neq i}^m A_{ij} x_j(k) + f_i(x(k), k) + B_i U_i(k) \quad (i=1,2) \quad (14)$$

where,

$$E_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix},$$

$$A_{11} = \begin{pmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix}, A_{12} = \begin{pmatrix} \frac{1}{9} & 0 \\ \frac{1}{9} & 0 \\ 0 & 0 \end{pmatrix}, A_{21} = \begin{pmatrix} \frac{1}{9} & 0 & \frac{1}{9} \\ \frac{1}{9} & 0 & -\frac{1}{9} \end{pmatrix},$$

$$A_{22} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix},$$

$$f_1(x(k), k) = (f_{11}, f_{12}, f_{13})^T, f_2(x(k), k) = (f_{21}, f_{22})^T,$$

$$f_{11} = \frac{\sin \sqrt{x_{11}^2(k)} \cdot \cos(kx_{12}(k))}{7 + \cos^2(kx_{22}(k))},$$

$$f_{12} = \frac{\sin(x_{13}(k)) \cdot \sin(kx_{12}(k))}{7 + e^{x_{11}(k)x_{12}(k)}},$$

$$f_{13} = \frac{\sin x_{21}(k) \cdot \cos(kx_{12}(k))}{7 + k \sin^2(kx_{22}(k))},$$

$$f_{21} = \frac{\sin \sqrt{x_{11}^2(k) + x_{13}^2(k)} \cdot \cos(kx_{12}(k))}{7 + \cos^2(kx_{22}(k))},$$

$$f_{22} = \frac{\sin x_{21}(k) \cdot \cos(kx_{11}(k))}{7 + k \sin^2(kx_{12}(k))}$$

We choose the control law $U_i(k) = -K_i x_i(k)$ ($i = 1, 2$),

$$K_1 = \begin{pmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \end{pmatrix}, K_2 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix}, \text{ and } W_1 = I_3, W_2 = I_2, \mu = \delta = \frac{1}{37}$$

$$\text{then } V_1 = \begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix}, V_2 = \begin{pmatrix} \frac{4}{3} & 0 \\ 0 & 0 \end{pmatrix}.$$

It is easy to test that (8) and (9) are holded, then we know this system (14) is satbilizable from Theorem 1.

CONCLUSION

In this paper, the state feedback stabilization of a kind of nonlinear discrete singular large-scale control systems is investigated by using generalized Lyapunov function method. According to the bound limit parameter of interconnecting terms, there gives some sufficient conditions for determining the asymptotical stability and unstability of the singular closed-loop large-scale system while the subsystems are regular, causal, and R-controllable.

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