Spurious Relationship of Long Memory Sequences in Presence of Trends Breaks

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Abstract
This article extends the theoretical analysis of spurious relationship and considers the situation where the deterministic components of the processes generating the individual series are long memory sequences with structural changes. Show it by using the ordinary least squares estimator, the t-statistics become divergent and pseudo correlation. However, two long memory time series having change points can produce spurious regression. In the presence of structural change points, confirm the rate of t-statistic tends to infinity increased with the increase in sample size. Numerical simulation results show that when structural changes are a feature of the data, the presence of spurious relationship is unambiguous. And the spurious regression not only depends on long memory indexes, but also for trend of model is also very sensitive.

Key words: Spurious relationship; Long memory sequences; Structural changes; t-statistics; Numerical simulation

INTRODUCTION
Many years ago, economists had been found that there exists spurious regression phenomenon in economic variables. However, in what circumstances will be spurious regression phenomenon, there was no unified recognition for a long time. As the early work of Monte Carlo study of Chow (1960) and Quant (1960), more and more effort has been taken to understand the nature of spurious regression and numerous studies have been undertaken an upsurge of interest in various models with an unknown change point. Issues about the distributional properties of the estimates in particular those of the break date, have been considered by Bai (1997). These tests and inference issue have also been addressed in the context of multiple change points in Bai and Perron (2003). Recent contributions include Gombay (2010), Jin et al. (2011), Beran and Shumeyko (2012) and Aue and Horváth (2013).

Moreover, the least squares estimates from the conventional spurious regression are inconsistent and have random limits. Numerical simulation has been used to study the spurious regression phenomena caused by the independent I (1) variables, such as Granger and Newbold (1974). Spurious relationships are shown to occur in random walks and linear trends by Durlauf and Phillips (1988). For the case of a random walk with drift, Entorf (1997) showed that t-statistic diverges to infinity as the sample size grows. Recently, Stewart (2006) argued that spurious relationship would generally occur in a regression of an I (1) dependent variable on an I (0) regressor, with or without another I (1) regressor. On the other hand, Noriega and Ventosa-Santaulària (2007) presented an analysis of the spurious phenomenon when there are a mix of deterministic and stochastic nonstationarity among the dependent and the explanatory variables in a linear regression model. More research in spurious relationship is appropriate include Kim and Lee (2011) and García-Belmonte.
However, there are more and more evidences showing that many economic and financial time series have the characteristics of long memory series. And some studies reveal that spurious regression do not only for independent random walks but also for long memory processes, such as Marmol (1998) and Tsay and Chung (2000). It demonstrated that spurious correlations are evident in regression involving combinations of long memory I(d) series, with d being a fractional number. However, it has been acknowledged that structural changes in the mean of time series can easily be confused with long memory dependence. Hence, the purpose of this paper is to investigate the possible existence of spurious relationship with a pair of stationary long memory processes with mis-specified change points, which could take place at different points in time.

1. THE MODELS AND ASSUMPTIONS

Similarly to these previous studies, we examine the regression, but assuming that the series are independent long memory processes with mis-specified change points. In order to investigate the spurious regression effects caused by structural changes, we specify the DGPs of three variables, and $y_{t}, r_{t} \in \mathbb{R}$ as

$$x_{t} = \mu_{t} + \delta_{t}(\sum_{j=1}^{\tau} \xi_{j}) + \epsilon_{t}, \tag{1}$$

$$y_{t} = \theta_{t} + y_{t-\tau_{t}} + \epsilon_{t}, \tag{2}$$

where $x_{t}$ contains structural breaks in means, and $y_{t}$ involves trends changes; $\mu_{t}$ and $\delta_{t}$ are, respectively, the permanent means and the transitory intercepts, resulting from a change, of the process $x_{t}; \theta_{t}$ and $\epsilon_{t}$ are, respectively, the permanent trends and the transitory trends, resulting from a change, of the process $x_{t}; \tau_{t}$ and $\epsilon_{t}$ are change points of $x_{t}$ and $y_{t}$; $\tau_{t}$ is the indicator function. These sequences $\epsilon_{t}$ and $\xi_{t}$ are the stochastic part of the processes with long memory.

Before introducing our models, we would briefly review some basic properties of the long memory processes. The series $\epsilon_{t}$ is a fractionally integrated process of order, denoted as $I(d)$ if $(1-L)^{d}\epsilon_{t}=\zeta$ is white noise with zero mean and finite variance, where $L$ is the back-shift operator. The fractional differencing operator $(1-L)^{d}$ is defined as follows:

$$(1-L)^{d} = \sum_{j=0}^{\infty} \binom{j}{k} L^{j-k} d^{k}.$$

The $I(d)$ processes are natural generalization of the $I(1)$ processes that exhibit a broader long memory characteristics. The main feature of $I(d)$ process is that its autocovariance function declines at a slower hyperbolic rate (instead of the geometric rate found in the conventional ARMA models)

$$\gamma(j)=O(j^{d-1}).$$

Where $\gamma(j)$ is the autocovariance function at lag $j$. In particularly when $0<d<0.5$, the stationary $I(d)$ process is said to have long memory since it exhibits long-range dependence in the sense that $\sum_{j=0}^{\infty} \gamma(j) = \infty$. For $-0.5<d<0$, the process has short memory. We have independence or standard short memory when $d=0$. If $0.5<d<1$, the process is nonstationary but still mean reverting. The relevant lines of research on long memory models may be found in Horváth and Kokoszka (2008) and McElroy and Politis (2011).

We focus on the case that $\epsilon_{t}$ is a stationary $I(d)$ process with $0<d<0.5$, and require it obeying a functional central limit theorem, as stated in the lemma below. Let “$\overset{\sim}{=}$” and “$\overset{\ast}{=}$” stand respectively for the weak convergence and convergence in probability. $B_{d}(\tau)$ denotes the fractional Brownian motion (of “type I” in the terminology of Marinucci and Robinson [1999]) with

$$B_{d}(\tau) \equiv \left\{ \begin{array}{ll} \int_{0}^{\tau} (\tau-s)^{d} W(s) + \int_{\tau}^{\infty} [(\tau-s)^{d} - (s-s)^{d}] dW(s) \end{array} \right\}.$$

Where $\Gamma(\cdot)$ is the Gamma function and $W(s)$ is a standard Brownian motion.

**Assumption 1.1** The strictly stationary symmetrical innovations $\epsilon_{t}$ and $\zeta_{t}$ are assumed to be mean zero with long memory indexes $d_{1}, d_{2} \in (0,0.5)$.

Our method relies on the results derived by Davidson and Jong (2000).

**Lemma 1.1** If Assumption 1.1 holds, then as $T \rightarrow \infty$,

$$(T^{-0.5+d}) \left[ \sum_{t=1}^{T} \epsilon_{t} \right]^2 = (\kappa_{d}B_{d}(\tau), \sigma_{d}^{2}) \overset{\ast}{\longrightarrow} (\kappa_{d}B_{d}(\tau), \sigma_{d}^{2}).$$

Where $\Gamma(\cdot)$ is the Gamma function and $W(s)$ is a standard Brownian motion.

**Lemma 1.2** Suppose $x_{t}$ and $y_{t}$ are respectively generated by (1) and (2) with break point $\tau_{t}, \tau_{t} \in (0,1)$.

The long memory series $\epsilon_{t}$ and $\zeta_{t}$ satisfy Assumption 1.1. Then as $T \rightarrow \infty$,

a) $T^{-1} \sum x_{t} = \mu_{x} + (1-\tau_{x}) \delta_{x} \overset{\ast}{\longrightarrow} \Gamma_{x}$

b) $T^{-1} \sum y_{t} \overset{1}{\rightarrow} \left[ \theta_{y} + (1-\tau_{y}) \gamma_{y} \right] \overset{\ast}{\longrightarrow} \Gamma_{y}$

c) $T^{-2} \sum x_{t} \overset{1}{\rightarrow} \left[ \mu_{x} + (1-\tau_{x}) \delta_{x} \right] \overset{\ast}{\longrightarrow} \Gamma_{xx}$

d) $T^{-2} \sum y_{t} \overset{1}{\rightarrow} \left[ \theta_{y} + (1-\tau_{y}) \gamma_{y} \right] \overset{\ast}{\longrightarrow} \Gamma_{yy}$

e) $T^{-1} \sum x_{t} \overset{1}{\rightarrow} \left[ \mu_{x} + (1-\tau_{x}) (2\mu_{x} \delta_{x} + \delta_{x}^{2} + \sigma_{x}^{2}) \right] \overset{\ast}{\longrightarrow} \Gamma_{xx}$

f) $T^{-1} \sum y_{t} \overset{1}{\rightarrow} \left[ \theta_{y} + (1-\tau_{y}) \gamma_{y} \right] \overset{\ast}{\longrightarrow} \Gamma_{yy}$

g) $T^{-1} \sum x_{t} \overset{1}{\rightarrow} \left[ \mu_{x} \theta_{x} + \delta_{x} \gamma_{x} (1-\tau_{x})^{2} + \mu_{x} \gamma_{x} (1-\tau_{x})^{2} + \delta_{x} \gamma_{x} (1-\tau_{x})^{2} + \mu_{x} \gamma_{x} (1-\tau_{x})^{2} \right] \overset{\ast}{\longrightarrow} \Gamma_{xy}$
\[ \sum T^{-1} x_t y_t = \frac{1}{2} \left[ \mu_2 \theta_y \delta_x (1 - \tau_x)^2 + \mu_2 \theta_y (1 - \tau_y)^2 + \delta_x \theta_y ((1 - \tau_x)^2 + \tau_D (1 - \tau_M)) \right] \equiv \Gamma_{xy}. \]

Where, \( \tau_M = \max(\tau_x, \tau_x) \), \( \tau_Y = \min(\tau_x, \tau_y) \), and \( \tau_D = \tau_M - \tau_y. \)

Hence, the proof of Lemma 2.2 is completed.

### 2. MAIN RESULTS

We consider the regression model given by

\[ y_t = \alpha + \phi t + \beta x_t + \eta_t, \quad (3) \]

where \( x_t \) and \( y_t \) are respectively the regressand and regressor, and \( \mu \) is the regression error. Let \( \hat{\alpha} \) and \( \hat{\beta} \) denote the ordinary least squares estimates from a regression of \( y_t \) on a constant, the trend \( t \), and \( x_t \) respectively. Their respective ‘variances’ are estimated by \( \hat{s}_{\alpha}^2, \hat{s}_{\beta}^2 \), and \( \hat{s}_{\eta}^2 \), from which we have the \( t \)-ratios \( \tau_{\hat{\alpha}} = \hat{\alpha}/\hat{s}_{\alpha} \), and \( \tau_{\hat{\beta}} = \hat{\beta}/\hat{s}_{\beta} \).

As stated the following theorems, we could denote \( \tau_{\alpha} = \max(\tau_x, \tau_y) \) and \( \tau_D = \tau_M - \tau_y. \) The proofs of these theorems are omitted since they are similar to the line of proofs in Tasy (2000). But full proof is available on request.

The next theorem interprets the asymptotic behavior of the estimated parameters, associated \( t \)-ratios and \( R^2 \) in model (3).

**Theorem 2.1** Suppose \( x_t \) and \( y_t \) are respectively generated by (1) and (2) with break fraction \( \tau_x, \tau_y \in (0,1) \). The long memory series \( \xi_t \) and \( \xi_x \) satisfy Assumption 2.1, then as \( T \to \infty \):

\begin{enumerate}
  \item \( T^{-1} \sum T^{-1} x_t y_t \equiv \alpha, \)
  \item \( (\frac{1}{2} \Gamma_{xx} - \frac{1}{2} \Gamma_{tx}) \Gamma_y + \left( \frac{1}{2} \Gamma_{tx} - 3 \Gamma_{tx} \right) \Gamma_{xy} \equiv \beta, \)
  \item \( (\frac{1}{2} \Gamma_{xx} - \frac{1}{2} \Gamma_{yx}) \Gamma_x + (\frac{1}{2} \Gamma_{tx} - \frac{1}{2} \Gamma_{tx}) \Gamma_{xy} \equiv \varphi, \)
  \item \( (\frac{1}{2} \Gamma_{xx} - \frac{1}{2} \Gamma_{yx}) \Gamma_x + (\frac{1}{2} \Gamma_{tx} - \frac{1}{2} \Gamma_{tx}) \Gamma_{xy} \equiv \beta, \)
  \item \( T^{-1/2} \tau_D = \alpha \left( \sigma^2 \cdot \frac{\Gamma_{xx} - \Gamma_{tx}}{\sigma^2} \right)^{-1/2}, \)
  \item \( T^{-1/2} \tau_D = \beta \left( \sigma^2 \cdot \frac{\Gamma_{xx} - \Gamma_{tx}}{\sigma^2} \right)^{-1/2}, \)
  \item \( R^2 \Rightarrow 2 \cdot \frac{\Gamma_{xx} \Gamma_{xy} - \Gamma_{tx}^2}{\Gamma_{xy}^2 - \Gamma_{xx} \Gamma_{yy}}. \)
\end{enumerate}
where

\[ D_X = \Gamma_x \Gamma_{tx} - \frac{1}{3} \Gamma_x^2 - \Gamma_{tx} + \frac{1}{12} \Gamma_{xx} , \]

\[ \sigma^2 = \Gamma_{yy} + \alpha^2 + \frac{1}{3} \varphi^2 + \beta \Gamma_{xx} - 2(\alpha \Gamma_y + \varphi \Gamma_{ty} + \beta \Gamma_{xy}) , \]

\[ \varphi \Gamma_{ty} + \beta \Gamma_{xy} + \alpha \varphi + 2 \alpha \beta \Gamma_{x} + 2 \varphi \beta \Gamma_{tx} . \]

**Proof** Write the regression model \( y_t = \alpha + \varphi t + \beta x_t + \eta_t \), in matrix form from:

\[ Y = \Theta \phi + U. \]

The vector of OLS estimators is \( \hat{\theta} = (X'X)^{-1}X'Y \), and we define

\[
X'X = \left( \sum x_t \right) \left( \sum x_t \right) = \left( \begin{array}{ccc} a & b & c \\ b & d & e \\ c & e & f \end{array} \right) .
\]

Hence, one could rewrite \( \hat{\theta} = [\text{det}(X'X)]^{-1} \left( \begin{array}{ccc} d - e^2 & ce - bf & be - cd \\ ce - bf & a - c^2 & bc - ae \\ be - cd & bc - ae & ad - b^2 \end{array} \right) \left( \begin{array}{c} \sum y_t \\ \sum t y_t \\ \sum x_t y_t \end{array} \right) \). \]

Where

\[ \text{det}(X'X) = 2bc \alpha + d \alpha^2 - c^2 \beta - \beta^2 b \]

\[ = 2 \sum t \sum x_t + T \sum t^2 \sum x_t^2 - \sum t^3 \left( \sum x_t \right)^2 . \]

Using the results from Lemma 2.2, one could get

\[ T^{-5} \cdot \text{det}(X'X) \Rightarrow \Gamma_x \Gamma_{tx} - \frac{1}{3} \Gamma_x^2 - \Gamma_{tx} + \frac{1}{12} \Gamma_{xx} \equiv D_X . \]

To prove a) - c), one can derive expression for \( \hat{\alpha} \), \( \hat{\varphi} \), and \( \hat{\beta} \) as follows:

\[ \hat{\alpha} = [\text{det}(X'X)]^{-1} \cdot \left( \begin{array}{c} d - e^2 \\ ce - bf \\ be - cd \end{array} \right) \left( \begin{array}{c} \sum y_t \\ \sum t y_t \\ \sum x_t y_t \end{array} \right) \]

\[ \Rightarrow D_X^{-1} \cdot \left[ \begin{array}{c} \frac{1}{2} \Gamma_{xx} - \frac{1}{2} \Gamma_{tx} \\ \Gamma_x \Gamma_{tx} - \frac{1}{2} \Gamma_{xx} \end{array} \right] \Gamma_{ty} + \left( \frac{1}{2} \Gamma_x - \Gamma_{tx} \right) \Gamma_{xy} \equiv \alpha , \]

\[ T \hat{\varphi} = [\text{det}(X'X)]^{-1} \cdot T \left( \begin{array}{c} ce - bf \\ (af - c) \end{array} \right) \left( \begin{array}{c} \sum y_t \\ \sum t y_t \\ \sum x_t y_t \end{array} \right) \]

\[ \Rightarrow D_X^{-1} \cdot \left( \Gamma_x \Gamma_{tx} - \frac{1}{2} \Gamma_x \right) \Gamma_{ty} + \left( \Gamma_x - \Gamma_{tx} \right) \Gamma_{xy} \equiv \varphi , \]

\[ \hat{\beta} = [\text{det}(X'X)]^{-1} \cdot \left( \begin{array}{c} be - cd \\ (bc - ae) \end{array} \right) \left( \begin{array}{c} \sum y_t \\ \sum t y_t \\ \sum x_t y_t \end{array} \right) \]

\[ \Rightarrow D_X^{-1} \cdot \left( \frac{1}{2} \Gamma_x - \Gamma_{tx} \right) \Gamma_{ty} + \left( \frac{1}{2} \Gamma_x - \Gamma_{tx} \right) \Gamma_{xy} \equiv \beta . \]

To prove d) - f), one could write the t-statistics as:

\[ t_{\alpha} = \hat{\alpha} \cdot \left[ s^2 \cdot (X'X)^{-1} \right]^{-1/2} , \]

\[ t_{\varphi} = \hat{\varphi} \cdot \left[ s^2 \cdot (X'X)^{-1} \right]^{-1/2} , \]

\[ t_{\beta} = \hat{\beta} \cdot \left[ s^2 \cdot (X'X)^{-1} \right]^{-1/2} . \]

Where

\[ s^2 = T^{-1} \sum (y_t - \hat{\alpha} - \hat{\varphi} t - \hat{\beta} x_t)^2 \]

\[ = T^{-1} \sum (y_t - \hat{\alpha} + \hat{\varphi} t + \hat{\beta} x_t)^2 - 2 \hat{\alpha} \hat{y}_t - 2 \hat{\varphi} \hat{t} y_t - 2 \hat{\beta} x_t y_t \]

\[ + T^{-1} \sum (2 \hat{\varphi} \hat{t} + 2 \hat{\beta} x_t + 2 \hat{\beta} \hat{x}_t) \]

\[ \Rightarrow T_{yy} + \alpha^2 + \frac{1}{3} \varphi^2 + \beta \Gamma_{xx} - 2(\alpha \Gamma_y + \varphi \Gamma_{ty} + \beta \Gamma_{xy}) + \alpha \varphi + 2 \alpha \beta \Gamma_{x} + 2 \varphi \beta \Gamma_{tx} \equiv \sigma_a^2 . \]

and \( (X'X)^{-1} \) is the \( i \)th diagonal element of \( (X'X)^{-1} \), as:

\[ T \cdot (X'X)^{-1} \]

\[ = T^{-1} \sum \left( \begin{array}{ccc} y_t^2 - \hat{\alpha}^2 - \hat{\varphi}^2 t^2 - \hat{\beta}^2 x_t^2 \\ -2 \hat{\alpha} \hat{y}_t - 2 \hat{\varphi} \hat{t} y_t - 2 \hat{\beta} x_t y_t \end{array} \right) \]

\[ + T^{-1} \sum (2 \hat{\varphi} \hat{t} + 2 \hat{\beta} x_t + 2 \hat{\beta} \hat{x}_t) \]

\[ \Rightarrow \Gamma_{yy} + \alpha^2 + \frac{1}{3} \varphi^2 + \beta \Gamma_{xx} - 2(\alpha \Gamma_y + \varphi \Gamma_{ty} + \beta \Gamma_{xy}) + \alpha \varphi + 2 \alpha \beta \Gamma_{x} + 2 \varphi \beta \Gamma_{tx} \equiv \sigma_a^2 . \]

Hence, the proof of Theorem 2.2 is completed.

### 3. SIMULATION

This section adopts the method of Monte Carlo simulation to verify the correctness of the theoretical derivation. In the simulated data, the sample size respectively by \( T=100, 500, 1,000, 10,000 \). For each simulated sample, the percentage of rejection are obtained by 1, 000 replications at 5% nominal level, i.e., the percentage of \( t \)-ratios such
that $|\tau|>1.96$.

We consider the process with mean change points (1) and with trend change points process (2),
\begin{align*}
x_t &= \mu_x + \theta_1 x_{t-1} + \varepsilon_t, \\
y_t &= \theta_y t + \gamma_y (t - [T \varepsilon_y]) 1_{[t>\varepsilon_y]} + \xi_t,
\end{align*}

where, $\mu_x = 0.8$, $\theta_y = 0.3$ we give $\delta_y = 0.6$, $\gamma_y = 0.2$, other cases have similar results. Information process of $\varepsilon_t$ and $\xi_t$ are independent of each other, the long memory index $d$ and $d_t$ adopted the value in $\{0.1, 0.2, 0.3, 0.4\}$.

### Table 1

**Regressing Between Two Long Memory Sequences With Structural Changes, $|\tau|>1.96$**

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<td>5.42</td>
<td>5.84</td>
</tr>
<tr>
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<td>2</td>
<td></td>
<td>5.26</td>
<td>4.88</td>
<td>5.24</td>
<td>4.84</td>
<td>5.34</td>
<td>4.80</td>
<td>5.66</td>
<td>4.98</td>
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<td></td>
<td>4.74</td>
<td>4.98</td>
<td>5.40</td>
<td>5.10</td>
<td>5.04</td>
<td>4.56</td>
<td>5.12</td>
<td>5.20</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td></td>
<td>5.32</td>
<td>5.46</td>
<td>5.50</td>
<td>4.98</td>
<td>5.32</td>
<td>5.24</td>
<td>5.32</td>
<td>5.00</td>
</tr>
</tbody>
</table>

In order to study the asymptotic properties of the $t$-statistics. We set the change point of $\tau_x, \tau_y \in \{0.01, 0.5, 0.9\}$. When $\tau_x=\tau_y=0$ and $\tau_x=\tau_y=1$, the regression rate is negligible for long memory index $d$. For example, when $d_1=d_2=0.2$ and $\tau_x=\tau_y=0$, the regression rate is $5.86\%$, $5.54\%$, $5.08\%$ and $4.76\%$ for $T=100$, 500, 1000, 5000. Particularly, if one of these two series does not exist change points, the rejection power is always closed to its nominal level. What is surprising is that, the spurious regression relationship very strong if the observed sequence is driven by a linear trend. And the rejection frequency significantly increases as the sample size growing. However, it is not true for $\tau_x=\tau_y=0.5$ case, its rejection frequency is closed to the nominal level. Finally, the Spurious regression relationship still is sensitive to the long memory index $d$, but its effects are negligible for long memory index $d$. For example, when $\tau_x=\tau_y=0.1$, $d_1=0.2$ and $T=1,000$, the rejection rate are $96.18\%$ and $87.16\%$ for $d_1=0.2, 0.4$. When if $\tau_x=\tau_y=0.1$, $d_1=0.4$ and $T=1,000$ the rejection frequency are $87.16\%$ and $86.8\%$ for $d_1=0.2, 0.4$. Therefore, the presence of spurious regression in Mean-Trend case is dominated by the long memory index $d$.
Table 2
Regressing Between Two Long Memory Sequences With Structural Changes, \(|\rho|>1.96\)

<table>
<thead>
<tr>
<th>(d_\xi=0.4)</th>
<th>(r_\varepsilon=0)</th>
<th>(r_\varepsilon=0.1)</th>
<th>(r_\varepsilon=0.5)</th>
<th>(r_\varepsilon=0.9)</th>
<th>(r_\varepsilon=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T/d_\varepsilon)</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
<td>0.4</td>
<td>0.2</td>
</tr>
</tbody>
</table>

0 | 100 | 4.98 | 9.22 | 4.44 | 8.56 | 5.00 | 8.24 | 5.20 | 8.56 | 4.62 | 8.58 |
| 500 | 4.74 | 5.74 | 4.86 | 6.04 | 5.38 | 5.46 | 5.06 | 5.82 | 5.36 | 5.10 |
| 1000 | 5.20 | 5.30 | 4.56 | 5.72 | 5.64 | 5.20 | 5.50 | 5.60 | 5.00 | 5.84 |
| 5000 | 5.16 | 5.64 | 4.88 | 5.50 | 5.06 | 4.50 | 5.04 | 5.34 | 4.78 | 5.26 |
| 100 | 6.66 | 16.54 | 7.86 | 19.64 | 5.22 | 12.12 | 5.56 | 13.98 | 6.40 | 14.82 |
| 500 | 9.68 | 12.68 | 60.14 | 57.98 | 4.46 | 4.58 | 4.92 | 7.04 | 5.82 | 9.22 |
| 1000 | 7.70 | 9.60 | 87.16 | 86.80 | 5.52 | 5.20 | 4.42 | 4.78 | 7.60 | 8.24 |
| 5000 | 5.66 | 5.64 | 100 | 100 | 37.02 | 36.98 | 22.16 | 21.32 | 5.84 | 5.64 |
| 100 | 6.42 | 14.44 | 12.04 | 23.36 | 6.54 | 12.34 | 4.58 | 5.84 | 6.00 | 12.96 |
| 500 | 4.86 | 6.40 | 32.84 | 33.90 | 5.42 | 6.02 | 16.84 | 16.00 | 5.52 | 6.24 |
| 1000 | 5.92 | 5.28 | 51.58 | 51.72 | 4.52 | 5.06 | 36.26 | 35.94 | 4.46 | 5.16 |
| 5000 | 4.48 | 6.02 | 98.68 | 98.68 | 4.76 | 5.14 | 97.84 | 97.68 | 5.00 | 4.72 |
| 100 | 4.30 | 11.18 | 4.62 | 10.86 | 4.48 | 10.52 | 5.72 | 6.44 | 4.52 | 8.62 |
| 500 | 4.48 | 6.02 | 7.08 | 8.80 | 8.22 | 10.26 | 35.44 | 20.44 | 4.90 | 5.88 |
| 1000 | 5.54 | 5.42 | 9.52 | 10.78 | 14.06 | 14.54 | 72.48 | 63.66 | 5.06 | 5.20 |
| 5000 | 4.38 | 5.22 | 31.32 | 31.24 | 48.30 | 47.84 | 100 | 100 | 4.60 | 4.30 |
| 100 | 4.32 | 9.48 | 5.02 | 9.48 | 5.06 | 8.30 | 4.90 | 8.78 | 4.56 | 9.60 |
| 500 | 5.50 | 6.38 | 4.94 | 5.90 | 5.06 | 5.44 | 5.14 | 5.74 | 5.10 | 5.50 |
| 1000 | 5.14 | 5.08 | 5.20 | 5.14 | 5.24 | 5.60 | 4.84 | 5.46 | 5.34 | 5.42 |
| 5000 | 4.80 | 5.08 | 5.46 | 5.46 | 4.94 | 4.52 | 4.96 | 4.64 | 4.88 | 5.14 |

To give intuitive idea for the influence of stable indexes and breaks fractions, we provide the rejection frequency with sample size \(T=1,000\) in Figures 1-2. As expected, Figures 1-2 clearly show that spurious relationship is present when the long memory sequences contains breaks. There are two peaks, and at the same time there are high rejection power. On the other hand, there are higher rejection frequency occurs near \(r_\varepsilon=r_\varepsilon=0.9\) than other cases; while the rejection rate is the lowest at the center and edges of the sample. We can get a conclusion that the smaller long memory index provide higher rejection rate.

Figure 1
The Rejection Frequency as \(T=1,000, d_\xi=0.2\) and \(d_\xi=0.1,0.2,0.3,0.4\), Respectively

Figure 2
The Rejection Frequency as \(T=1,000, d_\xi=0.4\) and \(d_\xi=0.1,0.2,0.3,0.4\), Respectively
In general, it is clear from these simulations that if the sequence undergoes the structural breaks, the spurious relationship is present.

**CONCLUSION**

In this paper, we use the least squares estimation to study the spurious relationship of long memory time series. Our model includes two different model, mean model and trend model. In the presence of structural change points, confirm the rate of t-statistic tends to infinity increased with the increase in sample size. Numerical simulation results show that when structural changes are a feature of the data, the presence of spurious relationship is unambiguous. The presence of spurious relationship will always be more severe if the sequences involve multiples breaks. And the spurious regression not only depends on long memory indexes, but also for trend of model and the sample size are also very sensitive.

**REFERENCES**


